

NOTES FROM THE FIELD

A Reflective Homework System in Mathematics Courses

Wiktor Mogilski
Utah Valley University

Alan Parry
Utah Valley University

KEYWORDS *reflective abstraction, metacognition, formative assessment, calculus education*

Introduction

Reflection is a valuable way of thinking that is an important part of learning (Moon, 2013) and professional development (Schön, 2017). Students often write reports, compile personal development portfolios, write reflective essays/journals, etc. When completing such reflective assignments, the students must analyze what they did, how they did it, and why they did it. In this process they also think about what they learned and how their knowledge and skills developed.

The authors have taught at a total of nine higher education institutions between them. In our experience teaching mathematics at those institutions, assessments in traditional mathematics courses are typically not reflective or have no reflective component. Typical assessments consist of regular homework assignments and/or quizzes at least partially graded for correctness, chapter or midterm exams, and a final exam. All these assessments are summative in that they grade for correctness and check whether learning has already occurred (Blankman, 2024). Homework assignments are peculiar here because they occur during learning and students have very little time to acquire mastery before they receive a permanent grade. Instead, homework assignments should be formative and allow students to make mistakes and identify ways to improve without risking permanently damaging their grade (Blankman, 2022; Vatterott, 2018). Summative assessments should be reserved for their traditional role in other subjects

as exams or quizzes given at the end of a chapter, unit, course, or topic.

In order to ensure that homework assignments are formative and include a reflective component, we propose the adoption of a reflective homework system, which we will describe in detail in the next section. The homework format described can be used and adapted to many different contexts and courses as desired by the instructor. This homework system possesses a reflective component that follows the theoretical framework of reflective abstraction described by Dubinsky (1991). Reflective abstraction is the ability to arrive at new knowledge by reflecting on knowledge one already possesses without the need for additional information.

After elaborating on our rendition of a reflective homework system, we turn our attention to investigating potential student experiences with such a homework system in the realm of mathematics education. This is a good setting for such a homework system, since mathematics is perceived as one of the most difficult subjects among students, particularly those who are not mathematically inclined (Kajamies, et al., 2010). Previous research has indicated that important issues to address in mathematics education are retention (see e.g., Rohrer & Taylor, 2006) and student anxiety (see e.g., Adamu, 2014). Thus, there is a need to develop homework systems that strengthen knowledge retention and reduce student anxiety. We also think that it would help to provide a procedure that is easy for students to follow and increases student motivation in completing homework.

We sought to better understand the impact that a reflective homework system had on our students at our home institution. A more rigorous quantitative study would be needed to establish a causal relationship between the observations we made here and the homework method proposed.

To this end, we administered a reflective homework system in a number of our undergraduate calculus courses and then implemented anonymous surveys to evaluate the impact.

Specifically, the reflective homework system was administered in the undergraduate courses Calculus 1 and 3 during the Spring 2022 and Fall 2022 semesters. These courses took place at a large university in the Western United States, and each course had roughly 30 students. These courses were chosen for two reasons: (1) the curriculum structure of these courses is similar; and (2) there is a different level of mathematical experience between students in each course.

A survey was administered four times during the semester that each course was offered. While each of the four surveys was different, all four surveys had common questions that allowed the progression of student responses throughout the semester to be tracked.

Calculus 1 and 3 have a similar curriculum structure in the sense that they cover the same notions but in two different settings: Calculus 1 strictly analyzes *single-variable* functions while Calculus 3 focuses on *multivariable* functions. The curricular similarity between Calculus 1 and 3 was relevant to structuring the four surveys so that each survey contained calculus-specific questions uniformly across the two courses.

Considering different levels of mathematical experience is also relevant to this discussion. First, including experiences of students of varying levels of mathematical exposure provides valuable information to determine whether the observed effects of students the homework method were due to mathematical experience or the method itself. Second, comparing the responses of students with different levels of mathematical experience justifies considering the implementation of a reflective homework system in different levels of the undergraduate mathematics curriculum. This will be discussed in more detail later.

The Reflective Homework System

As mentioned above, the authors observed in teaching at several institutions that it seems that most assessments typically given in mathematics courses at all levels are summative assessments. Summative assessments

are high-stakes assessments that check for student learning and penalize students for falling short. These are used in most subjects as periodic checks for student competence that occur after learning (Blankman, 2024). Because of the high-stakes nature of these assessments, students are more inclined to find shortcuts or even to cheat to improve the grade they will receive on the assessment. Setting the cheating issue aside, looking for shortcuts or cramming to improve performance on a summative assessment does not seem to encourage students to reflect on or improve their understanding. As a result, in a course with only summative assessments, a student's likelihood of producing deep learning about a subject appears limited (Vicente, et al., 2021). Even worse, the high-stakes nature of these assessments tends to increase student anxiety, at least about the topic at hand and even potentially about the entire academic subject as well (Ismail, et al., 2022).

Formative assessments, on the other hand, are low- or no-stakes checks for student learning that offer opportunities for students to make mistakes without penalty, encourage students to recognize their own learning deficiencies, and provide a chance for students to improve their learning. These assessments are designed to encourage deeper learning about a subject, teach students how to identify their own weaknesses and improve, and encourage a growth mindset (National Council of Teachers of Mathematics, 2013).

In many subjects, students are assessed via a combination of formative and summative assessments. Some assessments, such as quizzes and exams, are typically summative regardless of the subject, but the homework and practice assessments are usually formative (Mogboh & Okoye, 2019). However, in traditional mathematics courses, even homework assignments are almost always summative. In such courses, a typical homework assignment consists of a list of problems for students to solve (or in more advanced settings, sometimes a list of theorems for students to prove). Students are graded for accuracy in these assignments, lose points if they get problems wrong, and typically do not have any means of improving those grades after the fact. While instructors may intend homework assignments to be practice for students, homework becomes a high-stakes performance to exhibit learning rather than a tool to induce learning when the grade is based on accuracy. This is akin to permanently deducting points from a drama student's grade if that student does not perfectly perform a part in a play after just one rehearsal. It is no wonder that many students exhibit math anxiety.

This inability for the typical mathematics homework assignments to encourage and induce student learning can be further complicated by a delay in receiving feedback if grading takes a long time. When a student submits a homework assignment, there can be a multi-day delay before they receive their graded assignment back, with not just a score, but hopefully some written feedback from the instructor for each problem the student did incorrectly. Writing feedback that is effective for student learning takes time, and high student-to-instructor ratios can yield a large number of assignments to grade, resulting in several hours of grading for each homework assignment. Depending on the complexity of the course, students may even turn in the next homework assignment before they receive feedback on the previous one. As a result, students may continue to make similar mistakes from one homework assignment to the next. Furthermore, the delay in receiving feedback means students must try to process that feedback well after they completed an assignment and have moved on to thinking about other problems. Thus, the feedback students get may not be on problems that are recent enough to be remembered clearly or as meaningful to them as when they first attempted to solve them.

Given these drawbacks to the typical mathematics homework model, we sought an alternative that would be more beneficial to mathematics students. Reflecting upon our own experiences as successful mathematics students, as well as current research on the subject (e.g. Booth, et al., 2013), we noted that one of the things that helped us was the ability to see a worked-out solution to a problem that we just attempted. This allowed us to check whether our understanding was correct and, if it was not, to search for an error and understand how we made that error. This helped us to (1) identify misconceptions that we had about how to solve a problem and correct them before they became a habit, (2) internalize the kinds of mistakes that we might be prone to, especially if we noticed a pattern, which would help us avoid those mistakes in the future, and (3) learn how to evaluate our own work and look for common errors even when there were no solutions available. We thought then that the best way to do homework in a mathematics class was to create a formative assessment that encouraged students to have a similar experience.

To facilitate that similar experience for students, homework assignments were designed in the following way. This method can be summarized as having students *Attempt, Compare, and Revise* their homework. Each part of the process is described below identified by the emphasized words in each corresponding paragraph.

First, students were provided with a list of problems to try in the form of a worksheet and instructed to *attempt* each problem on their own. They were encouraged to use whatever resources for help they needed such as their textbooks, instructor office hours, working together, tutoring, etc.

Second, students were provided a set of completely worked out solutions to each problem. These solutions were intended to be instructive and so were quite detailed in their explanation, even providing potential alternate solutions to given problems. Students were instructed to *compare* their completed first attempts to the solutions and “self-grade” their work. Students then identified any errors they made and made comments about where they went wrong. Students were encouraged to do this in a different color than their original work or on a separate piece of paper to distinguish the comments from the original work. If students found that they did the problem completely correctly, they were instructed to indicate that they did so to make it clear that they compared their correct work to the solutions.

One might argue that the solution key might not be sufficiently self-contained, and that the student might need additional information to successfully arrive at new knowledge via the process of reflection. Sometimes students may find that they cannot identify their error even though their answer is incorrect, or students may have another question about a problem to which they do not know the answer. In these cases, students were instructed to write their questions down in a different color to draw attention to the question when the instructor reviewed the work later. The instructor answered these questions directly when reviewing student work, which allowed the instructor to provide targeted feedback where it was most optimal in helping students. This element of allowing students to easily access direct feedback from the instructor either utilized reflective abstraction by reminding students of information they already learned in class but were not connecting to the problem at hand or it filled any information void that students were not able to fill themselves through this process. Moreover, instructors were able to provide this feedback while grading for completeness which only adds minimally to the time required to grade the assignment.

Students were usually provided with the list of problems and solutions at the same time. For their first attempts, students were instructed that they were not allowed to use the solutions even though they were encouraged to use any other resource. The advantage to this approach is that students can access the solutions

immediately after their first attempts allowing them to get immediate feedback while the problem is still recent enough to be remembered clearly.

Third, students were instructed to *revise* all their first attempt solutions that they found were incorrect. Revisions were required to be separate from first attempts and should have resulted in complete and correct solutions. This ensured that students reinforced the correct way to solve problems more than just simply reading a correct solution. Furthermore, this entire process ensured that students considered each homework problem deeply at least twice, and three times if their initial attempt was incorrect. This part of the system is particularly designed to strengthen retention of new knowledge and skills. It is noteworthy that there is research that supports that such a practice will help with retention: (Brown, et al., 2016) showed that students that had the correction opportunity for a mid-term exam performed better on the final exam compared to students that were just given an answer key for the mid-term exam.

Finally, students collected their first attempts, comparisons/corrections, and revisions together and submitted them for grading. Instructors responded to all questions indicated by students in their comparison comments and graded the assignments for completeness, that is, so long as the students performed all the above steps for each problem, they receive full credit for the assignment. Since students have a completed set of solutions, the need for detailed instructor feedback is significantly reduced. Instead, instructors needed only to identify that students completed the work and to

answer students' specific questions, providing feedback that was more targeted to where students needed help the most. This minimized the time from student submission to instructor response. Moreover, the fact that the homework was graded for completeness is the key to making these assignments formative and low-stakes. Students were not penalized for making mistakes or for getting answers wrong. Instead, students were rewarded for spending time learning and improving their ability. Because instructors also answered questions that students had, students received active feedback on problems that they cared about most and hence were more likely to review that feedback afterward.

Figures 1 and 2 provide an example of a student response to a homework question using this method. The first image of Figure 1 depicts an actual first attempt and comparisons/corrections from a student, and the second image is the same student's subsequent revision with the correct answer. The first image of Figure 2 is an example of an instructor-provided solution, while the second image is an example of actual student work that includes a question for the instructor to respond to.

This is not dissimilar to how many writing classes employ multiple rounds of editing and revision when assigning writing projects for students. Thus, the pedagogical idea of this method is not novel and certainly has been used for many years in non-mathematical contexts (Graham, Harris, & Hebert, 2011). However, this method *is* novel in mathematics courses and yields a new way of providing formative assessments to mathematics students.

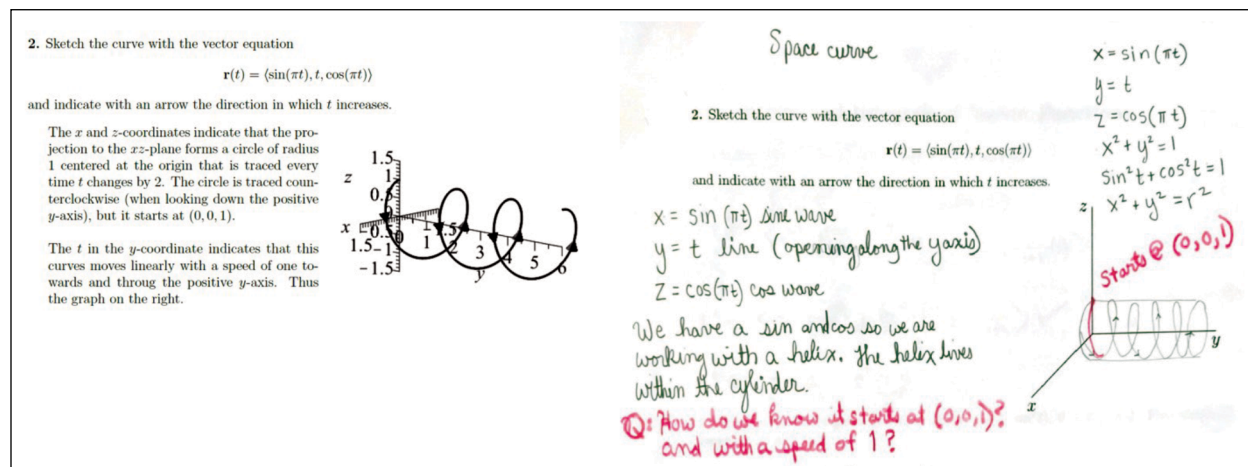
FIGURE 1

The Compare and Revise Portion

The image shows handwritten student work for a calculus problem. The problem is: "6. Find parametric equations for the tangent line to the curve at the point $(2, 4, 1)$ ". The curve is defined by $x = t^2 + 1$, $y = 4\sqrt{t}$, and $z = e^{t-1}$. The student's initial attempt shows $x' = 2t$, $y' = 4(t)^{1/2}$, and $z' = e^{t-1}(2t-1)$. They then find the tangent vector at $t=1$ as $\vec{r}'(1) = \langle 2(1), 4(1)^{1/2}, e^{1-1}(2(1)-1) \rangle = \langle 2, 4, 1 \rangle$. The point is $\vec{r}(1) = \langle 2, 4, 1 \rangle$. The parametric equations are $x = 2, y = 4, z = 0$. Corrections are written in red: "I accidentally used y instead of y' should be $4/2$ " and "should be $1(1)=1$ ". The final boxed answer is: "Parametric equations $x = 2, y = 4, z = 0$ ".

FIGURE 2

In-line Student Question



Surveys and Progress Checking

To assess student perception and experience with the homework system, anonymous surveys were implemented in the courses Calculus 1 and 3 during 2022 at a large Western open enrollment institution. In each class, the surveys were administered four times during the academic semester. The surveys had a proper disclaimer indicating that they were anonymous, optional, and did not impact student grades. The four surveys were titled: Beginning of Semester Survey, Differentiation Survey, Integration Survey, End of the Semester Survey.

Each survey consisted of 11-13 questions, all of which have the same answer options: Strongly Agree (1), Agree (2), Neutral (3), Disagree (4), Strongly Disagree (5). While each survey was different and contained specific topic-related questions, there were seven questions repeated across the surveys. The purpose of this approach is to provide longitudinal analysis: how student responses evolved over the course of the semester as the students gained more experience with and exposure to the homework system.

The following are the questions repeated across the surveys to track progress. For reference and discussion purposes, a label was assigned to each of these questions.

- (Q1: innovation) I find the homework system to be innovative.
- (Q2: learning advancement) The homework system is designed to help me better learn the material.
- (Q3: content retention) The self-grading and revision portion of the homework system will help me

retain the material.

- (Q4: feedback) The fact that I can directly ask the instructor questions within the homework assignments is helpful.
- (Q5: anxiety) The grading scheme for the homework system alleviates anxiety.
- (Q6: motivation) The homework system in this course motivates me to complete the homework assignments.
- (Q7: student success) The homework system in this class sets me up for success.

Q1 was implemented to see whether students have been exposed to a similar type of homework system before. This is important to consider, as previous exposure might impact student perceptions and survey responses. Of particular interest were new student experiences.

Q2 and Q3 were chosen to see whether students felt that they were better able to learn and retain the course material. Retention is the cognitive information processing of the learner which involves understanding, information processing, and storing within memory (Lutz & Huitt, 2018).

Q4 was incorporated to see if instructor feedback in the homework system would lead to positive student experiences. This would provide evidence that having the instructor fill any information void would help students successfully arrive at new knowledge via the process of reflection. It would also solidify the instructor's importance as part of the process.

Q5 and Q6 were crucial to the surveys as they deal with student anxiety and motivation.

Anxiety in mathematics is a well-known issue and well-researched problem. For example, Hembree states

that, “Mathematics anxiety is related to poor performance on mathematics achievement tests. It relates inversely to positive attitudes toward mathematics and is bound directly to avoidance of the subject” (Hembree, 1990). If students felt that this homework system relieved anxiety, then just that itself may justify implementing it.

Motivation in mathematics is just as well-studied. Hannula explains that “Motivation is conceptualized as a potential to direct behavior through the mechanisms that control emotion. As a potential, motivation cannot be directly observed. It is observable only as it manifests itself in affect and cognition, for example as beliefs, values, and emotional reactions” (Hannula, 2006). This suggests that a good way to observe whether the homework system impacts motivation is by including a question concerning the topic in our surveys.

Finally, Q7 was included to see if students perceived the homework system as promoting success in the course. Since typical calculus courses like these are heavy on examinations (summative assessments), success in such a course can be quantified by performing well on exams. We believe this is how students will measure their success in the courses.

In addition to the seven questions that repeated throughout the surveys, each survey contained specific questions, some of which were related to content covered in calculus. Because Calculus 1 and Calculus 3 both cover a similar curriculum in different flavors, these topics were uniform across both courses. The reason for including course content specific questions is due to the importance of calculus in a typical STEM curriculum. Calculus courses have been described as “gateway” courses in many recent calculus education research studies (e.g. Bressoud, Carlson, Mesa, & Rasmussen, 2013). A gateway course serves as a stepping-stone for students to complete their primary degree plan. Because of the obvious significance of promoting student success in calculus, content-specific questions were added to see if implementing an innovative formative assessment would help students learn the challenging topics covered in these courses. These additional questions for each survey are listed below.

Beginning of Semester Survey (Survey 1)

- I expect to do well in this class.
- The course has a balanced grading scheme between tests and homework.
- I expect this course to strengthen my mathematics computational skills.

- I expect this course to strengthen my conceptual and theoretical mathematics skills.
- I expect this course to introduce me to real-life applications of Calculus.
- The procedure in the homework system is easy to follow.

Differentiation Survey (Survey 2)

- The homework strengthened my computational skills with problems involving differentiation.
- The homework had a good balance of conceptual problems and computation problems involving differentiation.
- The homework had a sufficient amount of applications of differentiation.
- I feel confident with the differentiation portion of this course.

Integration Survey (Survey 3)

- The homework strengthened my computational skills with problems involving integration.
- The homework had a good balance of conceptual problems and computation problems involving integration.
- The homework had a sufficient amount of applications of integration.
- I feel confident with the integration portion of this course.

End of Semester Survey (Survey 4)

- I did as well as I expected to do in this course.
- This course strengthened my mathematics computational skills.
- This course strengthened my conceptual and theoretical mathematics skills.
- This course exposed me to real-life applications of Calculus.
- The homework system directly impacted my performance in this course.
- The homework system helped me perform better on the written examinations.

Many of these questions were chosen to see if the homework system was well-designed, for example, to see if the instructions for the homework were easy to follow. The supplemental questions were also used to see if students found the chosen homework problems as sufficient for the mastery of the given topic. Note that in the “End of the Semester” survey, two final questions were added to see if students perceived the homework system as positively impacting their performance in the course.

Survey Results

In this section, the results of the surveys are summarized. We start with the results of the repeated questions (Q1-Q7), which are central to this article. Recall that participants had the following answer options: Strongly Agree (1), Agree (2), Neutral (3), Disagree (4), Strongly Disagree (5).

Figure 3 shows the means of the responses to the repeated questions across all the courses in which the survey was administered. Observe that all the means are below 2.0 (Agree). This indicates a positive reception to all aspects of the homework system. Furthermore, there is a significant trend of increased agreement in the responses to Q5 and Q6 across the surveys. Figure 4 illustrates the distribution of responses to these two questions across the surveys via a box-whisker plot.

In Table 1 are listed the means of the responses to other selected survey questions. These indicate that students find the procedure of the homework system straightforward and find that the homework system benefited their course performance.

To compare and contrast the two courses, the Calculus 1 and Calculus 3 groups were separated and considered individually. This helped gauge the reception of the homework system between students of different levels of mathematical experience. Figure 5 shows the collective distribution of student responses to Q1-Q7 in each course. Note that both histograms have roughly the same shape. However, it appears that overall student perception and attitude was more positive in the Calculus 3 group.

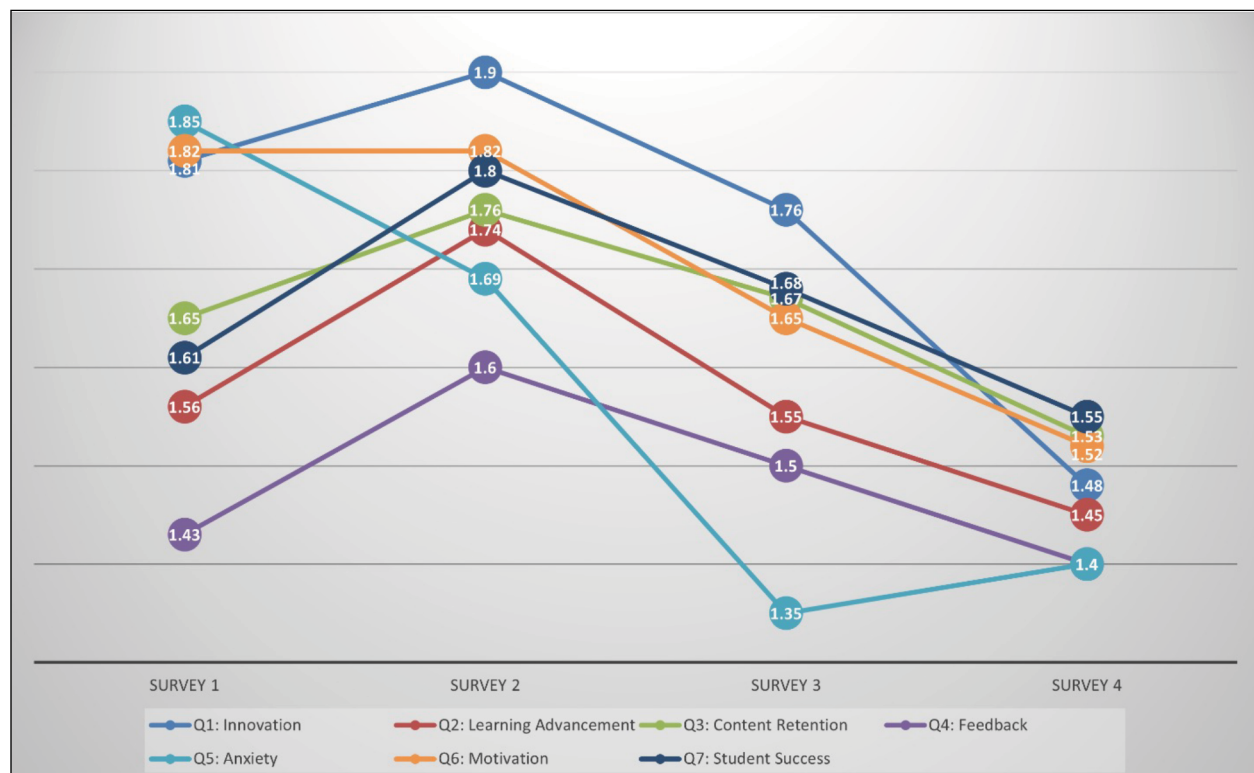
Table 1

Student Responses to Survey Questions about Homework System.

| Survey questions regarding the homework system and its procedures | Descriptive statistics |
|---|-----------------------------|
| The homework system directly impacted my performance in this course. | Mean: 1.48 Std Dev: 0.83 |
| The homework system helped me perform better on the written examinations. | Mean: 1.62 Std Dev: 0.95 |
| The procedure in the homework system is easy to follow. | Mean: 1.54 Std Dev: 0.76 |

FIGURE 3

Mean Trends of Repeated Questions



Figures 6A and 6B shows the mean trends for Q1-Q7 in both courses separately.

Note that among the Calculus 3 group, each of Q1-Q7 shows a significant trend toward increased agreement, especially near the end of the semester.

On the other hand, among the Calculus 1 group, all questions except Q5 appear to initially trend toward greater disagreement and then stabilize to roughly the same level they had originally. Thus, the trend for those questions is essentially flat. The initial trend toward greater disagreement might be explained simply by the initial shock some students experience in Calculus 1 due to it commonly being the first highly conceptual mathematics course they have taken. However, this survey does not provide enough evidence to address that claim.

Q5 in the Calculus 1 group dealt with student perceptions of their level of anxiety. This question is the only one in the Calculus 1 group where the student responses showed an overall trend toward increased agreement. This is similar to the trend observed for Q5 among the Calculus 3 group. Thus, while this homework system may not have had as pronounced of an effect in other aspects for Calculus 1 students compared to Calculus 3 students, it did lower their anxiety towards the course. As mentioned before, the reduction of anxiety on its own may make this system worthwhile, especially since it does not seem to adversely affect any of the other questions in Q1-Q7. Since Q5 shows a significant trend toward increased agreement in both courses, respective response distributions of Q5 are compared in Figure 7.

FIGURE 4
Overall Response Distribution for Q5-Q6

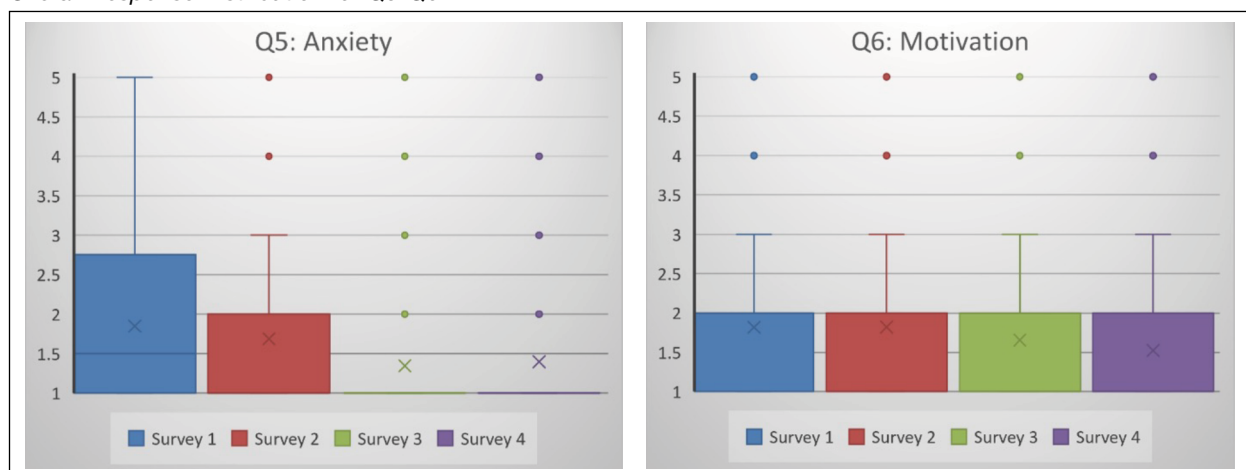


FIGURE 5
Q1-Q7 Collective Response Distribution in Calculus 1 and 3

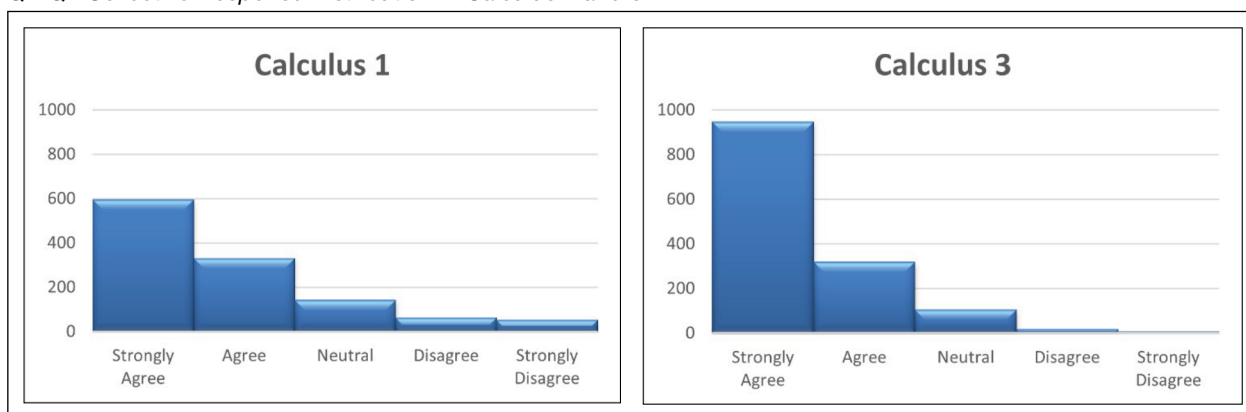


FIGURE 6A
Individual Course Survey Mean Trends

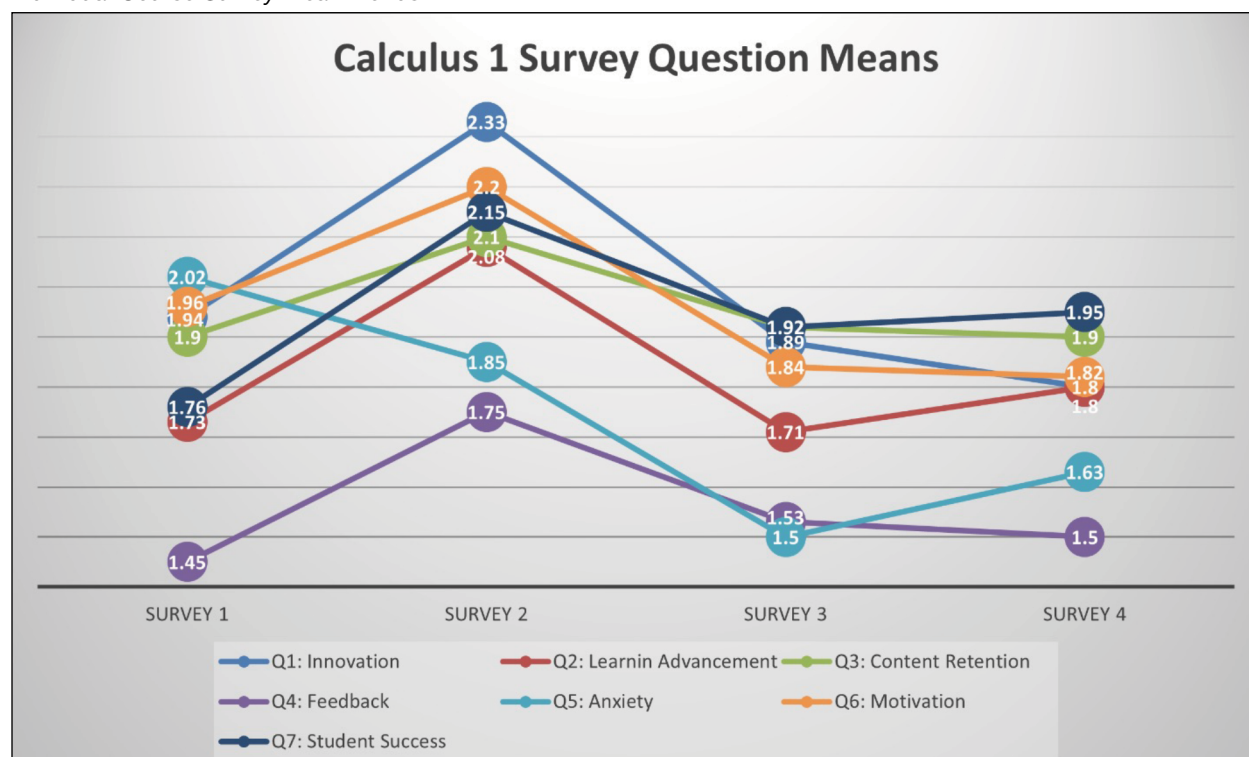


FIGURE 6B
Individual Course Survey Mean Trends

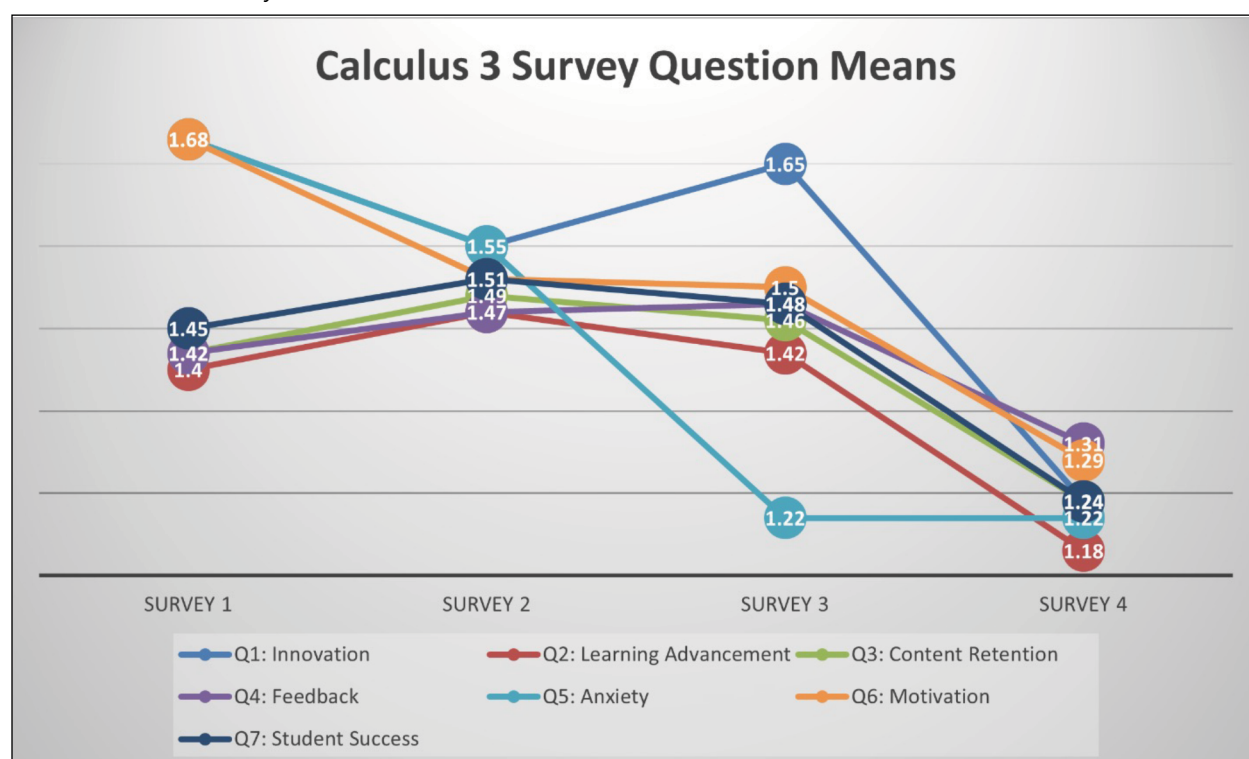
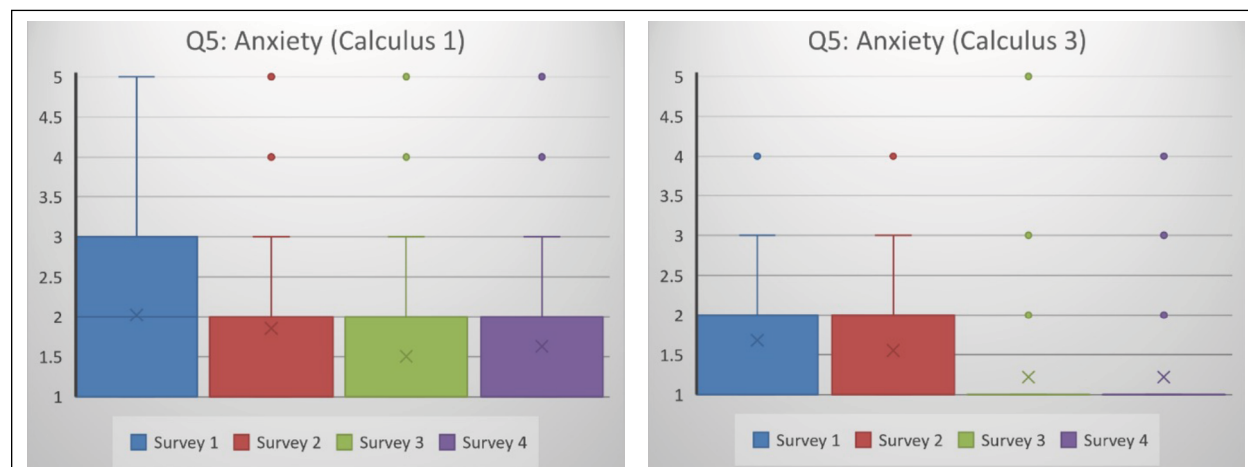


FIGURE 7
Overall Response Distribution for Q5-Q6



Conclusion

We conclude from the surveys that the reflective homework system had a generally positive impact on student attitudes in those courses, though this trend is more pronounced among the Calculus 3 group. In each of Q1-Q7, the collective (Calculus 1 and Calculus 3 combined) means of the responses in the final survey arrived at a lower number (recall that lower is better) than the initial survey, though some decreased significantly more than others. More specifically, Calculus 3 had significant trends toward increased agreement in each of the Q1-Q7, while Calculus 1 only showed a significant trend toward increased agreement for Q5. Thus, both groups experienced a significant reduction in anxiety. Overall, while the reflective homework system was well received by both groups, our survey indicates that the system had a larger impact for the more advanced students (the Calculus 3 group).

The contrast between the effects of the reflective homework system between Calculus 1 and Calculus 3 could be due to several factors. We suspect the main culprits are the curriculum itself and how the reflective homework system was utilized by students.

Curriculum-wise, Calculus 1 is often the first highly conceptual mathematics course that undergraduate students take. Thus, this course initially has an adjustment period, and students typically require a few weeks of acclimation. As noted, this might also explain some of the initial upward trends that one sees in Figure 6.

The other suspected factor is the utilization of the reflective homework system by students. A large portion of students taking Calculus 1 are freshmen/

sophomores, and hence they are still honing their study skills. This may mean that they are more prone to procrastination or are initially less comfortable with an assessment that requires more procedures to follow. Some modifications to the homework method could potentially address how students utilize it. For example, instructors could delay providing the solutions until after a first attempt is completed.

These surveys focused mainly on intangibles in student experience such as opinions on feedback, levels of anxiety, and motivation. Another question to be considered with this homework method is how it affects performance in a math class, such as exam scores or grades. While our survey included student perceptions of these, a study that better quantifies the effects of this homework method on academic performance would be an interesting next step toward understanding the impact of this homework method.

It is also an interesting question of how a similar homework method to this would function in a more advanced proof-based course. The authors utilize a similar method for those courses, but it does come with other considerations. These include the much wider variance of correct responses to a question asking students to prove a statement and the fact that seeing a solution too soon can potentially stifle the creative process needed to create a valid proof. The authors have had to make modifications to this reflective homework system to make it viable for these courses. A study similar to the one in this paper or a more detailed one that quantifies the impact of this kind of reflective homework system on those upper division courses would be useful.

While the results of these surveys do not prove that this method will produce better outcomes for students compared to more traditional homework methods, they do suggest that this homework method has a generally positive impact on student experiences, particularly in reducing anxiety and improving motivation. Given these positive results, we encourage mathematics teachers to try implementing a reflective homework system in their math classes akin to the one described in this paper.

References

- Adamu, G. S. (2014). Mathematics anxiety among engineering students and its relationship with achievement in calculus. *International Journal of Psychology and Counseling*, 6(1), 10-13.
- Blankman, R. (2022, May 9). *Math formative assessment examples*. Retrieved from Shaped: <https://www.hmhco.com/blog/math-formative-assessment-examples>.
- Blankman, R. (2024, July 12). *Math summative assessment: Explanation and examples*. Retrieved from Shaped: <https://www.hmhco.com/blog/summative-assessment-examples-in-math>.
- Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*, 25, 24-34.
- Bressoud, D. M., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The calculus student: insights from the Mathematical Association of America national study. *International Journal of Mathematical Education in Science and Technology*, 44(5), 685-698.
- Brown, B. R., Mason, A., & Singh, C. (2016). Improving performance in quantum mechanics with explicit incentives to correct mistakes. *Physical Review Physics Education Research*, 12(1), 010121.
- Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In *Advanced mathematical thinking* (pp. 95-126). Dordrecht: Springer Netherlands.
- Graham, S., Harris, K., & Hebert, M. A. (2011). *Informing writing: The benefits of formative assessment*. A Carnegie Corporation Time to Act report. Washington, DC: Alliance for Excellent Education.
- Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational studies in mathematics*, 63, 165-178.
- Hembree, Ray. "The nature, effects, and relief of mathematics anxiety." *Journal for research in mathematics education* 21, no. 1 (1990): 33-46.
- Ismail, S. M., Rahul, D. R., Patra, I., & Rezvani, E. (2022). Formative vs. summative assessment: impacts on academic motivation, attitude toward learning, test anxiety, and self-regulation skill. *Language Testing in Asia*, 12(1), 40.
- Kajamies, A., Vauras, M., & Kinnunen, R. (2010). Instructing low-achievers in mathematical word problem solving. *Scandinavian Journal of Educational Research*, 54(4), 335-355.
- Lutz, S., & Huitt, W. (2003). Information processing and memory: Theory and applications. Educational Psychology Interactive, 1, 17. Moon, Jennifer A. *Reflection in learning and professional development: Theory and practice*. Routledge, 2013.
- Mogboh, V. E., & Okoye, A. C. (2019). Formative and summative assessment: Trends and practices in basic education. *Journal of Education and Practice*, 10(27), 39-45.
- National Council of Teachers of Mathematics. (2013, July). Formative assessment: A position of the National Council of Teachers of Mathematics. Reston, VA: NCTM
- Rohrer, D., & Taylor, K. (2006). The effects of overlearning and distributed practise on the retention of mathematics knowledge. *Applied Cognitive Psychology: The Official Journal of the Society for Applied Research in Memory and Cognition*, 20(9), 1209-1224.
- Salas Vicente, F., Escuder, Á. V., Pérez Puig, M. Á., & Segovia López, F. (2021). Effect on procrastination and learning of mistakes in the design of the formative and summative assessments: a case study. *Education Sciences*, 11(8), 428.
- Schön, D. A. (2017). *The reflective practitioner: How professionals think in action*. Routledge.
- Vatterott, C. (2018). *Rethinking homework: Best practices that support diverse needs*, 2nd edition. Alexandria, VA: ASCD.