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NOTES FROM THE FIELD

Manipulatives in a Mathematics Classroom: The Case of Frankards

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Introduction

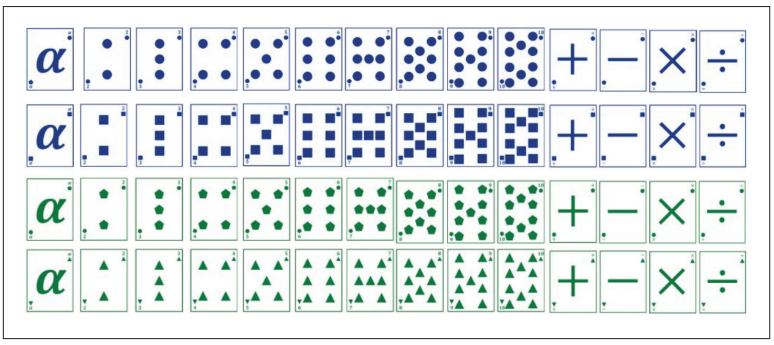
Teachers are tasked with providing opportunities that cultivate and maximize learning. A wide array of approaches can be used to achieve this goal, one of which is manipulatives. We present a teacher-developed manipulative and explain its use in the classroom. We illustrate how this manipulative can be used to teach the concepts of mutually and non-mutually exclusive events.

According to Bartolini and Martignone (2020), mathematical manipulatives are artifacts utilized in teaching mathematics. These are used to explore and investigate mathematical concepts and processes, particularly problem solving. Manipulatives can either be concrete or virtual. The *Encyclopedia of Mathematics Education* defined concrete manipulatives as "physical artifacts that can be concretely handled by students and offer a large and deep set of sensory experiences." In contrast, virtual manipulatives are "digital artifacts that resemble physical objects and can be manipulated in a similar way as their authentic, concrete counterpart" (Bartolini & Martignone, 2020, p. 365). The teacher-developed manipulative, *Frankards*, is "a set of 56 cards specifically crafted to assist teachers in teaching probability concepts through concrete manipulation" (Fran, 2021, p. 2). The deck of *Frankards*, as shown in Figure 1, is a copyrighted work in the National Library of the Philippines with registration number O2021-01.

Typically, teachers use a standard deck of cards to illustrate some ideas of probability. However, they are banned to prevent students from engaging in gambling-related activities. As a result, the first author developed *Frankards* (see Figure 1), which is more mathematical in form than the standard deck of cards. Divided into blue and green cards, *Frankards* is composed of four alpha cards, thirty-six arithmetic cards, and sixteen Multiplication-Division-Addition-Subtraction (MDAS) cards. This was designed to provide students a more engaging learning experience through concrete manipulation as they learn the mathematical concepts.

Figure 1

Deck of Frankards



Note: This figure is adapted from Fran (2021, p. 2).

Teaching and Learning with Manipulatives

Mathematics teachers have long utilized manipulatives, and some have been proven to be effective in teaching some important concepts in mathematics. Students who use manipulatives, specifically algebra tiles, performed better in algebraic operations than students who never worked with the same material (Larbi & Mavis, 2016). Salifu (2022) also suggested that algebra tiles be utilized in teaching algebraic processes such as solving linear equations. Thus, it is recommended that educators use manipulatives in teaching algebra.

Gurung and Chaudhary (2020) showed that learning with concrete manipulatives in a mathematics class results in a more meaningful learning experience for students. Similarly, concrete manipulatives encourage teachers to be more creative in teaching abstract concepts. For manipulatives to be used effectively, learners' real-life experiences must be considered (Eby, 2022). Teachers should also utilize a variety of instructional manipulatives, depending on the topic being discussed. Teachers should take note of how the lessons are designed when using manipulatives. Manipulatives provide an avenue for learners to explore and understand the topics in a meaningful and enjoyable way. This was supported by Furner and Higgins (2022), who pointed out that manipulatives aid and motivate students to learn mathematical concepts and skills.

In cases where students experience anxieties in learning mathematics, Stoehr and Olson (2021) highlighted that teachers' use of manipulatives helps reduce the anxieties through engaging learning activities. With the positive effects of using manipulatives, Kontas (2016) recommended that teachers use these instructional materials more frequently to maximize student learning. It was also noted that manipulatives are instrumental in giving meaning to abstract mathematics concepts.

Aside from helping students create their own representation of abstract mathematical concepts, manipulatives also assist learners in communicating what they have understood to their teachers and peers (Odum, 2022). As with previously studied manipulatives, *Frankards* holds promise of aiding students' comprehension when entering an unfamiliar land, in this case, one involving probabilistic concepts (Fran, 2021).

Using *Frankards* in the Mathematics Classroom

The use of *Frankards* in teaching probability has been shown to contribute to students' achievement with a significant increase in test scores (Fran, 2021). This report will discuss the applications of *Frankards* in teaching probability through illustrative examples. These problems were anchored on the learning competencies set by the Philippines' Department of Education for Grade 10 Mathematics, focusing on mutually and non-mutually exclusive events. Mutually exclusive events are events that cannot occur at the same time. On the other hand, non-mutually exclusive events can happen simultaneously. If the problems involve mutually exclusive events, equation (1) is used. On the other hand, equation (2) is used if the problem involves non-mutually exclusive events.

$$P(A \text{ or } B) = P(A) + P(B) \quad (1)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) \quad (2)$$

Example 1. Mutually Exclusive Events

Let us consider the following problem:

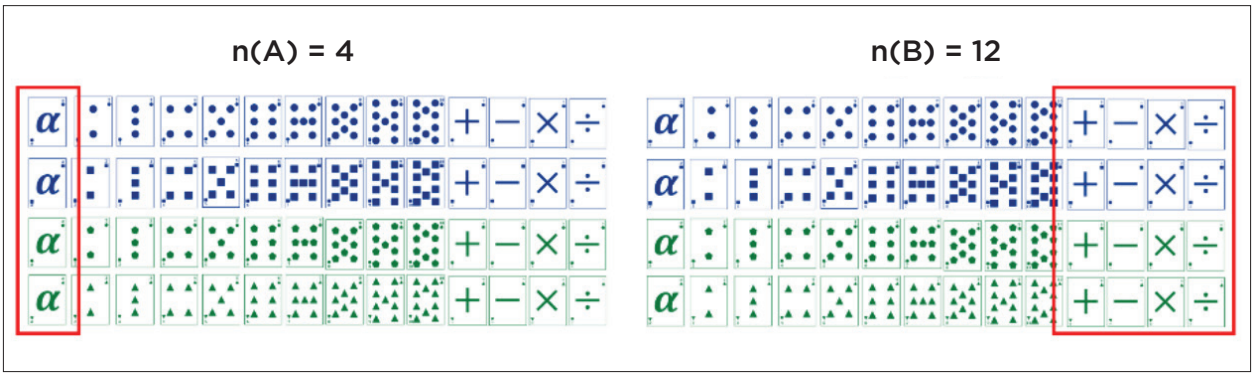
In a standard deck of Frankards, find the probability of getting an alpha or MDAS card.

Solution:

To solve this problem, let A be the alpha cards and B be the MDAS cards. Then, *Frankards* will be utilized, as shown in Figure 2.

Figure 2

Illustration for Example 1



As shown in Figure 2, getting an alpha card and an MDAS card cannot happen simultaneously. Since the problem is a mutually exclusive event, the equation to be used is (1). Hence,

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ or } B) = \frac{4}{56} + \frac{12}{56}$$

$$P(A \text{ or } B) = \frac{16}{56} \text{ or } \frac{2}{7}$$

Example 2. Non-mutually Exclusive Events

Let us consider the following problem:

In a standard deck of Frankards, find the probability of getting a blue or arithmetic card.

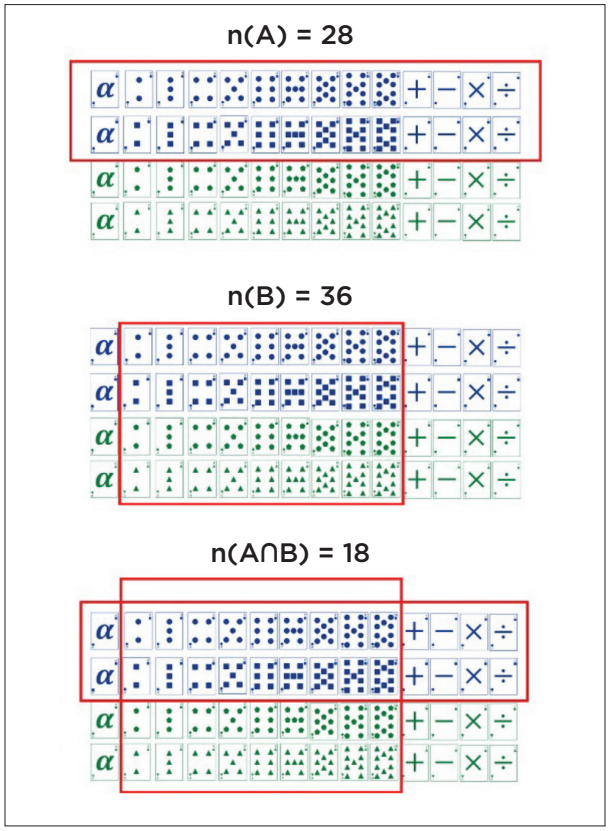
Solution:

To solve this problem, let A be the blue cards, and B be the arithmetic cards. Then, *Frankards* will be utilized, as shown in Figure 3.

As shown in Figure 3, getting a blue card and an arithmetic card shows that the problem is a non-mutually exclusive event. It can be shown that one can draw a blue arithmetic card from the deck. Hence, the equation to be used is (2). Using (2), then,

Figure 3

Illustration for Example 2



$$P(A \text{ or } B) = P(A)+P(B) - P(A \cap B)$$

$$P(A \text{ or } B) = \frac{28}{56} + \frac{36}{56} - \frac{18}{56}$$

$$P(A \text{ or } B) = \frac{46}{56} \text{ or } \frac{23}{28}$$

Feedback from Students and Teachers

The use of *Frankards* in the classroom does not only allow learners to understand the abstract concepts of mathematics. It also engages students in class activities through concrete manipulation (Fran, 2021). As mentioned by one of the students:

"*Frankards* inspired us, students, to love math and to eradicate our math anxiety."

- Student 1

When used in teaching manipulatives, *Frankards* also changes the perspectives of the learners in studying probability concepts. As noted by Student 2:

"Frankards really changed my point of view for cards because who would have thought that playing cards can be used as teaching materials."

- Student 2

Teachers also found *Frankards* to be helpful to them and, more importantly, to the students. As stated by one of the teachers:

"Their [Frankards] use will be more interesting to the students because they look more mathematical in form as the suits that were used are addition, subtraction, multiplication, and division symbols."

- Teacher 1

Generally, both teachers and students found *Frankards* useful in teaching and learning probability.

Concluding Statements

As educators, it is imperative that we provide learners with opportunities to allow learners to reach their full potential. One way is by immersing them in activities that utilize manipulatives. *Frankards*, as discussed in this short report, showcased its potential to be an effective instructional manipulative in teaching probability concepts and mastering numerical skills such as fractions. Hence, it is suggested that *Frankards* be used as instructional material to engage and motivate students in learning probability and other related concepts such as permutation and combination.

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