NOTES FROM THE FIELD

Online Investigations of the Quadrilateral Hierarchy Using "Launch-Explore-Summarize"

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Introduction

As three mathematics educators who have been working for years within different facets of online teaching, we often wonder about transitioning face-to-face methods to online platforms (Fernández & Leszczynski, 2019; Lahiere, n.d). For us, upholding constructivist philosophies that center on "the personal construction of meaning" and the negotiation of learners' "taken-as-shared" meanings (Wood et al. 1991, p. 591) presents new challenges when participants are interacting from distinct locations, introducing additional temporal and technological concerns. Nevertheless, we appreciate the opportunities that online learning provides to busy practicing teachers pursuing continuing education (Huang & Manouchehri, 2019).

One of our favorite face-to-face teaching frameworks is "Launch, Explore, Summarize" (LES) (Shroyer, 1984), which embraces constructivist principles by supporting inquiry and investigations of mathematical ideas through classroom interactions (Michigan State University, 2018). In the first part of this paper, we describe considerations for moving a face-to-face LES lesson on creating a *quadrilateral hierarchy* (Figure 1) to an online format in two geometry classes of 24 and 25 middle-school teachers. The mathematics teacher educator (MTE) (an author) used her institution's *synchronous* conferencing platforms (Conferences and Zoom) to run the lessons. In the second part of this paper, we use the online lesson recordings to describe implementation attempts, learning opportunities, and challenges that arose in the lessons to inform others who are interested in such transitions for their own practice.

FIGURE 1

Quadrilateral Hierarchy
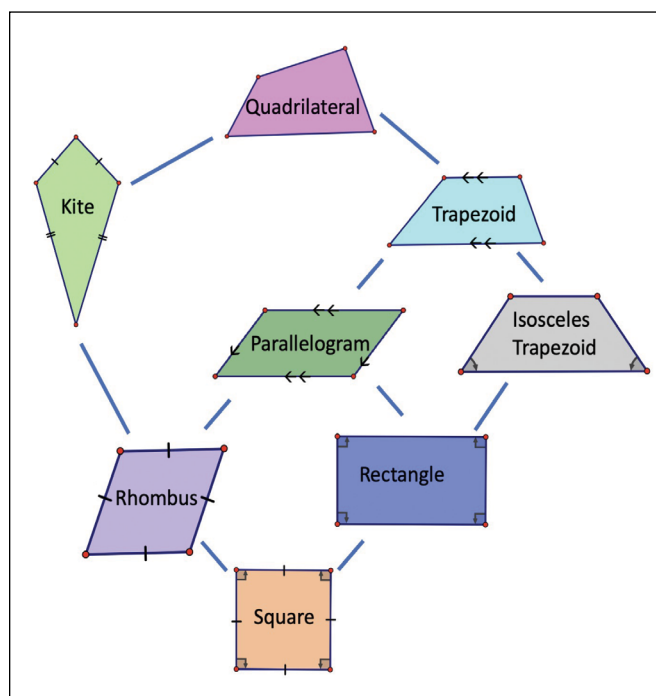
Moving the Lesson Online

Table 1 outlines the MTE's lesson transitioning, including preparation of resources. The overall idea for the lesson was for teachers to acquaint themselves with a smaller portion of the hierarchy (Launch), use that exercise as a foundation to create the general hierarchy in cooperative groups (Explore), and discuss and debate findings in a whole-class setting until a final hierarchy was reached (Summarize).

Launch

A Launch phase is initiated when an educator reviews prior ideas and presents new ones, preparing for the Explore task. In both the face-to-face and online lessons, the Launch included reviewing quadrilateral definitions and investigating a whole-class *mini-challenge* to determine whether a square is a rectangle or a rectangle a square. In the online classes, the MTE utilized screen-sharing capabilities to lead these discussions. Using a Powerpoint (PPT) slide containing a moveable rectangle and square, she employed PPT's *editing* mode to insert text reflecting teacher thinking, move shapes, and insert line segments between shapes to indicate hierarchical relationships suggested by teachers. In their respective locations, the teachers could use the definitions sheet and arrange the paper cutouts to reason

about the challenge. Depending on the conferencing platform, teachers raised their hands digitally or physically or used chat messaging to communicate (these features were also utilized in the Summarize phase).

Explore

In the Explore phase, learners interact cooperatively to investigate an activity initiated by the Launch. The educator supports these interactions by listening in, raising questions, and gathering information for the Summarize phase. Although this is readily accomplished in face-to-face settings, the conferencing programs compelled other arrangements. In the first class, the MTE's limited familiarity with Conferences' breakout rooms meant that teachers worked *individually* with paper cutouts. In the second class, the Zoom Breakout Room feature provided an analogue of face-to-face groups. The MTE supported this groupwork by preparing Jamboard slides containing moveable quadrilaterals, and instructing teachers in breakout rooms to arrange (and rearrange) shapes to create a hierarchy (some teachers preferred paper cutouts). The MTE invited teachers to elicit help in all settings, and visited some but not *all* breakout rooms.

Summarize

The Summarize phase is devoted to sharing conjectures, and organizing and generalizing findings into a culminating message. Throughout the phases, the educator is

Table 1

Moving LES from Face-to-Face to Online

	Face-to-Face	Online, Spring 2020	Online, Fall 2022
LMS & Platform Used	Classroom	Canvas & Conferences	Canvas & Zoom
Resources Provided	Paper handouts of shapes & definitions distributed on day of lesson	Powerpoint (PPT) slides of printable shapes & definitions posted to LMS in advance of lesson	PPT slides of printable shapes & definitions posted to LMS in advance of lesson; Jamboards containing moveable shapes
LAUNCH: MTE facilitates whole-class discussion on "Is a rectangle a square?" mini-challenge	MTE arranges shapes on doc cam and writes on whiteboard	MTE shares screen with a rectangle and square, rearranging shapes on a slide in PPT's editing mode	
EXPLORE: Teachers create full hierarchy using the challenge completed during the Launch phase as a model	MTE circulates the classroom; teachers work in small groups using paper shapes	Teachers arrange hierarchy with paper shapes, working individually in their respective locations	Teachers work in small breakout rooms arranging shapes in Jamboards
SUMMARIZE: MTE facilitates whole-class discussion based on work from the Explore phase, building a hierarchy as participants navigate to consensus	MTE arranges shapes on doc cam and writes on whiteboard	MTE shares screen of PPT slide containing all eight quadrilaterals, rearranging shapes on the slide in PPT's editing mode	

discouraged from "telling," and instead encouraged to guide and challenge learners as they question and justify their mathematical findings. Irrespective of format, the MTE utilized turn-taking to elicit a single quadrilateral location and the corresponding reasoning from an individual teacher or group spokesperson. As teachers provided their contributions, the MTE displayed their suggestions using a shared PPT slide with moveable quadrilaterals for both online lessons. The MTE intended the evolving shared representation as a point of reference to provoke thinking. She listened to teachers, moving quadrilaterals where they indicated, regardless of correct placement. Subsequent groups could challenge a quadrilateral's location, but the challenge needed to be justified and could be rebutted. The MTE continued the discussion until a shared hierarchy was reached.

Implementing the Lesson: Opportunities and Discoveries

The MTE noted that "nothing was lost" in moving the LES lesson online. Whether using chat or verbal communication, whole-class discussions were active, lively, and filled with naturally occurring questions, challenges, confirmations, and argumentation. During the Launch, teachers' insights provided the basis for a hierarchical representation used throughout the Explore and Summarize phases (Figure 2).

FIGURE 2

Launch Slides Co-created between MTE and Teachers
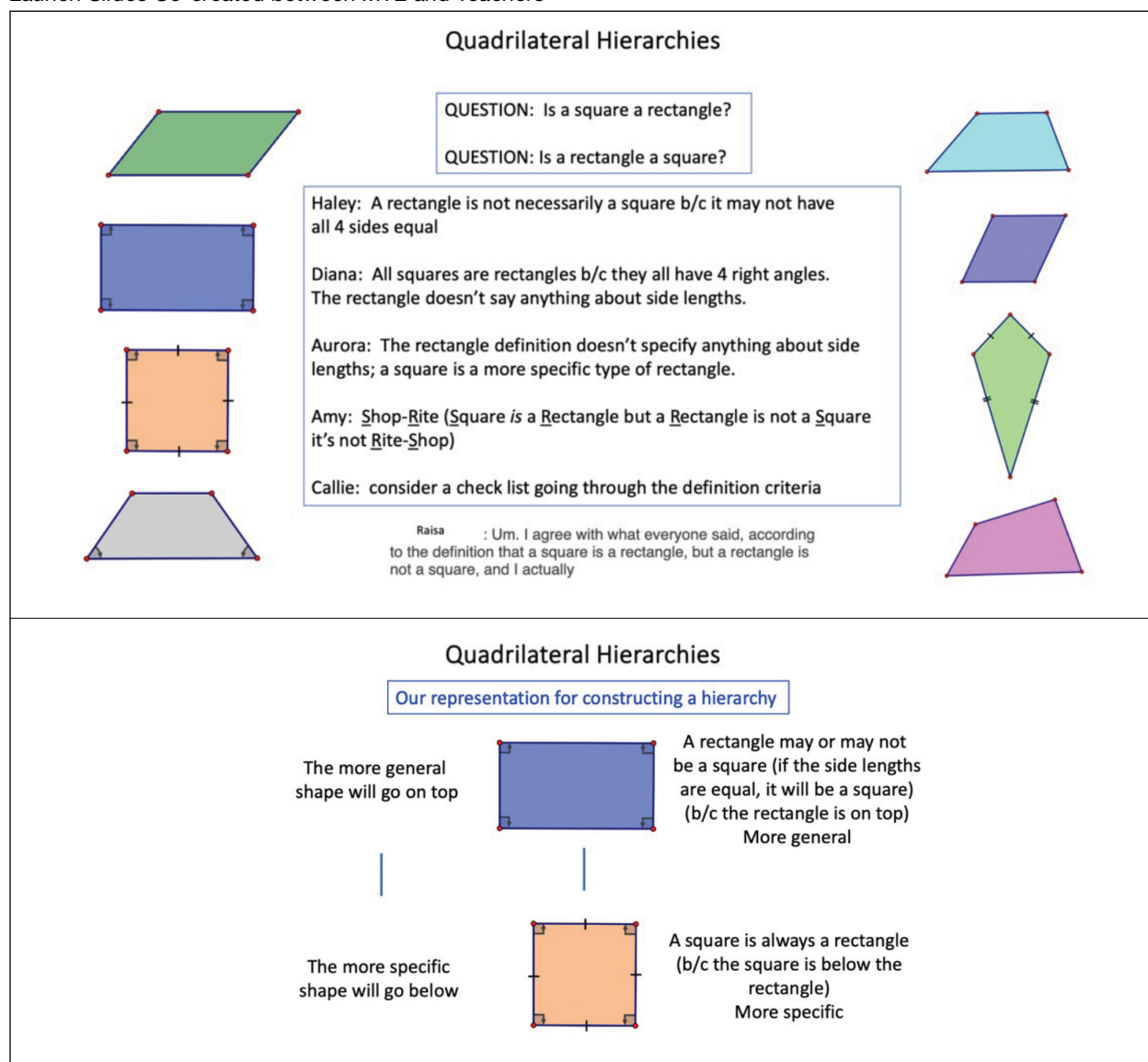
In the Exploration phase, the Jamboard activity mirrored the collaborative nature intended for group-work. For the reader wondering whether teachers were authentically engaged in breakout rooms that are hidden from the MTE's sight, Jamboard's version history function provided an accounting of teachers' activities. During the 24 minutes of groupwork, multiple teachers per group rearranged shapes and typed notes into Jamboard slides. Four out of ten groups created mostly accurate hierarchies (missing one or two shapes). Teachers transitioned between placing shapes above or below the connections they had already established (eg., placing a parallelogram above a rectangle or a rhombus below the kite). Two groups inverted the hierarchy (placing the square on top) and one group struggled with horizontal arrangements (eg., placing a rectangle and parallelogram alongside each other).
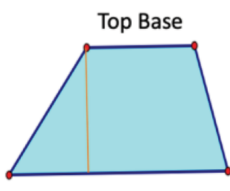
During the Summarize phase, the online discussion was as rich as any face-to-face implementation. Some teachers' contributions stemmed from the lesson design. For example, the definitions for a rhombus (four equal sides) and rectangle (four right angles) do not immediately suggest they are parallelograms. This purposeful design prompted teachers to identify missing implications as they reasoned with *only* the given definitions. In both classes, teachers questioned the rhombus-parallelogram connection, with Daniel typing, "the implications of equal sides, would be something with parallel lines, right?" In the other class, Sandra wondered aloud,

"I still don't know what, if I feel like the rhombus and the parallelogram should be connected. We connected it, but it just, it doesn't match the definition. So it's bothering me." These teachers are seeing or remembering the rhombus as a parallelogram, but realizing the need for a rigorous argument based only on the provided information. Their scrutiny reflects their mathematical prowess, and is precisely the outcome the MTE hoped for. Moreover, it provides a teacher-initiated springboard to prove that a quadrilateral's four equal sides or angles imply it is a parallelogram.

Other teachers' observations went beyond the lesson design highlighting constructivism in action and the theory's recognition of the value of unexpected questions (Wood et al, 1991). For example, Fiona raised the issue of inherited hierarchical properties when she typed into the chat, "So does the area formula for a trapezoid work for a parallelogram?" This question was a pleasant surprise for the MTE, and generated a rich discussion in which teachers proposed multiple promising methods for investigation. Fiona was able to resolve the conjecture when she noted that "bases are the same length in a parallelogram" and the MTE picked up this chat entry. If $b1 = b2$, this suggested that the trapezoid's formula $\frac{1}{2}h(b1+b2)$ becomes $\frac{1}{2}h(2b)=bh$. As teachers typed their excitement and encouragement at this discovery, Ariana courageously articulated, "I'm sorry I'm really confused." The MTE reassured her and highlighted salient points about Fiona's argument in a
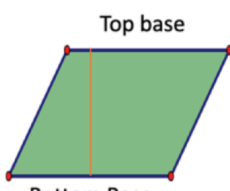
FIGURE 3

Quadrilateral Hierarchy

new PPT slide (Figure 3). Ariana's responsive chat messages during the MTE's explanation ("ahhh", "wow", "I'm very impressed") indicated her understanding.

In this online setting, the richness and value of this unexpected investigation culminated in two teachers describing a newfound belief in the hierarchy's integrity: Fiona typed, "I did not believe a parallelogram was a trapezoid and now I do" and Carmen followed with, "I feel like that further supports the hierarchy (...) I had a hard time believing it too." These observations speak to the teachers' unvoiced doubts about the relationships represented in their hierarchy, which were only discovered through this unexpected investigation.

Challenges and Conclusion

The MTE noted that challenges, like navigating streams of incoming text messages or worrying whether teachers were on-task during individual or Jamboard sessions, were comparable to other synchronous and face-to-face lessons. A novel online challenge concerned the advanced preparation and sharing of handouts to give teachers a week's notice to print (or draw) and cut shapes. The MTE further recognized the compromised collaborative experience during the first Explore session, but noted that this did not adversely affect the lesson's Summarize discussion. Moreover, with evolving technologies and familiarity, the collaborative activity was restored during her second online iteration. The MTE appreciated that technologies that typically present final products (e.g., PPT, Jamboard) can be repurposed (see Mishra & Koehler, 2006) to reflect and document evolving and responsive thinking. Such technologies, along with an appropriate activity, can serve to uphold critical LES features in online synchronous settings, suggesting the promise of LES to the work of online mathematics teacher education.

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