[2] See, for instance, William W. Abbott, *Certain aspects of the sonata-allegro form in piano sonatas of the 18th and 19th Centuries* (Unpub. Diss., Indiana University, 1956) where a number of movements of this kind by these composers are listed.

Mayer Joel Mandelbaum—*Multiple division of the octave and the tonal resources of 19-tone temperament*

Ann Arbor: University Microfilms (UM order no. 61–4461), 1961. (479 p., Indiana University diss.)

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While some recent developments in music have tended to minimize the role of defined pitch as a structural element, there nevertheless are theorists and composers who are concerned with creating a new music in which the melodic and harmonic orders are still fundamental organizing parameters. Rejecting both the extremes of 12-tone combinatorial serialism and the psychological ambiguities of the aesthetic of indeterminacy, these musicians seek to organize sound by means which, though grounded in history, are consistent with the results of modern acoustical research. Such endeavours may be subsumed under the somewhat unsatisfactory term *multiple division*,[1] the division of tonal space into steps which deviate significantly from the 300-year-old norm of twelve equal tones. In a field as large and complex as this, no general attempt at a theoretical treatment has appeared since Bosanquet in 1876.[2] The relevant material that has appeared in the meantime has tended to suffer from relative obscurity and either unsympathetic or idiosyncratic treatment by its authors. In correcting these long standing deficiencies in the literature and in proposing a new and rational theory of composition in the 19-tone system, Dr. Mandelbaum has created the definitive work in the field.

The dissertation is divided into three sections, the first of which is devoted to the various reasons for advocating multiple division grouped under the general categories: acoustical, historical, and evolutional. The last, since it deals in part with concrete aspects of style and performance, may have the greatest immediacy for the contemporary musician although the speculative theorist, as Dr. Mandelbaum considers himself, must also build his case on traditional foundations.

Musical acoustics, or, one is tempted to say, harmonics, is developed from a consideration of tonal consonance and its relation to the ratios of small whole numbers. Dr. Mandelbaum's elaboration of these concepts through a thorough discussion of tone lattices is particularly valuable. Explained in this manner, the functional relationship of rather complex pitch ratios is made clear. The voluminous literature on this aspect of acoustics is well-covered. Nonharmonic and nonoctaval scales, fields of active interest today,

are briefly introduced, as are a few examples from physical acoustics such as inharmonicity. In general, however, the emphasis is on what is usually referred to by the term *musical acoustics.*

Multiple division finds its greatest support historically in that it affords a means of re-creating the glories of past musical cultures, while retaining the advantages of more recent styles. Preeminent among such cultures is that of the Greeks. Mandelbaum provides an interesting analysis of how the Pythagorean system breaks down when applied to the enharmonic genus. This may lend support to those who believe that the various tunings catalogued by Ptolemy enjoyed actual use. On the other hand, Kathleen Schlesinger's subharmonic harmoniai deserve more attention than given here. Despite their controversial status, they form an imposing system which is capable of fascinating development beyond that of her modal tuning for the piano.[3] Senary just intonation is well-handled, with the usual reservations made about its employment even in the pure scale era. The often passed over mean-tone system and its analogues are gratifyingly discussed, since unequal systems are not only interesting per se, but are very closely related to certain important equal divisions such as 31, 43, and 19 itself. Oriental and folk musics are included in the historical section, since some theorists hope to assimilate them too into a world-wide synthetic musical culture.

Most writers on the future evolution of music have expressed a profound dissatisfaction with current practices, although few have been as thorough as Yasser in *A theory of evolving tonality*, a major landmark in the literature. In addition to a consideration of Yasser's theories, Mandelbaum discusses quarter and other aliquot divisions of the tone. One important aspect of these systems is that of interperformability. While one may grant the performability of the existing duodecimal literature on quarter-tone instruments, the same cannot be said for those tuned to 19-tone or other systems which are not multiples of twelve. Since most of the multiples of twelve have little acoustic advantage over twelve itself, this could be interpreted as a further reason not to advance to a new system. However, the wide acceptance of electronic music suggests that the time of change has come and that the necessary readjustments are not insuperable.

Having introduced the main concepts of multiple division, Mandelbaum turns to a more detailed examination of the writers advocating specific systems. Since many of these writers are poorly known and not easily obtainable in this country, this section is extremely useful.

These writers can be roughly divided into four groups. The first consists of those primarily interested in equal temperaments. Of these, Bosanquet is the most important. His classification scheme and generalized keyboard are highly relevant today, nearly a century after first publication. Any keyboard designer or notation reformer would do well to read his *Elementary treatise.* More widely known for their music are two 20th-century theorists and composers, Ivan Wyschnegradskii and the late Julián Carrillo. The music of the latter is now available on discs from Mexico.[4]

The second group comprises those concerned with unequal systems built on some version of just intonation. The best known of this group is Harry Partch, theorist, composer, and instrument builder, whose own book, *Genesis of a music*, has recently gone out of print.

By far the most attention is paid to those writers advocating 19-tone temperament, although many of these have made other noteworthy contributions as well. The two most vocal partisans of 19 are "Ariel" in Germany and Yasser in the United States, though the latter's theories are applicable to other systems too. Kornerup is perhaps better known for his Golden System, which is of note today because of the growing interest in musical application of the Golden Section.[5] While he has championed 31 more than 19, A. D. Fokker's contributions to multiple division are so pervasive that his work may be conveniently discussed here, as he has helped greatly to clarify the position of 19 among other proposed temperaments.

The final group of authors are those presenting generalized methods for finding optimal equal temperaments. In general, two approaches are used. One, which might be termed melodic, simply examines the errors in representation of a given set of just intervals. This is basically "Ariel's" method, and is as valid as his choice of intervals. The other is a harmonic one, based on the accuracy of certain generating harmonic intervals such as the third, fifth, and harmonic seventh. Fokker has developed an interesting method which depends on the choice of certain intervals which Würschmidt termed defining intervals.[6] If the factors of these intervals are taken as components of a set of vectors, the resulting octave division corresponds to a periodicity in the corresponding tone lattice. Professor Viggo Brun, a Norwegian mathematician, has applied the Euclidean algorithm to the problem of finding series of temperaments which represent the generating intervals with increasing accuracy. Its chief limitation is the difficulty of differentially weighting the factors. In both these cases, one assumes that some just intonation is the ideal for which the required temperament is a cyclic approximation. While these approaches may be criticized, they serve to stress the point that there is no one ideal temperament, but only temperaments more or less well-suited to individual musical needs. The frequency with which certain temperaments are arrived at by different methods could be interpreted as evidence of similar intentions among theorists, as well as of the universal value of these tunings.

Mandelbaum's contribution to this literature is an analysis and evaluation of 19-tone temperament relative to its possible competitors. He goes about this by defining a quantity to be known as the combined error factor (CEF), the product of the absolute error in an interval's representation and its fractional error in scale degrees. CEF's for various intervals are combined to form an overall harmonic evaluation. In addition, the full range of equal temperaments is subdivided in order not to have to compare such incommensurables, from a performance viewpoint, as 19- and 53-tone temperament.

According to his scheme, the temperaments of number twelve through twenty-four will be considered in one group. Employing only the perfect fifth and the major third as generating intervals, Mandelbaum computes the total CEF in several ways with the advantage usually going to the 22-tone system. If, however, he adds the minor third into the calculations, the favor turns strongly to 19. One might ask, though, whether the minor third is really independent of the other two intervals, and whether its inclusion is objectively justified. One might also take exception to Dr. Mandelbaum's exclusion of any intervals derived from the seventh harmonic in this group of temperaments. Granting that the CEF of 7/4 is quite large even in 22 (the only possible tuning in this area), this system can articulate a dominant seventh formation, making a one-unit distinction between the 6/5 and 7/6 minor thirds. It is thus possible that such a melodic phenomenon would permit 22 to be used as a musical system embodying the 4 : 5 : 6 : 7 tetradic structure, despite its acoustic errors. The other temperaments are treated in a similar but less controversial manner.

Turning specifically to 19 in the last section, Dr. Mandelbaum explains his method of composition in this system. The basic gamut is considered to be derived from cycles of perfect fifths and minor thirds, the first because of this interval's overwhelming importance, the second because of the great accuracy with which it is represented in this tuning. Here, as in twelve, consonance and dissonance function as opposites; there are, however, six new neutral intervals peculiar to 19. Consonances include the traditional thirds and fifths and their inversions with the addition of the major second and minor seventh. Major sevenths and minor seconds are dissonant. Scales are formed by projecting cycles of each of the nine intervals of 19 until only two interval classes differing by one unit remain. When arranged symmetrically, these scales are termed "quasi-equal-interval-symmetrical". Such scales occur in pairs—typical ones are the 12+7, 13+6, 14+5, and 11+8, to name the most important. Since each of these scales is formed by analogy to the diatonic scale of seven fourths, modulations will be made most naturally by the generating interval. Thus, each scale pair demands a unique harmonic treatment. Chords are classified according to their intervals; consonant chords are composed of consonances, while dissonant and imperfectly consonant formations contain dissonant and neutral intervals respectively. A pentadic basis for harmony is postulated in which no chordal interval contains less than three nor more than five units of the temperament. The discussion closes with a consideration of the type of resolution demanded by certain of the neutral and dissonant intervals and the relationship of these requirements to the harmonic context.

In addition to proposing a theory of composition, Dr. Mandelbaum has realized his concepts in nine preludes for two specially tuned pianos. The scores are included as part of the dissertation, and since 19 is a system for which standard notation can be used, they should present few problems to the reader. The tuning of the two pianos is a reasonable solution to the prob-

lem of assigning 19 tones to two 12-tone octaves. Considering the 19 pitches as derived by a series of altered fifths one piano is tuned to sharps and naturals, the other to flats and duplicated sharps. The music is of a vigorous, rather percussive nature and makes free use of the unique sonorities and progressions available in 19. A particular attachment is seen for intervals derived from the Golden Section—the interval C to F-flat is very close to .382 octaves. In other places, sonorities reminiscent of Yasser are seen. Of particular interest is an attempt to treat 12 of the 19 tones in serial fashion, reserving the other seven for the purpose of modulation. Little use seems to be made of pentadic harmony; this may be partly due to instrumental limitations. A tape containing the Preludes and a number of textual illustrations accompanies the manuscript. As it is really an integral part of the dissertation, the prospective reader should make an effort to obtain it.

The publication of this work would fulfill an important function, as there is no good reference book available on multiple division. Anyone considering a seminar on the subject would be advised to locate some copies before plunging into Partch and others. However, since six years have passed, a number of additions and revisions would be desirable. A considerable amount of new material has appeared, so far unpublished, from an informal but very active group in Los Angeles.[7] This group has also been active in building instruments and performing locally. In the general literature, important suggestions have been made in psychoacoustics as well. Some exploration into the serial properties of 19 as 19 should be offered for the sake of composers who lean toward this technique. Since it is a system with a prime number of tones, 19 has certain properties not shared by 12. For example, neither an all-interval row nor an all-interval set is possible. Nonetheless, in spite of its age, this work is an indispensable addition to anyone's library on multiple division and should be made available by better channels than interlibrary loans and University Microfilms.

FOOTNOTES

[1] Ivor Darreg has suggested the term *xenharmonics*.

[2] This is his general theory of the division of the octave, appearing in his book *An elementary treatise on musical tuning and temperament*, London 1876.

[3] For example, Ervin Wilson has developed what he calls "diaphonic cycles", which combine arithmetic division with tetrachordal articulation.

[4] A collection of records is currently available from Cruzada Pro-Sonido 13, Santísimo 25, San Angel, D.F. Mexico.

[5] Two other examples of golden ratio systems are Schroer's "aureotonality" and Walter O'Connell's ingenious and elaborate system which is to appear in Volume 8 of the English edition of *Die Reihe*.

[6] Defining intervals are small intervals set equal to zero degrees of a temperament. They comprise the various commas and dieses.

[7] This group is made up of colleagues and students of Ervin Wilson, Harry Partch, and others.