JOURNAL OF
MATHEMATICS EDUCATION
AT TEACHERS COLLEGE

*A Century of Leadership in Mathematics and Its Teaching*

# Beliefs and Perceptions of Learners and Teachers in Mathematics Education

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2023 | VOLUME 14, ISSUE 2



NOTES FROM THE FIELD

# Briefly Recalling the Early Teaching and Learning of Arithmetic in America: Revisiting the Influence of Colburn's *First Lessons* Two Hundred Years Since its Publication

Brian Darrow, Jr.
*Southern Connecticut State University*

**KEYWORDS** *history of mathematics education, curriculum, American education, arithmetic teaching*

## Introduction

The first mathematical subject to enter school curriculum in the United States was arithmetic, which was later followed by algebra in secondary schools and colleges in subsequent years (da Ponte & Guimarães, 2014; Kilpatrick, 2014; Kaestle, 1983). In the earliest years of the United States, exact definitions of arithmetic were not generally or formally agreed upon or stated explicitly. However, several common elementary mathematical activities, such as numeration and calculation with the four operations, were ubiquitous in early writings on the subject (Slocomb, 1831; Colburn, 1821; Bjarnadóttir, 2014; Cohen, 2016). Additional components of common arithmetic included the calculations with, and the properties of fractions, decimal numbers, proportions, measurement, and elementary accounting (Bjarnadóttir, 2014; Cohen, 2016; Jones & Coxford, 1970; Karpinski, 1940). There also seems to have been a distinction made in school curriculum at the turn of the nineteenth century between arithmetic, which was characterized by concrete quantity and calculation, and algebra, which included the consideration of unknown and variable entities.

## Early Teaching of Arithmetic in America

Consistent with the nature of education in the earliest years of the United States, the teaching of arithmetic began informally. Kaestle (1983) notes that in these early years, "Most children attended school at some time, but much education also came through the family, the church and the workplace" (p. 4) and that many others "did without schooling, remaining illiterate or picking up the three R's from parents or friends" (p. 4). The three R's (reading, writing, and arithmetic) formed the foundation of the curriculum in early informal and formal schooling in the early United States. However, information on the teaching of arithmetic has received less attention in historical texts due to the emphasis placed on developing students' ability to read and write, primarily to prepare them to engage with religious texts (Kaestle, 1983; Tyack, 1967).

The basic four operations (adding, subtracting, multiplying, and dividing) and related calculations formed the core of early arithmetic learning in America (Cohen, 2016). The teaching of more rigorous topics was generally subject to the widely varying proficiencies of individual educators, which ranged from family members, tutors, schoolteachers, to mentors (Cohen, 2016). Many students did not reach learning beyond calculations with the four operations, and most students' learning "ended with the Rule of Three" (Cohen, 2016, p. 122), which is a staple of early arithmetic teaching and refers to the solving of fractional proportions. The reasons for this ceiling of arithmetic learning include the public sentiment that arithmetic learning was "regarded as too difficult for children younger than ten or twelve to study" (Cohen, 2016, p. 118) and that mathematical learning was of little use to most of America's workers.

Although several of the founding fathers and early statesman such as Benjamin Franklin, Daniel Webster, and, perhaps most vocally, Thomas Jefferson, received instruction in arithmetic and advocated for its expansion in curriculum in the 1700's, widespread formal arithmetic instruction in schools was not realized until well into the 1800's (Cohen, 2016). However, during the early years of the United States and into the nineteenth century, formal arithmetic was rarely, if ever, taught to females and non-white males (Cohen, 2016). Females were generally excluded from mathematical learning except at the "most elementary level" (Cohen, 2016, p. 139), partly because girls did not progress in school past the first several years. Another factor was the prevailing and widely accepted notion that females could not comprehend arithmetic or other forms of mathematics (Cohen, 2016). For non-white males, the pervasive racist societal structure of the country and active institution of slavery prevented many from participating in any formalized learning whatsoever (Cohen, 2016; Kaestle, 1983).

## The Prominent Pedagogy in Early American Arithmetic

The prominent pedagogy of the informal and formal teaching of arithmetic at the turn of the nineteenth century revolved around tenets of mental discipline theory. This theory, Kliebard (2004) notes, has its roots in antiquity and is characterized by the assertion that "certain subjects of study had the power to strengthen faculties such as memory, reasoning, will and imagination" and that "certain ways of teaching these subjects could further invigorate the mind and develop these powers" (p. 4). Kliebard continues by noting the famous analogy of the mind as a muscle; and this muscle is strengthened by "vigorous exercise" (p. 4) often in the form of "monotonous drill, harsh discipline and mindless verbatim recitation" (p. 5).

Historians and mathematics educators agree that this was quite visible in the teaching of arithmetic, and that the subject was particularly vulnerable to being naturally aligned with the theory. An instantiation of mental discipline in mathematics education practice was the "rules" or "rule method" of teaching, where students were required to repeatedly memorize and apply numerical facts and procedures (Bidwell & Clason, 1970; Cohen, 2016). Cohen (2016) notes that the learning "deliberately relied on memory, not on understanding" (p. 121). One pedagogical tool that illustrates this pedagogy was the copybook or ciphering book, which were "widely used in the eighteenth century as substitutes for textbooks, ever in short supply" (p. 120). The students were dictated rules and calculations that they dutifully copied into these books with little to no attention paid to the understanding of such mathematical work.

## A Shift in Pedagogical Approach

The majority of school children experienced arithmetic under the "rule method" through the turn of the nineteenth century and well into the common school movement. Although it would take years, and in some cases decades, for schoolchildren and teachers in the United States to see it, a shift in mathematics education and research began taking place during the first fifth of the nineteenth century—one which focused on developing conceptual mathematical understanding.

Due to the delay in reaching the classroom, this shift was arguably most visible in the development of textbooks. The textbooks from the colonial period, such as the widely used *Cocker's Arithmetick* (1677), embodied the "rule method" and the associated characteristics of mental discipline (Bjarnadóttir, 2014). One of the earliest texts that contrasted this viewpoint, was Samuel Goodrich's *The Child's Arithmetic* (1818), which encouraged the use of "manipulatives" or tangible objects to be used in the development of arithmetic understanding. According to Cohen (2016), Goodrich argued "that learning by rules and rote actually prevented children from comprehending arithmetic" (p. 134).

Goodrich's text and his sentiments were significant, which were of the first to offer a consideration of pedagogical alternatives to the "rule method" which nearly completely characterized formal arithmetic teaching in the United States since the country's inception. Widely considered to be the most significant in this regard was the "inductive method" developed from the pedagogical theories of the Swiss philosopher Johann Pestalozzi and championed in America by textbook author Warren Colburn (Cohen, 2016; Kilpatrick, 2014; Karp & Furinghetti, 2016). Colburn's seminal text, *First Lessons in Arithmetic on the Plan of Pestalozzi, with Some Improvements*, was originally published in 1821, and was followed by the 1826 edition, the title of which was often adjoined with Colburn's name: Colburn's *First Lessons. Intellectual Arithmetic, Upon the Inductive Method of Instruction* (1826). The texts provided opportunities for students to discover arithmetic rules and develop

an understanding of the related concepts. Also notable was the text's emphasis on "mental arithmetic," which encouraged computation without pencil and paper, and the complete omission of other classic elements of "rule method" pedagogy such as the "Rule of Three" (Bjarnadóttir, 2014).

The historical significance of this conceptual change in pedagogy was recognized immediately. Colburn's first and subsequent texts were an "instant sensation among educators in the 1820's" (Bjarnadóttir, 2014, p. 447). Bjarnadóttir (2014) notes that this shift in pedagogy was so profound that "the vast diffusion of numerical skills in the United States from the 1820's to 1900 is owed to [Colburn's] influence" (p. 447). It is important to note that Colburn's work is viewed historically as marking "the beginning of widespread concern with pedagogy in arithmetic teaching" (Bidwell & Clason, 1970, p. 1). Cohen (2016) notes that shortly after its publication, the *North American Review* "prophesied 'We have no doubt that Mr. Colburn's book will do much to effect an important change in the common mode of teaching arithmetic" (p. 134). Additionally Bjarnadóttir (2014) noted that it "electrified educators with the startling notion that children could learn arithmetic basics even before they could read and write" (p. 447), which stood in direct contrast to what had been the fundamental educational arrangement in American schooling since its inception.

This moment in the history of mathematics education characterizes one of the first widespread considerations of how students develop understanding and how pedagogy contributes to this. Moreover, it marks a shift in the aims of mathematics education. For it had been implicitly assumed that students should understand the mathematical content of arithmetic; however, the explicit statement of this as a goal and the academic treatment of how it should be attained was groundbreakingly new (Bjarnadóttir, 2014; Cohen, 2016). Another metric of the significance of this development is the fact that despite its beginning just over two hundred years ago, it "continues to resonate in educational theory and practice in the twenty first century" (Bjarnadóttir, 2014, p. 447; Cohen, 2016). Therefore, one may trace the roots of many contemporary pedagogical debates in mathematics education to this moment in time, particularly with respect to the debate of best practices regarding the interplay between mathematical skills and procedures and the development of conceptual understanding.

## Connections to Today

In the two hundred years since Colburn's seminal text, the mathematics education community has reached a consensus that mathematics teaching which focuses solely on the memorization and the application of rules and procedures is generally bad practice. This is because tasks that focus solely on these elements of mathematics learning have been found to require low levels of cognitive demand and do not alone help students form rich connections among the mathematical concepts at hand (Stein et al., 2000). Although procedural learning should not be the sole focus or aim of mathematics learning, mathematics education research has also established that such learning is an essential component of mathematical understanding and proficiency (NCTM, 2000; NRC, 2001). Therefore, it is clear that the pedagogy of the colonial period did address some elements of mathematical learning; however, it did not contribute to the holistic development of conceptual understanding on which such a high pedagogical value has since been placed.

Pedagogy that develops mathematics knowledge through deductive logic and proof, problem solving, and discovery or inquiry-based learning tasks has been shown to require the highest levels of cognitive demand and develop rich conceptual understanding (NRC, 2001; NGA, 2010; NCTM, 2000; Stein et al., 2000). However, such learning is not always present in many American classrooms today. Most often criticized are pedagogies that still value elements of the mental disciplinarian model and do not provide such opportunities for rich conceptual learning. The reasons for why such teaching is still prevalent are numerous and complicated, but among the most common are restrictions of time, extensive curricular demands, inequitable access to instructional resources, and a resistance to changing traditional practice.

Although mathematics pedagogy has evolved substantially since the early nineteenth century, the field still wrestles with many of the issues that were present then. The impact of the mental disciplinarian approach on learning can still be seen in classrooms across America. Despite this, the last two hundred years have produced tremendous advances in the quality of mathematics teaching as well as our understanding of what it means to learn mathematics. Such a series of advances began in earnest with our forebears of the early 1800's.

## References


Bidwell, J. K. & Clason, R. G. (Eds.). (1970). *Readings in the history of mathematics education*. National Council of Teachers of Mathematics.

Bjaranadóttir, K. (2014). History of teaching arithmetic. In Karp, A. & Schubring, G. (Eds.), *Handbook on the history of mathematics education* (pp. 431-458). Springer.

Cocker, E. (1677). Cocker's *arithmetick*. London Publishing.

Cohen, P. C. (2016). *A calculating people: The spread of numeracy in early America*. Routletdge.

Colburn, W. (1821). *Arithmetic on the Plan of Pestalozzi, with Some Improvements*. Cummings and Hilliard.

Colburn, W. (1826). *Colburn's first lessons. Intellectual arithmetic, upon the inductive method of instruction*. William J. Reynolds & Co.

da Ponte, J. P. & Guimarães, H. M. (2014). Notes for a history of the teaching of algebra. In Karp, A. & Schubring, G. (Eds.), *Handbook on the history of mathematics education* (pp. 459-472). Springer.

Goodrich, S. (1818). *The child's arithmetic*. The University of Michigan Press.

Jones, P. S. & Coxford, Jr., A. F. (1970). Mathematics in the evolving schools. In National Council of Teachers of Mathematics (Eds.) *A history of mathematics education*. National Council of Teachers of Mathematics.

Karp, A. & Furinghetti (2016). *History of mathematics teaching and learning*. Springer.

Karpinski, L. C. (1940). *Bibliography of mathematical works printed in America through 1850*. University of Michigan Press.

Kaestle, C. F. (1983). *Pillars of the republic: Common schools and American society*. Hill and Wang.

Kilpatrick, J. (2014). Mathematics education in the United States and Canada. In Karp, A. & Schubring, G. (Eds.), *Handbook on the history of mathematics education* (pp. 323-334). Springer.

Kliebard, H. M. (2004). *The struggle for the American curriculum*, 1893-1958. Routledge.

[NCTM] National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Lawrence Erlbaum, National Council of Teachers of Mathematics.

[NRC] National Research Council (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: The National Academies Press.

[NGA] National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common core state standards for mathematical practice*. National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press, Reprint 2014.

Schoenfeld, A. (2013) Reflections on problem solving theory and practice. *The Mathematics Enthusiast*, 10(1 and 2).

Slocomb, W. (1831). American calculator. William Davis.

Stein M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Tyack, D. B. (Ed.). (1967). *Turning points in American educational history*. Blaisdell Publishing Company.