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Presenting a New Model to Support the Secondary-Tertiary Transition to College Calculus: The Secondary Precalculus and Calculus Four Component Instructional Design (SPC 4C/ID) Model

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ABSTRACT Although the secondary-tertiary transition has been investigated in mathematics education research with different focuses and theoretical approaches, it remains a major issue for students in the transition. With success in a science, technology, engineering, or mathematics (STEM) major at stake, we investigated a novel approach to support the transition from secondary precalculus or calculus to tertiary calculus. Using the Four Component Instructional Design (4C/ID) model and empirical data from the United States (US) nationally representative FICSMath project, we mapped instructional experiences of students in the transition to theoretical components of the 4C/ID model. From exploratory factor analysis (n=6,140), we found six factors that mapped to the 4C/ID model components and created the new Secondary Precalculus Calculus (SPC) 4C/ID model. In this model, the Learning Task Component represents tasks to engage learners in meaningful problem solving; the Support Component grounds instruction in reasoning and understanding; the Procedure Component integrates group work and graphing calculators to connect concepts to procedures; and the Part-Task Component represents instruction to develop automaticity. The SPC 4C/ID model presents a unique support for precalculus and calculus teachers in the quest of teaching for learning and transfer of learning across the transition.

KEYWORDS *secondary mathematics, postsecondary mathematics, modeling instruction*

Introduction

Why do we need a(nother) model for teaching precalculus and calculus? Because many in the field—high school teachers and professors alike—have been trying to improve student learning of mathematics to disappointing ends. In the United States (US), for example, students in 12th grade have demonstrated unimproved mathematics scores on the National Assessment of Educational Progress (NAEP) from 2005-2019. Without testable instructional models, we are left with claims about teaching mathematics that are difficult to affirm or reject. We therefore present an empirically based model that focuses on the secondary-tertiary transition in mathematics. Exactly when this transition begins and ends is obscure, yet educational research suggests a period between two years before and after entering university (Gueudet, 2008). In the US, even if secondary students score a 3 or higher (grades range from 1 to 5) on the College Board Advanced Placement (AP) Calculus exams, they may still struggle to perform and persist in college calculus (Atuahene & Russell, 2016). Bressoud (2009) hypothesized that the College Board AP Calculus curriculum is so broad that students move through it learning procedures instead of concepts necessary for success in tertiary calculus. The most common transition support offered is that of bridge courses, e.g., high school-level courses, such as algebra and/or precalculus that are taken (or retaken) at the tertiary level. In the US, tertiary level enrollment in precalculus is well-populated with students who previously completed precalculus in high school. Perhaps surprisingly, retaking precalculus in college does not predict earning higher grades in college calculus (Sonnert & Sadler, 2014). With questionable support from bridge courses and with known variability of preparation quality, a look back to instruction and learning in secondary mathematics is a logical next step.

Theoretical Perspectives

Several theories in the field of mathematics education have been used to investigate the secondary-tertiary transition. Clark and Lovric (2008; 2009) situated the transition in an anthropological framework of the rite of passage. Within their work, they discuss the vast array of changes across teaching styles, the types of mathematics taught, the levels of conceptual understanding, and the advanced mathematical thinking required. Other researchers have investigated the actions, process, objects, and schemata(APOS) theory (Dubinsky & McDonald, 2001; Gueudet, 2008; Selden & Selden, 2001). This theory views mathematical knowledge as being constructed through mental actions that are organized in schemata to make sense of problem-solving situations. Gueudet (2008) discussed the APOS theory relative to students in the transition as they are shifting to more advanced mathematical thinking. While these perspectives have added to our understanding of the transition, the theory foundational to our work is Cognitive Load Theory (CLT).

A major premise of CLT is that working memory load is decreased when domain specific schemata are activated from long term memory. The three sources of working memory load are described as: extraneous cognitive load coming from how material is organized and presented during instruction; intrinsic cognitive load coming from element interactivity, or the interaction of the interconnected parts of the content; and germane cognitive load, which encodes, sends, and connects newly processed information to existing long term memory schemata. A major instructional challenge is how to limit extraneous and intrinsic cognitive load enough so that working memory has the resources to successfully encode information for storage into long term memory. When schemata are built from this process, then learning can occur. CLT defines learning as a *permanent change in long-term memory* (Sweller et al., 1998), and we refer to learning the same way. We also believe that instruction focused on easier-to-present procedures compromises learning (Curry, 2017). This phenomenon is referred to as the *transfer paradox* because such instruction may have an effect on short-term retention for test performance but not on learning (van Merriënboer et al., 2006). This, along with the Four Component Instructional Design (4C/ID) model, created from CLT to support instruction of a complex task (van Merriënboer et al., 2006), is what specifically attracts us to this theory. A complex task, in contrast to simple tasks, has many different solutions, real world applications, cannot be mastered in a single session, and poses a very high load on the learners cognitive system (van Merriënboer et al., 2006). The 4C/ID model was not created specifically for mathematics, however we are applying the model to mathematics because instruction and learning of mathematics is a complex task. For example, mathematics requires multiple solutions during problem-solving (e.g., numeric, algebraic, graphic, etc.), is replete with real world connections, requires time to learn, and—for many—creates a very high working memory load. The 4C/ID model was confirmed for mathematics using data from the Factors Influencing College Success in Mathematics (FICSMath) (Wade et al., 2020). However, there has never been an investigation into how well the theoretical components of the model correspond to the actual instruction of secondary precalculus and calculus teachers. Thus, the purpose of this paper is to explore the fit between the 4C/ID model and senior level high school students' perceptions of how their precalculus and calculus instructional experiences prepared them for college/university calculus.

4C/ID Components

The 4C/ID model components include Learning Task, Support, Procedure and Part-Task Components. These components help understand how to reduce cognitive load and support working memory during the learning of complex tasks. Table 1 presents the model components with their descriptions. Learning tasks ideally connect learners with constituent skills from the support and procedure components that make up the whole task (van Merriënboer et al., 2002). Working with the whole task is challenging yet required for making connections between prior knowledge and new learning. For example, when learning logarithms, the prerequisite concepts of exponents and functions must be used to support learning. Most learners are not cognitively prepared to learn logarithms when there are no schemata developed for exponents and exponential functions.

Figure 1 presents the model as conceived by van Merriënboer et al. (2006). What is important to grasp from the representation of the model is that the Support Component (overarching concepts) is foundational to learning complex tasks. The Procedure Component and the Learning Task Components are established upon the concepts. As presented in Table 1, the partially shaded

Table 1

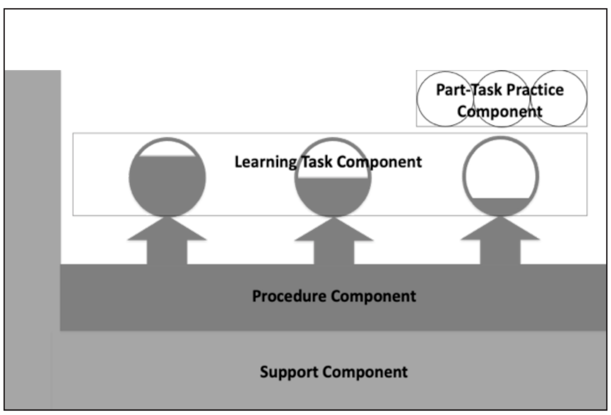
Description of the Theoretical Components of the 4C/ID Model (modified from 4CID.org)

4C/ID Component	Description and Goal of the of the Component
Learning Task Component	Integrates non-routine and routine skills and knowledge with authentic whole task learning experiences, is organized from simple to complex, and provides diminishing scaffolding support (represented by the partially shaded circles).
Support Component	Is foundational to learning tasks as it supports learning of non-routine aspects of learning tasks, explains how to approach problems using cognitive strategies, details how the domain is organized using conceptual models and is always available.
Procedure Component	Specifies how to perform aspects of the tasks through step-by-step instruction, is presented just-in-time and fades as learners acquire more expertise.
Part-Task Practice Component	Provides additional practice for routine aspects to reach a high level of automaticity, provides repetition, begins after routine aspects have been introduced in context of the whole task.

circles represent diminished scaffolding over time of the constituent skills that make up the whole task (van Merriënboer et al., 2002). The Part Task Component represents practice for automaticity, and such practice is known to reduce the working memory load.

Figure 1

Van Merriënboer's Theorized Components of the 4C/ID Model. From Four-Component Instructional Design. *4cid.org.*



Research Questions

The FICSMath project has collected a wealth of empirical data about US students' instruction and learning experiences from their last high school mathematics class before entering tertiary calculus. For this paper, the data analyzed latent constructs to determine how well actual instructional practices, as reported by students in the transition, align with components of the 4C/ID model. Our research questions are thus two-fold: What is the fit of students' perceptions of their instructional experiences, as reported on the FICSMath survey, with the theoretical components of the 4C/ID model? If there is a fit, how can the 4C/ID model be modified to better align with secondary precalculus and calculus instruction?

Data and Methods

The FICSMath Project

The FICSMath project, conducted at the Science Education Department of the Center for Astrophysics | Harvard & Smithsonian remains the most recent US study of high school preparation for college calculus success. Three sources of data were gathered for the development of the FICSMath survey. One source was a broad literature review of current issues in secondary and tertiary mathematics education. Another was a qualitative online survey sent to precalculus and calculus teachers and professors across the nation. Teachers were asked what they were doing, and professors were asked what teachers should be doing, to prepare students for tertiary calculus (for results, see Wade et al., 2016). Lastly, a focus group consisting of experts in secondary and post-secondary mathematics and mathematics education discussed the survey items. Together these provide evidence of content validity. To gauge test-retest reliability, we carried out a separate study in which 174 students from three different colleges took the survey twice, 2 weeks apart. Our analysis found that, for groups of 100, less than a 0.04% chance of reversal between the 50th and 75th percentiles existed (Thorndike, 1997). In the end, the FICSMath survey included 61 questions (many with multiple imbedded items) regarding demographics, course taking, performance levels, and instructional experiences from participants' most recent high school mathematics course before entering single variable college calculus. The survey was administered at the beginning of the Fall

Table 2

The Types of Instructional Questions (14) and Items (70) on the FICSMath Survey

Types of Instructional Questions on the FICSMath Survey	Number of Items	Scale
The amount of conceptual understanding and memorization of procedures required in the class.	2	0-5
The ways calculators were used in class.	7	0-1
Frequency of use of calculators and/or computers in class.	3	0-4
Emphasis on specific types of instruction in class.	7	0-5
Frequency of types of in-class, student-to-student and/or teacher-to-student questioning, responses, and interactions.	10	0-5 and 0-4
Types of problems investigated and solved in class.	9	0-7
How often calculations were checked for reasonable answer.	1	0-5
Types of in-class questions on tests or quizzes.	9	0-1
Use of specific teaching characteristics.	6	0-5
Ways mathematics was connected to real life in class.	4	0-4
Types of support for problem solving given in class.	3	0-4
Types of teaching manipulatives used in class.	3	0-4
Use of in-class assessments.	6	0-4

2009 semester to a stratified random sample of 276 small, medium, and large 2- and 4-year institutions (336 college and university calculus courses or sections). Students completed the surveys in college/university class, and, when the semester was over, the professors reported grades on the surveys before returning them to Harvard University. In the end, we obtained data from 10,437 students from 134 institutions that returned the surveys (73.6% response rate from those who agreed to participate). For the purposes of this study, we included only respondents who had precalculus and/or calculus their senior year in high school and were the next semester in single variable college or university single variable calculus. This reduced the sample to 5,985 respondents. Though the individual percentages of missing values for the variables used were small, multiple imputation was applied to prevent a compound loss of data. Our sample thus included 6,140 cases, a 2.6% increase from the 5,985 respondents under listwise deletion.

Instructional Questions and Items

Table 2 shows the various instructional questions from the FICSMath survey. Instead of choosing which instructional questions should be mapped to components of the 4C/ID model, we included all 70 items (from the 14 questions) in exploratory factor analysis (EFA). This allowed instructional experiences to be mapped empirically instead of theoretically.

Exploratory Factor Analysis

Exploratory factor analysis (EFA) is a widely used and broadly applied statistical technique in the social sciences (Costello & Osborne, 2005). We first investigated the Kaiser-Meyer-Olkin (KMO) statistic, which indicates the proportion of variance in the variables that may be caused by underlying factors. High values close to 1.0 indicate that factor analysis can be usefully applied to the data. The KMO value was 0.848, suggesting that EFA is a reasonable method to investigate the underlying constructs. The large number of participants in the FICSMath study allowed us to meet many of EFA established best practices. For reliable results, the total number of variables in EFA should be at least three to five times larger than the number of expected common factors (Fabrigar et al., 1999). With 70 variables and seven factors, we comfortably met this standard. Additionally, the recommended sample size should have a ratio of 10:1 of observations to factors (Costello & Osborne, 2005) which we also met comfortably with a sample size of 6,140 (after multiple imputation) and seven factors. Table 2 shows that the scales of the variables were different, so we standardized the variables before running EFA. We also selected Maximum Likelihood as the factor analysis method and used eigenvalues greater than 1 (Gorsuch, 1983) and the Scree test to determine the number of factors to keep (Cattell, 1966; Fabrigar et al., 1999). Because, according to Yong and Pearce (2013), larger sample sizes allow smaller loadings, we decided that items with a factor loading of 0.30 or higher would remain in the factors. We also viewed instructional variables as being correlated and hence used the Promax oblique rotational method because of its expedience with larger datasets and the simple structure it can achieve (Gorsuch, 1983; Yong & Pearce, 2013). In the end, 34 out of the 70 variables held together in seven factors. The factors explain 51.9% of the variance within the data. As seen in Table 3, Cronbach's alpha for the factors ranged from 0.618 to 0.873, indicating high internal consistency within each factor.

Table 3

Factors, Constructs, Loadings, and the FICSMath Items (n=6,140; 51.9% of variance explained).

Factors (Cronbach's Alpha)	Latent Constructs	Factor Loadings	FICSMath Items
Factor 1 (0.873)	Ways instruction connects mathematics to the real world and other subject areas.	0.936	Connected math to real life applications.
		0.850	Connected math to everyday life.
		0.705	Examples from everyday world were used.
		0.655	Connected math to other subject areas.
Factor 2 (0.837)	Instruction to support problem solving.	0.928	Teacher highlighted more than one way of solving a problem.
		0.794	Teacher explained ideas clearly.
		0.686	Teacher used graphs, tables, and other illustrations.
		0.599	Teacher presented various methods for solving problems.
Factor 3 (0.746)	Instruction to support mathematical literacy, reasoning, and conceptual understanding.	0.831	Emphasis on precise definitions.
		0.703	Emphasis on vocabulary.
		0.503	Emphasis on mathematical proofs.
		0.421	Emphasis on mathematical reasoning.
		0.412	Emphasis on functions.
		0.331	Extent of conceptual understanding.
Factor 4 (0.747)	Ways calculators were used in the course to support problem solving.	0.626	Allowed to use for trigonometric functions.
		0.614	Allowed to use on exams.
		0.590	Allowed to use to plot graphs of functions.
		0.570	Allowed to use for simple calculations.
		0.566	Allowed to use for homework.
		0.453	Allowed to compute derivatives and integrals.
Factor 5 (0.785)	Frequency of various types of problems solved in the course.	0.822	Frequency of word problems.
		0.706	Frequency of problems with multiple parts.
		0.566	Frequency of problems with written explanations.
		0.507	Frequency of problems with proofs.
		0.485	Frequency of problems being graphed by hand.
Factor 6 (0.712)	Student and teacher classroom interactions to support learning mathematics.	0.941	Classmates taught each other.
		0.775	You taught your classmates.
		0.375	Small group discussions were held.
		0.335	Students spent time doing individual work in class.
Factor 7 (0.618)	Instructional time spent on preparing for assessments and going over assignments.	0.610	Class time spent preparing for quizzes or tests.
		0.580	Time spent reviewing past lessons.
		0.489	Class time spent preparing for standardized tests.
		0.451	Tests or quizzes were given in class.

Table 4

Pearson Correlations Among the Factors (n = 6,140)

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Factor 1						
Factor 2	0.375**					
Factor 3	0.321**	0.454**				
Factor 4	0.231*	0.215*	0.299*			
Factor 5	– 0.031	– 0.006	– 0.071	– 0.067		
Factor 6	0.235*	0.178*	0.115*	0.138*	0.050	
Factor 7	0.260*	0.270*	0.204*	0.254**	– 0.025	0.255*
** Weak correlation (0.00 ≤ r ≤ 0.30); **Moderate correlation (0.31 ≤r ≤ 0.50)*						

Lastly, the Pearson product-moment correlations were computed to determine if the factors held cohesively together among one another. We used Onwuegbuzie and Daniel's (1999) guide of appropriate sample sizes of 800, 84, and 28 to detect small (r = 0.1), moderate (r = 0.3) and large correlation (r = 0.5) levels, respectively. With a sample size of 6,140 we used 0.1 and 0.3 as the cut off for weak and moderate correlations, respectively. As seen in Table 4, all factors were positively correlated with each another except factor 5.

Results

Being comfortable with the factors and how they held together, we determined if and how the factors mapped to the theoretical components of the 4C/ID model. Factors with at least a weak positive correlation (r > 0.1) and theoretical alignment with the components were mapped together. We now present how we perceive the 4C/ID components as they relate to the teaching of high school precalculus and calculus. Table 5 presents the

Table 5

Mapping of the Factors and Constructs to the Theoretical Components of the 4C/ID Model

Factor and Construct from FICSMath Data (correlation value if more than one factor).	4C/ID Component	Description of the Component
Factor 1: Ways instruction connects mathematics to the real world and other subject areas.	Learning Task Component	Integrate new content with necessary prior knowledge and connect with applied problems. Scaffold instruction by integrating the Support and Procedure Components and diminish scaffolding over time.
Factor 2: Instruction to support problem solving. **Factor 3**: Instruction to support mathematical literacy, reasoning, and conceptual understanding (r = 0.454).	Support Component	Support instruction of new learning tasks by explaining how to approach problem solving using cognitive strategies, concepts, reasoning, and mental models (graphs, charts, tables, patterns, etc.).
Factor 4: Ways calculators were used in the course to support problem solving. **Factor 6**: Student and teacher classroom interactions to support learning mathematics (r = 0.138).	Procedure Component	Connect problem solving tasks through step-by-step instruction while integrating prior knowledge with new content. Connection to applied problems create the stage for just-in-time instruction that fades as learners acquire more expertise.
Factor 7: Instructional time spent on preparing for assessments and going over assignments.	Part-Task Component	Provide additional practice through classwork, homework, group work, etc., for problem-solving tasks to reach a high level of automaticity. Repetition begins after new content has been introduced in the context of the whole task.

4C/ID components and addresses our first research question: What is the fit of students' perceptions of their instructional experiences, as reported on the FICSMath survey, with the theoretical components of the 4C/ID model? Factor 1 mapped to the Learning Task Component; Factors 2 and 3 being mapped to the Support Component; Factors 4 and 6 mapped to the Procedure Component; and Factor 7 mapped to the Part-Task Component. Factor 5, a measure of the quantity and variety of problems posed in the course, was negatively correlated and was thus not included in the mapping of factors to model components.

The Modified 4C/ID Model

We now address Research Question 2: How can the 4C/ID model be modified to better align with the actual instruction of secondary-tertiary mathematics? Table 6 presents the descriptions of the components of the modified Secondary Precalculus Calculus Four Component Instructional Design (SPC 4C/ID) model. The modified components were generated using Table 3 (factors generated from EFA) and Table 5 (mapping of factors with mathematics education language integrated into the 4C/ID components). The modified SPC 4C/ID model was designed for secondary precalculus and calculus instruction, with the ultimate goal of providing teachers with guidance on ways to better prepare students for tertiary calculus.

Table 6
Description of the Components of SPC 4C/ID model

SPC 4C/ID Component	Description of the SPC 4C/ID Components
Learning Task Component	• Present learning tasks that integrate new concepts by engaging learners in problem solving that integrates prior learning. • Present applied problems with associated mathematical tasks organized from simple to complex. • Scaffold instruction using the Support and Procedure Components. Decrease scaffolding over time.
Support Component	• Support learning by highlighting various ways of solving problems. • Use mental models to focus on mathematical concepts using graphs, tables, and other illustrations. • Focus on mathematical literacy (definitions and vocabulary), proofs, reasoning, functions, and conceptual understanding to present how the mathematical content holds together. • This component is paramount to building conceptual understanding that will be needed in university mathematics.
Procedure Component	• Use graphing calculators to connect concepts to procedures while being mindful that students will most likely not have access to graphing calculators in college level calculus. • Use small group discussions and group work where students can explain problem solving and where teachers can provide just-in-time guidance.
Part-Task Practice Component	• Provide additional practice to reach high levels of automaticity by reviewing past lessons, going over homework, and preparing for quizzes or tests. • Only use such review after concepts and procedures have been presented to reduce the over reliability on rote mathematical procedures.

Limitations and Future Work

The SPC 4C/ID model was generated from US students' instructional experiences from their senior level precalculus or calculus course, as they reported them on the FICSMath Survey the following semester in single variable college or university calculus. There is no voice, however, representing those who took precalculus and/or calculus in high school and did not proceed to college calculus the following semester. Likewise, this article is silent on instructional practices at the tertiary level—which are also worthy of examination (to see how tertiary calculus has addressed transition issues see Vandenbussche et al., 2018; Norton et al., 2019; & Viera et al., 2019). The SPC 4C/ID model may be unique to a US context, but we hope that mathematics educators and teachers from other countries will consider this model and investigate if it can support students in the transition to tertiary mathematics in their specific institutional structures. Lastly, while the SPC 4C/ID model is derived empirically, it is not yet tested for predictive validity. That is a clear next step for our work.

Discussion and Conclusion

The SPC 4C/ID model was generated by mapping the instructional experiences of students who proceeded to college or university calculus to theoretical components of the 4C/ID model. Initially Wade et al., (2020) used confirmatory factor analysis (CFA) to confirm the 4C/ID model using FICSMath data. In that work, we selected instructional experiences that aligned theoretically with the 4C/ID model. By comparison, this article presents a more empirically robust mapping of the FICSMath data to components of the 4C/ID model because we used no prior assumptions. This allowed all 70 pedagogical items to be used in EFA, rendering 29 items in six factors. The EFA items, correlations of the factors, along with the theoretical implications of the components of the 4C/ID model present an empirically traceable comprehensive instructional approach that aligns with the 4C/ID model. We encourage readers to review Table 6. This is where the SPC 4C/ID model translates to teaching precalculus and calculus. Each of the four l components is accompanied by concrete descriptions of teaching practices (e.g., focus on mathematical literacy using definitions and vocabulary). The basic application of the SPC 4C/ID model is that instruction can be organized around how cognitive load theory (CLT) suggests students learn, with attention to each of the four model components. The SPC 4C/ID model does not require a radical overhaul of what mathematics teachers do—it simply provides an empirically derived model for instruction based on CLT.[1]

Our brief footnote on homework—preserved in the model as one means for Part-Task Practice—illustrates the need to understand that it takes time and practice to learn. Secondary schools whose students cannot reliably find time and space to complete homework must find ways for students to practice.

One key—perhaps *the* key—in providing equitable opportunity are *teachers*. The skills, experience, and effectiveness of mathematics teachers necessarily varies; so too will their ability to teach using a model such as the SPC 4C/ID model. Designing instruction to support learning in precalculus and calculus, and equipping students to transfer that learning into more abstract tertiary calculus, is a complex task. It is also central to the rationale for the coherent model we present.

Mathematics teachers make daily decisions about curriculum, instruction, and assessment often without substantive feedback from students, colleagues, or school administrators. They must find ways, often on their own, to promote student learning. The SPC 4C/ID presents an organized empirical model to support the learning of concepts (Support Component) first and foremost, then other components are built upon the concepts. This model can be used by mathematics teachers to examine and improve their own practices. For example, when teaching trigonometric ratios they may face pressure to make abstract ideas concrete to address the common question of "when will we ever use this?" As a result, they may seek to connect trigonometric ratios to real life applications. The SPC 4C/ID model shows that real life connections must be supported by an understanding of concepts that undergirds the learning of procedures. Rather than answering the "when will we ever use this?" question in isolation, the SPC 4C/ID model suggests that teachers instead seek answers to a more robust question: How can I present students with meaningful problem-solving and real life tasks while providing support for learning overarching mathematical concepts with appropriate attention to procedures and the development of automaticity? It is not a small question, and the answers are not simple, but we hope the SPC 4C/ID model provides a framework that successfully addresses this question.

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[1] One interesting aspect of the model from an application standpoint is the preservation of homework as an item contributing to the Part-Task Practice Component. This is one area of teaching practice where we have seen substantial disagreements among teachers over time (Hansen & Quintero, 2017), especially from an equity standpoint. It is clear to us that, however one feels about homework, the provision of practice was part of the final model. If teachers are unable to assign homework (or students are unable to complete it), students may not get the practice they need. The SPC 4C/ID model suggests that mathematics teachers (and students) still need to find time for practice. This could happen via means other than homework—such as additional class meetings during the year, or calculus courses that take multiple semesters or years to complete—but, as far as we can tell, students need practice. Other components of the model, as outlined in Table 6, may have similar policy or practice implications. We encourage readers to review the model in light of their own contexts.

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