Beliefs and Perceptions of Learners and Teachers in Mathematics Education
How Pre-Service Teachers Define Good Mathematics Teaching and Learning

Christine M. Phelps-Gregory
Central Michigan University

Gabrielle Redfield
Western Governors University

Martha Frank
Central Michigan University

ABSTRACT Mathematics self-efficacy (beliefs about oneself as a learner of mathematics) and mathematics teaching efficacy (beliefs about oneself as a mathematics teacher) are important constructs that influence pre-service teachers’ (PTs’) learning and teaching (Bandura 1986). However, less is known qualitatively about how PTs define good teaching and learning when they make efficacy judgments such as, “I am good at learning mathematics” or “I am good at teaching mathematics.” This qualitative case study used journaling to examine 23 elementary PTs’ definitions of being good at doing and teaching mathematics. Our findings suggest PTs define being good at mathematics in a variety of ways, including receiving good grades, being fluid (quickly and successfully doing procedures), and having the ability to apply mathematics to new contexts. PTs also held a variety of definitions of good mathematics teaching, including focusing on student understanding, using group work and manipulatives, and having passion. These results have implications for researchers studying self-efficacy and teaching efficacy as well as for teacher educators hoping to engage PTs fully in their classrooms.

KEYWORDS Beliefs; mathematics self-efficacy; mathematics teaching efficacy; pre-service teachers; teacher education

Introduction

“I am bad at learning mathematics.” “I am a good mathematics teacher.” We have all heard such statements, that describe being good or bad at doing or teaching mathematics. But what do such statements mean? What do they mean to students? To researchers? To future teachers? Do these different groups hold similar definitions of good at learning, doing, and teaching mathematics?

The statements above are mathematics self-efficacy and mathematics teaching efficacy assertions. Mathematics self-efficacy is a student’s belief about their ability to do, perform, or learn mathematics (Bandura, 1986). Mathematics teaching efficacy is a teacher’s belief about their ability to teach mathematics, particularly to bring about student learning and engagement (Tschannen-Moran & Hoy, 2001). Much of the research on efficacy in mathematics learning and teaching has been quantitative (e.g., Enochs et al., 2000; Midgley et al. 2000). While valuable, such work often aggregates in a way that can lose nuance. Because of this, researchers understand much less about how students, particularly elementary mathematics pre-service teachers (PTs), define good teaching and learning when they say, “I am good at mathematics” or “I am good at teaching mathematics.” Understanding PTs’ definitions is important so teacher educators can better target their teaching to support PTs. In addition, recognizing PTs’ definitions is important to help us better understand the existing quantitative research and what PTs mean when they respond to Likert surveys on efficacy. In this paper, we present a qualitative case study that used journaling to examine 23 PTs’ definitions of being good at doing and teaching mathematics.
Conceptual Framework

This study centers on PTs’ beliefs; PTs’ beliefs are their judgments or notions about various ideas related to teaching and learning, including their beliefs about themselves as teachers and learners (Fives & Buehl, 2012; Gill & Fives, 2015). PTs’ beliefs are important because teachers’ beliefs guide their actions in teaching, influence their practice, and influence their students’ outcomes and beliefs (Buehl & Beck, 2015; Fives & Buehl, 2012; Tschannen-Moran & Hoy, 2001). PTs’ beliefs also influence their mathematical understanding and their future development as teachers (McLeod, 1994). And, importantly, teacher educators can influence PTs’ beliefs during their undergraduate classes (Fives & Buehl, 2012). We will refer to PTs’ beliefs about what it means to be good at doing or teaching mathematics as their “definitions.”

PTs’ definitions of being good at doing mathematics are related to their mathematics self-efficacy, their beliefs about their ability to do or learn mathematics (Bandura, 1986). PTs’ definitions of good mathematics teaching are related to their mathematics teaching efficacy. Mathematics teaching efficacy refers to teachers’ or PTs’ beliefs about their ability to teach mathematics (Tschannen-Moran & Hoy, 2001). For example, a PT could believe “I am good at mathematics” (high self-efficacy) and “I am good at teaching mathematics” (high teaching efficacy). However, in both statements, PTs’ definitions of good affect how we should interpret the meaning of their claims. “I am good at quickly solving procedures” is different from “I understand the main concepts” and yet both could be captured by “I am good at mathematics.”

Previous Research

Much work has been done on mathematics self-efficacy and mathematics teaching efficacy, generally showing that believing one is good at something (i.e., high mathematics self-efficacy and high teaching efficacy) leads to beneficial outcomes on effort, persistence, and achievement (e.g., MacPhee et al., 2013; Pendergast et al., 2011; Zeldin et al., 2008). High mathematics self-efficacy has been linked to higher achievement and a growth mindset, a belief that you can improve your mathematics skills (Komarraju & Nadler, 2013; Usher et al., 2019). High mathematics self-efficacy has also been linked to perseverance, grit (the ability to work to overcome obstacles), and self-regulation (Muenks et al., 2018; Tanner & Jones, 2003; Usher, 2009; Usher et al., 2019). In general, researchers agree that high self-efficacy is better for learning, though some researchers suggest that self-efficacy needs to be calibrated (that is, one should not be overly optimistic or pessimistic) (Pajares & Miller, 1994; Russell & Phelps-Gregory, 2022).

Research on elementary PTs finds they may have more mathematics anxiety, more negative attitudes, and more mathematics avoidance than other groups (e.g., Rech et al., 1993; Stoehr, 2017). This suggests that elementary PTs may have lower mathematics self-efficacy, however, little research has examined this specifically. Some research suggests both elementary and secondary PTs have high mathematics self-efficacy (Phelps, 2009; Zuya et al., 2016). Other research has found elementary PTs who take advanced mathematics classes have high mathematics self-efficacy, but those who do not take advanced mathematics classes have low self-efficacy (Xenofontos & Andrews, 2020). More work on understanding PTs’ mathematics self-efficacy beliefs is needed.

High teaching efficacy for mathematics teachers has been linked to their students’ achievement as well as their students’ own mathematics self-efficacy (Althauser, 2015; Chang, 2015). High teaching efficacy is also linked to teachers’ instructional practices, their willingness to implement challenging teaching strategies, and their beliefs in their ability to effect change in students (Bates et al., 2011; Bruce & Ross, 2008; Depaepe & König, 2018). Mathematics self-efficacy is positively correlated with mathematics teaching efficacy (Bates et al., 2011).

Most research on mathematics teaching efficacy has examined in-service teachers, with less focus on PTs. Some of the limited research has found PTs have high mathematics teaching efficacy (Zuya et al., 2016). In a qualitative study, Xenofontos and Andrews (2020) found that the majority of elementary PTs in their study expressed high mathematics teaching efficacy, stating they were confident they could fulfill their visions for teaching mathematics.

Unfortunately, previous research has often defined efficacy differently, using constructs ranging from mathematical competence to a belief in one’s ability to fulfill visions. Previous work has often been quantitative, using Likert scales to measure mathematics self-efficacy and teaching efficacy (Enochs et al., 2000; Midgley et al., 2000). These include survey items such as “I know how to teach mathematics effectively,” “I will typically be able to answer students’ questions,” or “I’m certain I can master the skills taught in class this year” (Enochs et al., 2000, pp. 200 – 201; Midgley et al., 2000, p. 19). While reliable and valid, these instruments assume PTs define
success in mathematics and mathematics teaching in the ways measured by the instrument. Quantitative studies may thus fail to capture PTs’ own definitions of teaching or being good at mathematics. To address this, our small-scale, qualitative study sought to capture PTs’ definitions using their own words. The work presented here addresses the following research question: What do elementary PTs believe it means to be good at mathematics and to be a good mathematics teacher?

**Methods**

To study the phenomenon of PTs’ beliefs about good learning and teaching in mathematics, we conducted a qualitative case study. Qualitative case studies involve the researchers studying one case, in this study a class of elementary PTs, in depth. A case study allowed us to examine PTs’ definitions of being good at mathematics and good at mathematics teaching in detail.

**Participants**

Participants in this study were elementary PTs who were majoring or minor ing in mathematics and were enrolled in an elective mathematics class for their major, which served as the case for this study. Two of the researchers served as instructors for this class. All PTs in the class were invited to participate in the study; 23 PTs agreed. Participants had previously completed one general mathematics class and at least two mathematics content courses for elementary teachers. Their responses in this study reflect those of mid-program PTs, who have completed some relevant mathematics and education classes but do not have significant teaching experiences. All participant names are pseudonyms.

**Data Collection**

This study used a journaling approach to explore PTs’ beliefs. Using journals is a common research practice because it offers an opportunity to explore past and present experiences in relation to beliefs (Bullough Jr., 2015). As part of the course, PTs were asked to write regular journal entries; the journal prompts were written collaboratively by the instructors based on research and readings from the course syllabus. This study focuses on the first two journal entries, completed during weeks one and two of the semester. Journal 1 broadly asked, “What does it mean to be a good math teacher?” Journal 2 first asked participants to name a time they were good at math and then asked, “What does it mean to be successful at learning mathematics?” Both journals also had optional sub-questions such as, “What makes being a good math teacher different from/similar to being a good teacher of another subject?” The goal of the sub-questions was to prompt PTs who might otherwise write less; however, participants were explicitly told they did not have to answer every sub-question. PTs were given the journal entries before instructors shared their own opinions (to avoid bias from instructor opinions) and PTs completed the journal entries on their own outside of class (to avoid peer opinion bias). PTs were given points for completing the journals but were not graded on their ideas, to encourage them to write freely. Journal responses were submitted electronically; each was generally one to two single-spaced pages.

**Analysis and Reliability**

Recall that our goal was to understand PTs’ definitions of being good at mathematics and teaching mathematics. We were interested in understanding what a PT meant when they said, “I am good at mathematics,” recognizing that this could mean a variety of ideas including understanding concepts, being better than peers at mathematics, or having procedural fluency. Our goal in coding was to capture PTs’ definitions of being good at mathematics and teaching mathematics.

To capture these definitions, the first author read all journal responses and then used a randomly chosen subset of the data to develop codes inductively. The use of an initial, smaller subset of data allowed us to withhold a portion of the data for checking the initial coding; after such checks, the first author coded the remaining data. Codes were created to capture PTs’ definitions, and thus we had codes such as, “good teaching means using group work,” “being good at mathematics means getting the right answer,” and, “being good at mathematics means you can help others with their work.”

We followed Campbell and colleagues’ (2013) analysis and reliability process. The units for analysis were meaning units; we unitized the data based on PTs’ main ideas. Codes could be applied to a single phrase or to several sentences to better capture participants’ meaning. To ensure reliability and to check against coding drift, the first author randomly chose three participants (13%) and stripped the codes from the analysis, leaving the unitization (to ensure consistent unitizing) (Campbell et al., 2013). The second author then coded the unitized data. The first two authors reached 80% reliability on their first coding, an acceptable level of reliability (Campbell et al., 2013).
Our findings suggest PTs define being good at mathematics in a variety of ways, including the ability to apply it to new contexts, to get good grades, and to do mathematics fluidly. PTs also showed variation in their definitions of being good at mathematics teaching with definitions including using group work and manipulatives, having passion, and focusing on understanding.

I was good at mathematics when...
Since past research has often shown some elementary PTs may have mathematics anxiety, negative math attitudes, and mathematics avoidance (Stoehr, 2017), we asked PTs to identify a time they were good at math, to help them think about what this meant to them. The majority of PTs (n = 13 PTs, 57%) responded that a time when they were good at mathematics was in algebra, with PTs citing both secondary and post-secondary algebra but with secondary algebra being the most common. Some PTs (n = 5, 22%) identified an experience in their university teacher education mathematics classes or their elementary school experiences (n = 3, 13%). Finally, one PT identified a secondary geometry course, and one said a secondary probability course.

Participants often described their positive mathematics experience in detail and connected it to their definitions of being good at mathematics. For example, Lucas wrote:

Sixth grade was when I was first introduced to algebraic equations, and it was a math concept that I really understood... I knew that I could always get the right answer with an algebraic equation given to me because I had exceptional basic math skills that always helped lead me to the solution.

Lucas went on to say that being good at mathematics meant having right answers and being fluid (quickly and successfully using procedures), both of which could be seen in his description of his positive experience.

I am good at mathematics when...
We now further explore PTs’ definitions of being good at mathematics. PTs had multiple meanings of being good at mathematics, some of which fit together and some of which were contradictory. PTs also often held multiple meanings at once, as Lucas did above.

I apply it to new contexts and situations.
The most expressed definition was that a person was good at mathematics if they could apply their learning to new problems or new contexts (n = 17, 74%). For example, Olivia said, “A successful learner can apply the concepts to different situations and can solve problems using a bank of different strategies.” Lily wrote:

There was a time when I realized I really understood a math concept. The concept was using algebra to figure out what “x” is. ... I learned this concept in a deeper way because I understood how it could be used to solve bigger problems and I understood how the smaller problems fit into this bigger concept.

One of the contexts mentioned by multiple PTs was applying mathematics to real-life contexts. For example, Charlotte express ed:

I learned it in a deeper way because I was able to relate it back to my life and things that I actually did in real life. I used algebra when I was selling pop and vegetables in high school, doing banking, managing time, you name it.

For PTs like Olivia, Lily, and Charlotte, being good at mathematics partially meant being able to use mathematics in new situations and for new problems.

I explain it to others.
Another common definition was that someone was good at mathematics when they could explain it to others (n = 16, 70%). For example, Mia said, “I can always tell how well I understand a concept by how well I can explain or teach it. If a student can do more than demonstrate a problem, and [can] actually explain why the answer is the answer and how they came on that answer then they will actually fully grasp the concept.

For PTs like Mia and Hannah, being good at mathematics partially meant being able to explain it well to others.

I get the right answers and good grades.
Some PTs (n = 13, 57%) expressed a belief that right answers and good grades showed you were good at mathematics. For example, Maya expressed, “I was enlightened when I wrote all my work out and got the right answer. That is when I decided math was my favorite subject because there is always a right answer and no what ifs.” In addition, some PTs (n = 5, 22%) described a belief that both right answers and understanding mattered. For example, Lucas wrote:
I felt that I was successful at learning in that math concept because I was good at it. In other words, I was performing well on my homework, quizzes, and tests. Based on my own experiences in learning, I believe that being successful at learning means not only performing well on classwork, quizzes, and tests but also showing confidence and understanding in the concept being taught.

However, an almost equal number of PTs (n = 11, 48%) expressed doubt that right answers or high grades alone meant you were good at mathematics. Mia said:

The grade that I receive is not always a reflection of how successful I was. I have completed many math courses and received an A without actually mastering the material because I was able to get good grades on exams by memorizing formulas and procedures.

Samantha wrote, “A student can solve every problem correctly and still not be proficient in math because he or she does not understand what each step means and why their method works.” Thus, for PTs like Avery or Lucas being good at mathematics partially meant getting the right answers and good grades. However, it also appeared as though some of the PTs doubted this definition of being good at mathematics.

I can do mathematics fluidly.
Some PTs (n = 12, 52%) also defined being good at mathematics as doing it “fluidly” or quickly and successfully, particularly for procedures. Olivia said that successful learning in mathematics is, “Fluid. If a person is successful at learning mathematics, then their ‘math actions’ are fluid.” Avery wrote, “In my elementary years I was drilled over and over with adding, subtracting, multiplying, and dividing numbers so that drilling and practice made me able to become fluent in those areas.” And Emma said, “I know I’m successful at learning mathematics when I don’t have to think about a problem before completing it and just automatically know what to do.” Thus, for PTs like Avery and Emma, being good at mathematics meant being able to do procedures and problems quickly and successfully.

I work hard at it.
Finally, some PTs (n = 6, 26%) expressed a definition that being good at mathematics is based on time, effort, and work. For example, Layla wrote, “Being successful at learning mathematics means you ... want to put the time into studying it. Without putting in the time and effort, you will not be a successful learner in any subject.” And Charlotte said, “Finally, I feel that being successful learning math means sticking with it because hard work does pay off in that subject area compared to others in my opinion.” For these PTs, being good at mathematics partially meant having the ability and willingness to study and learn.

I am a good mathematics teacher when...
PTs also had multiple meanings for being a good mathematics teacher, often holding several at once. Each definition of good mathematics teaching will be explored below.

I encourage students to work independently.
A common definition (n = 14, 61%) was that good mathematics teaching is when students are encouraged to do their own learning. Lily said, “Students best learn math when they are able to discover how to do problems in their own way.” And Anna wrote:

In a good math classroom, students are given problems that they have to figure out how to solve. They should use the tools they have been given and push themselves a little bit farther... Students learn best by doing, they can use problem-solving skills to enhance their higher-order thinking skills.

For PTs like Anna and Lily, being good at teaching mathematics meant encouraging students to problem-solve independently.

I use group work and manipulatives.
PTs also said good mathematics teaching involved the use of group work (n = 14, 61%) and manipulatives (n = 16, 70%). Charlotte said:

Because math should not simply be the teacher lecturing but rather students cooperatively learning in groups or students are teaching other students. These strategies/resources encourage students to share and find what will not work as well as what might/does work for solving the answer...

For PTs like Charlotte, being good at teaching mathematics meant encouraging students to problem-solve independently.
important in mathematics because of the nature of mathematics. Samantha wrote, “Most importantly, a great math teacher remains positive. Math is giving a ‘scary’ characteristic that remains with students throughout most of their education. Love math and show students that they too can love math!” Charlotte said, “Finally, a good math teacher differs from other subject areas by being able to make their subject fun due to the bad reputation math holds… so to combat that stereotype, teachers must try to make it fun…” For PTs like Avery and Charlotte, being good at teaching mathematics partially meant being passionate about mathematics and making your teaching interesting.

**I focus on understanding.**

PTs (n = 9, 39%) also believed that good mathematics teachers focus on understanding and did not move on if students did not understand (n = 8, 35%). For example, Mia said, “Math teachers specifically have to be good at recognizing when students are struggling and when students are ready to move on to a more challenging concept.” Harper wrote, “When I become a teacher, I want to make sure my students are all understanding the concept before I move on with the lesson.” For PTs like Mia and Harper, being good at teaching mathematics partially meant helping students understand before moving on.

**Discussion**

In previous studies, researchers often define being good at doing or teaching mathematics and then design instruments based on these definitions. In contrast, this study asked PTs about their own definitions. This flip allowed us to examine PTs’ unique definitions of mathematics self-efficacy and teaching efficacy. Our findings show PTs do define being good at mathematics teaching and learning in a variety of different ways. As a result of this finding, we suggest that, as teachers and researchers, we should not assume PTs mean the same things when they make statements like “I am good at mathematics” or “I am good at mathematics teaching.”

There are obviously several limitations to the study. First, it was a small-scale case study of a single classroom and thus the results are not generalizable. This small scale allowed us to collect detailed qualitative data but more work would be needed to see if these PTs’ definitions of good teaching and being good at mathematics were held by other groups of PTs. In addition, the study used journals to examine PTs’ beliefs. Journals allowed PTs time to think and write at home. However, they also prevented us from asking follow-up questions. Future studies using interviews that allow for follow-up questions could yield additional results.

Our findings have implications for researchers. Common quantitative scales and instruments may be measuring only one piece of mathematics self-efficacy or teaching efficacy. For example, our work found many PTs believe in the importance of group work, manipulatives, passion, and connecting mathematics to real life and define doing things like these as good mathematics teaching. However, the most commonly used mathematics teaching efficacy instrument, the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI),
rarely addresses these definitions (Enochs, 2000). While PTs’ definitions of being good at mathematics included concepts measured on instruments like the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, 1991), such as good grades, their definitions also included topics not commonly measured, such as the ability to explain mathematics or use it in new contexts and situations. These common scales and instruments may need to be adapted if it is found in larger studies that PTs and teachers define good teaching differently.

Our findings also have implications for teacher educators. PTs may have high mathematics teaching efficacy or self-efficacy but define it differently from us. Knowing this, we can make more informed instructional decisions. For example, knowing PTs may value real-life connections, we can include more of these in our instruction. By doing so, we can engage PTs more fully while also helping them build their self-efficacy and teaching efficacy.

A final implication for teacher educators is that PTs will bring these definitions of good learning and teaching with them when working with their future students. Teacher educators can build on this to help PTs construct lesson plans that further student learning. For example, since our findings suggest that PTs want to connect mathematics to real life and they believe in the importance of explanation, teacher educators could help them build lessons that do this in their student teaching and show them sources to draw on in their future teaching practice. This will hopefully translate to positive classroom experiences for their future students.

References


