About the Journal

Aim and Scope. The primary goal of the *Columbia Journal of Undergraduate Mathematics* is to provide undergraduate readers with high-quality, accessible articles on challenging topics, or novel approaches to teaching more familiar concepts. Articles published are purely expository; we do not accept research papers. Most are under 20 pages in length, with the primary exceptions being senior theses written by students at Columbia and other universities alike. The journal also accepts and publishes mathematical artwork with clear pedagogical value.

Contact. Email columbiajournalofundergradmath@gmail.com with any questions, or visit https://journals.library.columbia.edu/index.php/cjum for more information.

Letter from the Editors

Undergraduate mathematics students are frequently tasked with writing expository pieces accessible to their peers. By the time they graduate, most math majors will have been assigned a paper on a topic not explored by a class they took, assembled a summary of a research niche they explored for an REU, or simply decided to write about a result or topic they felt they had a fresh way of talking about. The *Columbia Journal of Undergraduate Mathematics* was created when we asked ourselves a simple question about these projects: where do they go? The answer at the time seemed to be 'nowhere': since undergraduate expository work does not present original results or research, this writing is not eligible for most mathematics publications. We decided to change that. Our belief was that, since papers written *by* undergraduates themselves are uniquely understandable *for* undergraduates, such papers represent an untapped intellectual resource. We were confident that if we created a journal to showcase this work, we would uncover a treasure trove of creative and insightful mathematical exposition just waiting to come to light.

To our delight, our call for submissions was answered by a wide variety of talented undergraduate expositors originating from schools across the country (and one or two from abroad). Throughout the editing process, we were inspired by how these submissions offered novel ways of explaining known mathematical concepts with target audiences ranging from first-semester undergraduates to the most advanced graduating seniors. This issue offers the best of the best of those submissions.

Our opening article, "A Topological Proof of the Riemann-Hurwitz Formula," approaches an algebro-geometric result through the lens of manifolds and algebraic topology, with exposition enhanced by diagrams drawn by the author herself. Without assuming much complex analysis, the paper introduces essential concepts in the study of Riemann surfaces and their branched covers, as well as the Euler characteristic, before proving the titular Riemann-Hurwitz formula. Our next piece, "Representations of Complex Tori and $\mathrm{GL}(2, \mathbb{C})$," explores the representation theory of algebraic groups, culminating in a classification of the representations for complex tori and $\mathrm{GL}(2, \mathbb{C})$. Along the way, the paper introduces representation-theoretic tools such as Hopf algebras, weight space decompositions, and the theorems of the highest weight, and should be accessible to those with a minimal background in algebraic varieties and Lie groups. Returning to geometry, "The Gauss-Bonnet Theorem" explains a classical topology result while only assuming the reader has a knowledge of linear algebra and multivariable calculus. We recommend this article to introductory readers.

For more advanced readers, "The Peter-Weyl Theorem & Harmonic Analysis on S^n" assumes only group and integration theory to introduce the representation theory of topological groups and its relation to functional and harmonic analysis. The article culminates in a proof of the Peter-Weyl theorem, a characterization of all the representations of a compact group in terms of the square-integrable functions on it with applications to Fourier-type decompositions on spheres. Finally, "Elliptic bootstrapping and the non-linear Cauchy-Riemann equations" introduces the essential technique of elliptic bootstrapping in geometric analysis. Assuming knowledge of manifolds, L^p spaces, and some familiarity with partial differential equations and complex analysis, the paper discusses almost-complex and symplectic manifolds and introduces Sobolev spaces to prove a regularity theorem for J-holomorphic curves, with an explanation of their importance to moduli spaces in symplectic geometry.

Now as the editors-in-chief of this inaugural issue, we want to highlight our wonderful team of content editors, copy editors, graduate student editors, and our faculty advisors. Creating, editing, and reviewing a brand-new journal is no easy task, and we want to thank each of you for all the time and effort you have put into making this first issue. We are particularly thankful for the contributions of our Head Copy Editor Jazmyn Wang and Chief Confidentiality Officer J Xiang, without whom this issue would not exist. We would also like to acknowledge the support from the Columbia Libraries and Department of Mathematics, who were the technical and financial backbone to make this project come to life.

Finally, on behalf of the editorial team, we would like to thank every undergraduate student who submitted to our journal. We know firsthand the hard work and dedication required to create a novel piece of exposition, and we truly appreciate every submission we received, even if they did not make it into the final issue. In particular, we want to provide a special thanks to our published authors, who submitted inspiring work and worked tirelessly with our editorial and copy teams to turn already-amazing papers into the polished versions you see now. Our undergraduate editors grew tremendously as mathematical thinkers and writers from reading and editing these papers. We hope our readers will grow in the same way from reading this issue, discover a new favorite theorem or two, and perhaps be inspired to produce some undergraduate expository work of their own.

Sincerely,

Aiden Sagerman, Zachary Lihn, and Lisa Faulkner Valiente

Editors-in-Chief, *Columbia Journal of Undergraduate Mathematics*

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