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SEPHI of Exoplanets Kepler-504 b, Kepler-315 b and Kepler-315 c

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ABSTRACT: The search for habitable exoplanets has improved with every passing year. New methods and advanced instrumentation with higher precision help find more habitable exoplanets and refine existing parameters of highly likely habitable exoplanets. This paper presents the Statistical-likelihood Exo-Planetary Habitability Index (SEPHI) values for Kepler-504 b (of star Kepler-504), Kepler-315 b, and Kepler-315 c (both revolving around star Kepler-315).[1,2,3] SEPHI is based on the geometric mean of the likelihood Gaussian estimation of four different comparison criteria with Earth as the only place we know harboring life: Telluricity, Atmosphere and Planet Gravity, Surface Liquid Water, and Magnetic Field.[4,5,6] The seven physical characteristics of exoplanets have been used to calculate those four criteria: planetary mass, planetary radius, orbital period, stellar mass, stellar radius, and stellar effective temperature. This is a follow-up to previously calculated ESI values for the three exoplanets mentioned above, with Kepler-504 b and Kepler-315 b having a high ESI of 71.23% and 69.44%, respectively.[7] It has been found that Kepler-504 b (with a host M-type star (small red dwarf)) has a SEPHI value of 0, Kepler-315 b (with a host G-type star) has a SEPHI value of 0, and Kepler-315 c (with a host G-type star) has a SEPHI value of 0. Thus, more than a combination of host star type, the orbital radius of exoplanet, and the final ESI to determine probable habitability, a further in-depth analysis through SEPHI can help us confirm its actual habitability for Earth-based life.

INTRODUCTION

Astronomers have been observing exoplanets for 32 years and studying their properties. In the last two decades, there has been immense progress in refining existing parameters through multiple missions by Kepler Space Telescope (KST), Transiting Exoplanet Survey Satellite (TESS), which have found 5500+ exoplanets, out of which 247 have an ESI > 0.5. 69 exoplanets are potentially habitable till date as per the Habitability Worlds Catalog.[8] However, calculating Earth Similarity Index, which compares the physical properties of the exoplanet to that of Earth, (ESI) is not enough to determine the likelihood to hold Earth-based life as habitability—an independent criteria of ESI.[9]

The ESI values provide a general idea of possible habitability, but habitability can't solely be concluded based on similar physical properties. Hence to confirm habitability, an independent criterion of the ESI, we need to conduct further in-depth analysis of the exoplanets by comparing more factors that contribute to life's existence, sustenance, and proper environment for it to thrive. Astronomers have used PHI to determine habitability, but certain problems are associated with it. The PHI is defined as the geometric mean of four sub-indexes:

$$PHI = (S.E.C.L)^{1/4}$$

where S is substrate presence, E is available energy, C is the appropriate chemical composition, and L is the presence of a liquid solvent on the particular exoplanet. Each of these variables is subdivided into different parameters (like atmospheric presence, magnetosphere, and redox chemistry) which are considered individually. The problem with PHI is that every parameter is determined through a modulus function (ad-hoc classification) and often combines complementary criteria. For example, the energy received from the star and tidal flexing are both considered together for the determination of E, but these events do not occur simultaneously. Furthermore, the PHI value for Jupiter or Saturn is 0.40 and for Mars, PHI is 0.56. This implies that Mars has a similar planetary habitability potential to that of Jupiter or Saturn, which is not true given the latter two are gas giants while Mars still has a surface. Hence, the SEPHI is used as a new form of PHI calculation that utilizes statistical likelihood. These likelihood functions are determined by Gaussian-like probability profiles (which is the exponential form). Unlike PHI which uses single variables, SEPHI uses comparison criteria (single variable/combination of them) and defines likelihood functions. Additionally, SEPHI does not take into account any free parameter which means that all the comparison criteria in the analysis have equal weight. Adopting SEPHI, with respect to ESI, the results for SEPHI values and ESI values for the exoplanets along with planets from our Solar System have been added to give A reference base for understanding SEPHI's relevance.

METHODS

For determining the four comparison criteria for the SEPHI, Mozos et al. (2017) focused on those describing the basic conditions allowing life on Earth and defined four sub-indexes ($\mathcal{L}$) for SEPHI. The final result was the geometric mean of these four sub-indexes, and it represented the statistical likelihood of a planet being potentially habitable with respect to Earth from an astrobiological perspective.[4] This definition was a statistic as the $\mathcal{L}_i$ were statistically independent:

$$SEPHI = \prod_{i=1}^{n} \mathcal{L}_i^{1/n}$$

The following are the four comparison criteria:

(i) Telluricity ($\mathcal{L}_1$): To be a telluric planet, with part of its surface composed of solid silicates similar to the general composition of Earth modeled as 17% Fe and 83% $MgSiO_3$

(ii) Atmosphere and planet gravity ($\mathcal{L}_2$): Existence of a dense atmosphere that can retain the necessary gasses to sustain life, and the presence of a proper planetary gravity that is compatible with life.

(iii) Surface Liquid Water ($\mathcal{L}_3$): Exoplanets having liquid water on its surface which is determined from the calculation of the Habitable Zone for the particular star system.

(iv) Magnetic Field ($\mathcal{L}_4$): Exoplanets having a strong magnetic field that will be able to protect life from solar radiation.

Participants/Organisms:

The data provided by NASA's MAST archive database was used along with the analysis of previous data on ESI for the three exoplanets. The main data used were the previously calculated parameters that were cross-checked for any updates in the parameters from the NASA Exoplanet Archive.[10,11]

Experimental Design:

Quantitative data from the archive database was used in this paper. This data was observed and collected by KST as part of its legacy Kepler and K2 missions. Additional data was also used from TESS. These data packets were already used in the previous ESI calculations. The calculations conducted in this paper were built on the data used previously such as calculation for relative planetary mass (m_p), relative planetary radius (r_p), relative escape velocity (v_e), stellar luminosity (L_n), relative density (ρ_p), Stellar Effective Temperature (T_{eff}), which forms the basis for calculating the four composition criteria used to determine SEPHI values for each of the exoplanets.[4] These particular criteria are essential to understand whether those basic conditions are compatible enough to allow Earth-based life (carbon-based life) on that exoplanet.

Measurements/Calculations:

Python programming language was used to calculate all four sub-indexes which were included in the SEPHI calculation for the exoplanets. Data, such as Habitable Zone coefficients, and coefficients for magnetic field likelihood calculation were taken from previously published papers.[12,13,14] Each of the four criteria used has the following likelihood function:

$\mathcal{L}_1$: Telluricity likelihood function

From Mozos et al. (2017), we know the following conditions for decay of the likelihood function:

$$\mu_{1,m_p} = r_{p,100\%\, MgSiO_3}$$

$$\mu_{2,m_p} = r_{p,[50\%\, MgSiO_3 - 50\%\, H_2O]}$$

$$\sigma_{1,m_p} = \frac{\mu_{2,m_p} - \mu_{1,m_p}}{3}$$

Figure 1: Relationship between a planet's relative radius and composition parameters. The mean radius for a 100% $MgSiO_3$ composition is denoted as μ_1, m_p, and for a 50% $MgSiO_3$ and 50% H_2O composition as μ_2, m_p . The standard deviation of the planet's telluric likelihood function, σ_1, m_p, is calculated from these mean radii.

where μ_1, m_p, is the relative radius (with respect to Earth) of a planet containing 100% $MgSiO_3$ composition, μ_2, m_p, is the relative radius (with respect to Earth) of a planet containing 50% $MgSiO_3$ and 50% H_2O composition, and σ_1, m_p is the standard deviation σ_1 equal to one-third of the

difference of the relative radius of the planet with the above two compositions which limits $\mathcal{L}_1$, m_p between 0 and 1.[4,15,16,17]

The telluricity likelihood function for a given relative planet mass (m_p) and relative radius (r_p) is:

$$\mathcal{L}_{1,m_p}(r_p) = 1 \qquad \text{for } r_p \le \mu_{1,m_p}$$

$$\mathcal{L}_{1,m_p}(r_p) = e^{-\frac{1}{2}\left(\frac{r_p - \mu_{1,m_p}}{\sigma_{1,m_p}}\right)^2} \qquad \text{for } \mu_{1,m_p} < r_p < \mu_{2,m_p}$$

$$\mathcal{L}_{1,m_p}(r_p) = 0 \qquad \text{for } \mu_{2,m_p} \le r_p$$

Figure 2: The likelihood function $\mathcal{L}_1$, $m_p(r_p)$ as Gaussian-like profiles for telluricity based on relative radius r_p with respect to Earth. It also shows the pre-condition for each likelihood function under which a planet can be assumed as telluric, with three conditions, one for r_p being less than or equal to μ_1, m_p, another one for a Gaussian decay of exponential form if r_p is between μ_1, m_p and μ_2, m_p , and lastly, if r_p is greater than or equal to μ_2, m_p .

$\mathcal{L}_2$: Atmosphere and planet gravity likelihood function

From Mozos et al. (2017), the likelihood function for planet gravity being compatible with Earth life is:

$$\mathcal{L}_2(v_e) = e^{-\frac{1}{2}\left(\frac{v_e - 1}{\sigma_{21}}\right)^2} \qquad \text{for } v_e < 1$$

$$\mathcal{L}_2(v_e) = e^{-\frac{1}{2}\left(\frac{v_e - 1}{\sigma_{22}}\right)^2} \qquad \text{for } v_e \ge 1$$

Figure 3: The likelihood function $\mathcal{L}_2(v_e)$ for calculating the ability of an exoplanet to maintain an atmosphere on the basis of relative

escape velocity v_e to that of Earth. The functions are based on Gaussian distribution profiles on two conditions of v_e, with $v_e < 1$ and $v_e >= 1$.

where v_e is the relative planet gravity measured by $\sqrt{g r_p}$, $\sigma_{21}=1/3$ and $\sigma_{22}=7.66/3$.

$\mathcal{L}_3$: Surface Liquid Water likelihood function

The Habitable Zone boundaries D_n depend on the following effective stellar flux (S_{eff}) with coefficients a, b, c and d depending on habitable zone and stellar effective temperature (T_{eff}),

$S_{eff} = S_{eff,\odot} + a(T_{eff} - 5780) + b(T_{eff} - 5780)^2 + c(T_{eff} - 5780)^3 + d(T_{eff} - 5780)^4$

and $D_n = \sqrt{\frac{L_n}{S_{eff}}}$ AU, where Ln is the stellar luminosity in solar units.

The likelihood function is defined on the basis of orbital major axis *a* (4). The following are the defined zones with respect to *a*:

Hot Zone: $a < D_1$

Inner-Transition Zone (ITZ): $D_1 \le a < D_2$

Green Zone: $D_2 \le a < D_3$

Outer-Transition Zone (OTZ): $D_3 \le a < D_4$

Cold Zone: $a > D_4$

The likelihood of holding liquid water if it is near (or within) the zone was calculated based on the planet's location with respect to the corresponding habitable zone:

$$\mathcal{L}_3(a) = e^{-\frac{1}{2}\left(\frac{a-\mu_{31}}{\sigma_{31}}\right)^2} \quad \text{for } a < D_2$$
$$\mathcal{L}_3(a) = 1 \quad \text{for } D_2 \le a \le D_3$$
$$\mathcal{L}_3(a) = e^{-\frac{1}{2}\left(\frac{a-\mu_{32}}{\sigma_{32}}\right)^2} \quad \text{for } a > D_3$$

Figure 4. The likelihood function calculates the probability of the exoplanet holding water in liquid form on its surface as a Gaussian-like profile distribution function on the basis of the exoplanet's orbital semi-major axis a. The function returns a constant 1 if the semi-major orbital axis is in the Green Zone of D_2 and D_3. The other two conditions give us the probability of containing surface liquid water, one being the Inner Transition Zone, where $a < D_2$ and the other Outer Transition Zone, where $a > D_3$.

where $\mu_{31}= D_2$, $\sigma_{31}= (D_2 - D_1)/3$ for the ITZ and $\mu_{32}= D_3$, $\sigma_{32}= (D_4 - D_3)/3$ for the OTZ (4).

$\mathcal{L}_4$: Magnetic Field

The likelihood function is defined as:

$$M_n= \alpha \rho^{1/2}_{0n}\, r^{10/3}_{0n}\, F^{1/3}_n$$

where r_{0n} (normalized radius) and F_n (normalized average convective buoyancy flux) are defined in terms of two corrections $\beta_1= R_p/R_r$ and $\beta_2= \rho_p/\rho_r$ (related to the size and density of the reference planet) and $\alpha= 1$.[14]

In case of Kepler-504 b, a super earth, $\rho_{0n}= 1$, $r_{0n}= \beta_1$, $Fn= \beta_2$ and in case of of Kepler-315 b and Kepler-315 c, two gas giants, $\rho_{0n}= 0.16$, $r_{0n}= 16\beta_1\beta_2$, $F_n= 100\beta_1\beta_2$.[18,19]

The likelihood function of having a strong magnetic field to protect life from strong solar and cosmic radiation depends on M_n:

$$\mathcal{L}(\mathcal{M}_n) = e^{-\frac{1}{2}\left(\frac{\mathcal{M}_n-\mu_4}{\sigma_4}\right)^2} \quad \text{for } \mathcal{M}_n < 1$$
$$\mathcal{L}(\mathcal{M}_n) = 1 \quad \text{for } \mathcal{M}_n \ge 1$$

Figure 5. The likelihood function $\mathcal{L}_4(M_n)$ calculates the probability, using a Gaussian-like distribution, as to whether an exoplanet has a strong dipolar magnetic field, which is required to protect life from cosmic radiation. The function calculates a Gaussian decay value for a relative magnetic moment less than that of Earth and returns a constant 1 for a relative magnetic moment greater than or equal to that of Earth.

After this, all the four likelihood function values are taken as a geometric mean to arrive at the SEPHI value = $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4)^{1/4}$.

Data Analysis:

In Table 1, the four criteria for the SEPHI along with the final SEPHI value for the exoplanets are shown.

Exoplanet	$\mathcal{L}_1$ Telluricity (in %)	$\mathcal{L}_2$ Relative Gravity (in %)	$\mathcal{L}_3$ Surface Liquid Water (in %)	$\mathcal{L}_4$ Magnetic Field (in %)	SEPHI (in %)
Kepler-504 b	28.52	1.11101015	0	100	0
Kepler-315 b	0	1.11101032	0	100	0
Kepler-315 c	0	1.11101087	82.186	100	0

Table 1: SEPHI values along with each of the likelihood function values of the comparison criteria for exoplanets Kepler-504 b, Kepler-315 b and Kepler-315 c. The values for $\mathcal{L}$ show the probability of the criteria existing for that exoplanet where $\mathcal{L}_1$ is the exoplanet's telluricity, $\mathcal{L}_2$, is the exoplanet's relative gravity, $\mathcal{L}_3$, is the probability to contain liquid surface water, $\mathcal{L}_4$, is the probability to have a strong dipolar magnetic field.

For comparison, the results of SEPHI values along with the corresponding ESI values, host star type, habitable zone, and orbital radius are shown in Table 2. For the calculation of values, Python's numpy library was used.

Exoplanet	ESI (in %)	ITZ (in AU)	OTZ (in AU)	Orbital Radius (in AU)	SEPHI (in %)
Kepler-504 b	71.23	0.1623	0.3250	0.0646	0
Kepler-315 b	69.44	0.9530	1.6803	0.402	0
Kepler-315 c	35.68	0.9530	1.680	0.791	0

Table 2. Comparison table for exoplanets Kepler-504 b, Kepler-315 b, and Kepler-315 c where the ESI, SEPHI along with the Inner Transition Zone (ITZ), Outer Transition Zone (OTZ), and the exoplanet's orbital radii values are shown. These values indicate the exoplanet's similarity to Earth to hold carbon-based life across multiple parameters.

RESULTS

Kepler-504 b, a super earth, has a telluricity likelihood of about 28.52%, while Kepler-315 b and Kepler-315 c being gas giants have 0 telluricity likelihood. For the exoplanet's relative gravity to that of Earth, all 3 of them had comparable values of 0.011110 or 1.111% that of Earth's gravity. Both Kepler-504 b and Kepler-315 b have 0 surface liquid water likelihood. Kepler-504 b having a surface does not harbor liquid water due to its high temperature of 114.84 Celsius (K), 14.84 degrees above the boiling point of water, and this is also confirmed by the $\mathcal{L}_3$ calculation. This high temperature is due to its close orbit with its host star as its orbital radius is at only 0.0646 AU. It was interesting to see that Kepler-315 c being a gas giant has a high liquid water likelihood of 82.1863%. Its temperature is 51.31 Celsius (K) which allows for water (if present) to exist in liquid form. All the 3 exoplanets were more massive than Earth which means that they have a strong dipole magnetic field which is confirmed by their magnetic field likelihood value of 1 (or 100%) for all. The SEPHI value for super earth Kepler-504 b is 0. Both of the gas giants Kepler-315 b and Kepler-315 c also have 0 SEPHI values. Getting even one value as zero among the four sub-indexes will lead to a 0 SEPHI since the geometric mean of these sub-indexes is taken. Hence, a combination of stellar properties and ESI value cannot be used to completely determine habitability. It is necessary to conduct more in-depth analysis on the particular exoplanet such as SEPHI, analyzing certain characteristics like telluricity, location in the habitable zone, surface temperature, surface liquid water, and strength of magnetic field, etc.

DISCUSSION

Previous papers selected G-type and M-type host stars since most of the discovered habitable exoplanets revolve around those star types.[7] The present data suggests that the chances of finding habitable exoplanets around these stars is extremely high and finding the ESI value confirms that. However, by including more variables and comparison criteria, a more detailed analysis like SEPHI shows us that it is not always the case. Habitability cannot be determined on the basis of stellar properties and ESI values. Further studies are required to determine habitability. The previous claim does give us a sense of general habitability, but, that is not a confirmation of life-supporting capacity for that exoplanet.

For telluricity, the corresponding relative radius of a planet with 100% $MgSiO_3$ and 50% $MgSiO_3$ - 50% H_2O were used given the exoplanet relative mass in terms of Earth. This was a grid search where corresponding radii can be determined as a function of relative earth mass.[15] For relative planet gravity calculation, 100% Fe was used as the conservative reference to get the calculation within the estimation of the highest relative gravity that can be compatible for life to exist.[20]

The most updated equations for determining habitable zones were used.[12,13,14] For magnetic field calculation $\alpha = 1$ was used since the exoplanets studied were more massive than Earth.[19]

The analysis used Python as the core program for calculations. Therefore, it includes rounding-off limitations floating point numbers. The code returns values only up to certain decimal points.

The study must be extended to a larger group of exoplanets. SEPHI should be checked for exoplanets around other star types as well since SEPHI being characteristic and planet-specific might bring out some interesting observations and conclusions about other host-star type exoplanets. There are also parameter restrictions like Telescope gathered data limitation, which introduces uncertainty in the data with regards to planetary properties like orbital radius, surface temperature, and telluricity.

CONCLUSION

This research wanted to find whether stellar parameters and ESI value is enough for determining habitability. However, upon finding the SEPHI values to be zero for all three exoplanets, it is determined that this is not always the case (even for exoplanets with host stars required for life to thrive, proper surface temperature but lack of surface soil telluricity). Hence, habitability should not be declared once we find the conforming stellar types and high ESI values. There is no doubt that a high ESI value suggests a high probability of finding life-supporting capacity. More comparison criteria must be checked for such that the exoplanet should fulfill for it to support carbon-based life.

For future work, both ESI as well as SEPHI values must be analyzed to completely determine habitability. Future advanced telescopes like the present JWST will be more capable of observing the atmosphere which in turn will make calculations more precise and allow for the inclusion of more comparison criteria in the SEPHI and determine habitability more accurately.

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NOTES

The code used for calculating the SEPHI values and each of the comparison criteria can be found in this Github repository: https://github.com/SattikBhaumik/Analyzing_Exoplanets/blob/main/SEPHI.ipynb

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infrared.

ABBREVIATIONS

ESI: Earth Similarity Index

PHI: Planet Habitability Index

SEPHI: Statistical-likelihood Exo-Planetary Habitability Index

KST: Kepler Space Telescope

TESS: Transiting Exoplanet Surveying Satellite

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