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Higher Arithmetic: A Textbook Analysis

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ABSTRACT This paper investigates the historical context, internal content, and educational significance of Higher Arithmetic, a 1919 mathematics text by David Eugene Smith and George Wentworth, which has been insufficiently studied. Through historical and content analysis, this study reveals the authors' intention to provide a review and extension of knowledge in a modern context of post-World War I industrial United States, intended for both students and pre-service teachers who need thorough teacher preparedness. Although Higher Arithmetic may not be well-recognized today, its progressive intentions for teacher training and the modernization of material remain goals that are as important today as in Smith and Wentworth's time.

KEYWORDS *Mathematics textbooks, Historical analysis, David Eugene Smith*

David Eugene Smith (1860-1944) is one of the most notable names in the history of mathematics education. During his life, he produced many works, including a significant number of textbooks, both independently, and in collaboration with his contemporaries and other peers. In his works, Smith always focused on historical perspective, modernization, and teacher preparedness, solidifying him as a leader in mathematics education. While some of his many textbooks have been studied in papers and student dissertations (Goodwin, 2012; Murray, 2012; "Notes and News," 1921), others have been less explored. In 1919, David Eugene Smith collaborated with fellow mathematics educator, George Wentworth (1868-1921), on one such underexplored textbook, *Higher Arithmetic*, published by Ginn and Company.

At the time of the textbook's publishing, the world (and the United States) was experiencing great change due to significant events in world politics and the United States economy, advancements in technology and manufacturing, and education. This paper aims to assess the historical context in which *Higher Arithmetic* was published, and to examine the contents of the text itself, to better understand its place in the progression of mathematics education. The current political climate, both nationally and internationally, consists of troubling economic realities, rising immigration numbers, and wars, which could affect education in similar ways to the climate of the early 1900s. Through a historical analysis of archival texts and an analysis of how *Higher Arithmetic* is structured for both thorough teacher preparedness and modern contextualization, we may gain a clearer picture of how to write educationally relevant textbooks today.

Historical Context

The 1890s to 1920s saw a progressive era of reforms in the United States, coinciding with a transition to industrialization and urbanization. Approaching the 1900s, the second largest economic depression in US history, the Panic of 1893, preceded the final stage of the Industrial Revolution. This brought a period of immigration, leading to an influx of children entering the public school system, and advances in industry and technology that affected

the educational system. Schools needed to teach labor specialization so workers and technicians could adapt to the shift from factories to machine-run, mass-production facilities. Early 1900s surveys on the arithmetic and practices needed in business occupations appeared (Goodwin, 2012). Smith himself emphasized omitting "obsolete" topics for modern education (Smith, 1903).

Numerous scientific innovations marked this era as well. The Wright brothers achieved powered flight in 1903, Einstein proposed his theory of Relativity in 1905, Henry Ford opened his first automobile assembly line in 1913, and the telephone (which had been patented in the late 1800s) became a more common household device. Amidst such innovation, however, the world fell into turmoil from 1914-1918 with World War I. The Child Labor Act of 1916 established the minimum working age of 14 years, extending the average time spent in school for children and teenagers when the number of students attending four-year high schools had already nearly doubled from 1890 to 1900 (Donoghue, 2006). The end of the war in 1918 saw a damaged postwar economy of disrupted trade, debt, and reparations in Europe, which would ultimately contribute to the Great Depression and economic downturn of the 1930s.

During all this time, David Eugene Smith continuously produced textbooks, motivated by his goal of modernization and his belief that effective mathematical education required better teacher preparation. Smith believed that excellent teachers needed not only rigorous content knowledge, but also pedagogical expertise, including historical, philosophical, and psychological perspectives, as well as experience teaching under supervision. Smith's texts, therefore, emphasized teacher preparation and historical context, and were often published in collaboration with other noted mathematics educators (both posthumously and with his contemporaries) such as Jekuthiel Ginsburg, Wooster Woodruff Beman, George A. Plimpton, Karl Fink, G. St. L. Carson, and George A. Wentworth (Ockerbloom, n.d.). *Higher Arithmetic* from 1919, authored by Smith and Wentworth, focuses on these areas Smith deemed so important.

Higher Arithmetic

Wentworth and Smith produced 32 unique publications (not including later editions) related to arithmetic, algebra, geometry, trigonometry, and more, written for primary, junior-high, and high-school levels, or specified for beginner, intermediate, and advanced mathematics. (Catalogs can be found through UPenn, "The online books page," and viewed at HathiTrust.) Wentworth and Smith's most recognizable textbooks include the first of the Wentworth-Smith mathematical series: *Arithmetic*—a three-book collection for an extensive elementary school arithmetic course. Considered "one of the most important efforts of recent times to produce a notably successful school arithmetical course that shall be scholarly, progressive, practical, and usable by all teachers" (Review, 1911a, p. 328), *Arithmetic* provided ample exercises so that teachers could be flexible with their selection, but would never lack practice work for students who required it. Wentworth and Smith notably omitted obsolete material and centered problems in the real-life of school children, such as farm work. The "Review" (1911a) stated, "David Eugene Smith has undoubtedly the most scholarly arithmetical equipment of anyone in America, and the Wentworth arithmetics have long been in the center of the stage, so that this combination brings into action the most complete scholarship and highly successful schoolroom methods" (p. 328). This review provides significant evidence for the importance both men held in the educational world, each revered for their work. An earlier "Review" (1909) on the first book concludes that it is the first arithmetic textbook to incorporate the best of both the old and new ideas, methods, and scientific principles. In a "Review" (1911) for their *Plane and Solid Geometry* textbook, a comparison is made to the trusted and widely-used texts of George A. Wentworth, citing a "simplicity of treatment, clearness of expression, symmetry of page" (p. 440) as the best of the old, which Wentworth and Smith combine with the new and modern.

On *Higher Arithmetic* itself, little review or analysis was found by this author. This lack of findings does not, however, preclude its popularity. While this author was unable to determine the success or amounts of use, it was recognized as part of the Wentworth-Smith Series, identified in journals of the time to be, "A series well known throughout the country for its sane pedagogy and sound service. Thousands are now using the series." (Font Matter, 1919, p. 2). *Higher Arithmetic* was mentioned one year later for its practicality and significance as a review of the fundamental principles and practices in arithmetic (Front Matter, 1920).

Smith and Wentworth

David Eugene Smith started a promising law career in 1884 when he took on a temporary mathematics teaching position. Having found a love for mathematics and

teaching, he took the role full-time, while still working in law and pursuing other educational degrees. In 1891, he became chair of mathematics at the Michigan State Normal School (i.e. teachers' college), where he first began writing textbooks with Wooster Beman, a noted mathematics educator, and innumerable papers in both mathematics education and its history. He also began building his extensive mathematics library. He took the position of chair in mathematics at Teachers College, Columbia University in 1901 before retiring in 1926 and donating his library of texts to Columbia University in 1931 (O'Connor & Robertson, 2015).

George Wentworth, born in Exeter, New Hampshire, took after his father, George A. Wentworth (1835-1906), by attending Phillips Exeter Academy as a boy and then Harvard University. G.A. Wentworth became a mathematics educator and worked at Phillips Exeter Academy as the head of the mathematics department, even serving as principal on two separate occasions, before resigning his position in 1891. G.A. Wentworth then dedicated himself full-time to the writing of approximately 50 math textbooks, many of which became the standard of the day, utilized in a vast number of schools and courses (American Council of Learned Societies [ACLS], 1936). His son, George Wentworth, left Harvard in his final year to do commercial work out west, but later returned to New Hampshire and began writing textbooks, like his father. He became the owner of his father's works upon his father's death, which would prove useful in the coming years. Wentworth and Smith officially entered a partnership, resulting in the *Wentworth-Smith Mathematical Series* and numerous other collaborations in the early 1900's. Although the official year of their partnership is not agreed upon--news sources of the time reported 1913 (Notes and News, 1921) while the *Journal of Education* wrote about the Wentworth-Smith Mathematical Series in 1911 (Review, 1911a)—their work was consistent throughout the 1910s.

Confusion in distinguishing between the two Wentworths does exist in the academic community. This is likely due to several factors: 1) George Wentworth's death in 1921 (Notes and News, 1921) is often absent from online and university websites, including the University of Pennsylvania and Teachers College libraries; 2) the distinction between father and son has been made using only a middle initial in the father's name; 3) both were notable in the mathematics education community and recognized to have collaborated with Smith (Ockerbloom, n.d.-a, Ockerbloom, n.d.-b); and 4) several examples of mistaken Wentworth attributions can be found through library searches for their publications and misinformed dissertations. Finally, although David Eugene Smith never collaborated with either Wentworth until *after* the death of G. A. Wentworth in 1906 (Ockerbloom, n.d.-a), owning his father's textbook rights made it possible for George Wentworth to give access and use of his father's work to Smith, resulting in publications (largely accessible through HathiTrust) (2024) from Smith and George Wentworth, posthumous publications from Smith and George A. Wentworth, and collaborations from all three.

The influence of all three men is evident in their collaborative works, and it is clear that a close relationship developed between Smith and Wentworth, fueled by their collaboration and mutual appreciation and respect for Wentworth's father's work. Smith became closely acquainted with Wentworth's entire family. Correspondences from both George Wentworth and his sister, Ellen Lang Wentworth, to Smith have survived through the Smith personal papers in the Columbia Rare Books Collection. Through George's correspondences, we gain a sense of his growing desire to collaborate with Smith over other authors with whom they had both collaborated previously, particularly G. St. L. Carson, a teacher of mathematics in England (Carson & Smith, 1913). Indeed, George Wentworth indicated in a letter to Smith in 1915: "To tell the truth I have much less interest in any of the books which we write with Carson than in any American book." The correspondence shows how Wentworth and Smith successfully worked together with detailed updates, frequently sent pages, 'casts' from the press, and thoughts on book structure and order. They write "send me the 1R of the Essentials and I will incorporate my changes with yours on the 1R. We ought to cast from the 2R at the latest" (Wentworth, 1915) or, when reflecting on the inclusion of a table of Logarithm Reciprocals, stating "The only reason I can see is that this table should be used for the logarithms of numbers in the denominator of a fraction– that is used in place of our American 'cologarithm.' If this was the reason for writing pages 340 and 341, why on earth do we not use this table in solving examples" (Wentworth, 1915). The subject of Carson's involvement in some of their projects becomes a point of humor on both men's parts, furthering this author's belief in their working relationship *and* in their personal, trusting friendship. Smith is quoted, "I sympathize with your desire to see Carson's [manuscripts] for the key to Algebra II. It will be here about three weeks ago and I will let you know when and that it came, whenever

this happened." (Smith, 2015) In response, Wentworth supplied, "I should also like to make a bet large or small and am willing to offer you long odds. If the Germans succeed in sinking any English mail steamers Westward bound I am willing to bet that the hold will turn out to be full of manuscripts from Carson" (Wentworth, 1915).

In Ellen's correspondence with Smith, we see evidence of Smith's appreciation for her father's work and the long-standing connection he held with the Wentworth family. Ellen Wentworth wrote to Smith in 1931, "I am very much pleased that you have recommended the name of my father in the Dictionary of American Biography," and, "I am glad that you saw Emily and her dear baby recently. Emily makes a lovely mother and we enjoy seeing them both wherever we can," referring to her niece, Emily Wentworth, the daughter of George Wentworth. The long-lived relationship with the entire family speaks to the closeness and respect Wentworth and Smith undoubtedly held for each other throughout their collaboration. As one of the final texts written by the two before Wentworth died, *Higher Arithmetic* is an excellent piece to analyze for evidence of Wentworth and Smith's relationship.

The Preface

The authors' intent for *Higher Arithmetic* is offered in the preface, written as a direct letter to the readers, in which Smith and Wentworth discuss their intended audience, prerequisite recommendations, the book's purpose, the approach for instructors of mathematics, the sequence of topics, and the overall focus of the content.

The textbook is directed toward students and teachers in "normal school" and high school mathematics and is intended for both review and extension of prior knowledge. This reflects Smith's educational desires, as normal schools, popularized during this era, were largely developed with his ideas in mind: focus on content knowledge and pedagogical understanding for future teachers (Wentworth & Smith, 1919, p. iii-iv). The authors further emphasize modernized arithmetic with some applied work relevant to day-to-day life, focusing on a thorough understanding of principles with commercial applications and only some formal definitions, rather than on memorization, drill, and technicalities of specific vocational trades. They further encourage teacher flexibility, to adapt the materials to match the needs and personal preferences of their students. No particular rules are provided for how to follow the text as the sequence of topics is considered conventional but intentionally non-binding.

The tone is informative and instructive, intent on preparing preservice teachers with knowledge beyond the elementary level and providing high school commercial students with the broad and thorough knowledge needed to enter the workforce. This includes attention to logarithms and the slide rule for the "growing demand" for arithmetic teachers and students planning to enter the business world to know these instruments (Wentworth & Smith, 1919, p. iii-iv).

The Contents

The structure of the Contents is in line with their previous works and the text's preface. The book includes twelve distinct chapters on different topics: Reading and Writing Numbers, Addition, Subtraction, Multiplication, Division, Percentage, Thrift and Investment, Mercantile Arithmetic, Corporation Arithmetic, Industrial Arithmetic, Arithmetic of the Bank, and Arithmetic of Civic Life. Each of these twelve chapters is written to house two pages of initial explanatory information on one subtopic followed by one page of exercises, then alternating single pages of written explanation with exercises for other subtopics, and finally a single page of exercises for cumulative chapter review. Consider, for example, Chapter IX: Corporate Arithmetic. This chapter holds subsections on Stocks (two pages of content, one page of exercises); Bonds, Wages, Graphs, and Circular Pictograms (each one page of content, one page of exercises); and Review of Chapters I-IX (one page of exercises). After these first twelve chapters is Chapter XIII: Advanced Problems. This chapter houses a brief explanation before it delves into seven and a half pages of advanced problem exercises. A section for Supplementary Work follows, focusing on square roots, logarithms, and the slide rule, and including written pages, tables, and exercises. The textbook concludes with a Tables for Reference section (measurement conversions) and an index. Specific topics included in each chapter can be seen in Figure 1.

This topic-related content is as reported: a review and extension of elementary work with a focus on the practical world applications, without being too technically specific to vocations. Starting with elementary topics of numbers, addition, subtraction, multiplication, division, and percentages, the concepts begin as a review, presented in the same order one might structure the elementary math curriculum to get progressively more difficult. However, within each section, the concepts move from the simple to more extended use that is average for an elementary school student.

Figure 1

Chapter subtopic content of "Higher Arithmetic," 1919

Chapter/Section	Subtopics
I. Reading and Writing Numbers	Notation, Number Names, Notation, Index Notation, RomanNumerals, Fractions, Per Cents, Ratio, Abstract and Concrete Numbers, Algebraic Notation, Review of Chapter 1
II. Addition	Integers and Decimals, Checks in Addition, Reduction of Fractions, Common Fractions, Compound Numbers, Short Methods, Algebraic Addition, Review of Chapters I and II
III. Subtraction	Methods of Subtraction, Complementary Method, Checks and Short Methods, Review of Chapters I-III
IV. Multiplication	Nature of Multiplication, Multiplication of Integers, Multiplication of Fractions, Multiplication of Decimals, Approximate Results, Finding Per Cents, Checks in Multiplication, Check of Nines, Short Methods, Compound Algebraic Numbers, Review of Chapters I-IV
V. Division	Nature of Division, Integers and Decimals, Checks in Division, Division of Fractions, Approximate Results, Short Methods in Division, Division of Compound Numbers, Algebraic Division, Review of Chapters I-V
VI. Percentage	Formulas, Formulas in Commission, Formulas in Discount, Formulas in Simple Interest, Review of Chapters I-VI
VII. Thrift and Investment	Personal Cash Accounts, Budgets, Household Accounts, Living within the Budget, Economy and Discount, Investing Money, Compound Interest, Simple and Compound Interest, Stocks and Bonds, Stocks, Other Kinds of Investment, Life Insurance, Review of Chapters I-VII
VIII. Mercantile Arithmetic	Cash Checks, Bills, Trade Discount, Invoices and Statements, Profit and Loss on Cost, Profit or Loss on Selling Price, Borrowing Money, Simple Interest, Property Interest, Review of Chapters I-VIII
IX. Corporation Arithmetic	Stock, Bonds, Wages, Graphs, Circular Pictograms, Review of Chapters I-IX
X. Industrial Arithmetic	Common Measures, Accuracy of Measurement, Plane Rectilinear Figures, Polyhedrons, Curvilinear Figures, Round Bodies, Measurement of Land, Metric System, Metric Length, Metric Area and Volume, Metric Capacity and Weight, Specific Gravity, Review of Chapters I-X
XI. Arithmetic of the Bank	Depositing Money, Drawing Money, Bank Books, Transmitting Money, Collecting by Draft, Borrowing from a Bank, Foreign Money, Review of Chapter I-XI
XII. Arithmetic of Civic Life	Tariff, State and Local Taxes, Internal Revenue, Review of Chapter I-XII
XIII. Advanced Problems	Micellaneous Problems
Supplementary Work	Advanced Theory and Practice: Square Roots, Logarithms and Logarithmic Properties, Slide Rule
Tables for Reference	Measurements
Index	

To this end, there are, as reported in other works of Wentworth and Smith, a vast number of exercises, intentionally ranging from simple to advanced. This not only expands the topics, but allows for flexible teaching, allowing a teacher to use what they need to fit the demands of their students and those students' personal needs, whether that be advanced materials only, or a lot of practice on every aspect, starting at a more beginner level. After percentages, the topics then increase in both difficulty and practicality by focusing on thrift, investment, mercantile, corporation, industry and banking arithmetics, and arithmetic for civic life. None of these go into depth on any particular vocation, rather, they are all aimed at real-world use and include expansion from simple to difficult problems.

In introducing content, the authors try to do so in a clear, concise manner. They identify important terms and principles, providing definitions (only sometimes formally), and occasionally light historical background. They do this before providing any examples or exercises to help in building the solid foundation and understanding desired. In addition, the natural progression of the book builds on notational awareness. Notation is introduced topic by topic, utilizing previously discussed notations.

Figure 2

Chapter I- Reading and Writing Numbers, Exercises 17-24.

Write in words, as on a check, the following numbers:

17. $225.	19. $1200.	21. $1950.	23. $10,500.
18. $250.	20. $1505.	22. $2775.	24. $15,708.

Figure 3

Chapter IV- Multiplication, Exercise 1 in Multiplication of Fractions.

1. Draw a line $\frac{3}{4}$ in. long, divide it into 3 equal parts, and then find the combined length of two of these parts. In other words, show that $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{1}{2}$, thus justifying the definition of the multiplication of fractions.

Figure 4

Chapter VIII- Mercantile Arithmetic, Bill Example.

JAMES McCREERY & CO.
Fifth Avenue Thirty-Fourth Street
New York

May 1, 1922

Mrs. David Dunham
501 West 120th St., City

	Account rendered		Charges		Credits			
Apr.								
7	3 doz. towels	7.50	22	50				
8	1 suit case		19	00				
	1 doz. towels				7	50		
12	1 pc. ribbon	.60						
	1 " "	.85						
	6 yd. linen 30	1.80	3	25				
			44	75	7	50	37	25

If this bill is paid by check and no further receipt is required, please detach this coupon and mail with check.
Folio 534 James McCreery & Co.

Name, Mrs. David Dunham Date, May 1, 1922
Address, 501 West 120th St. Amount, $37.25

Figure 5

Chapter VIII- Mercantile Arithmetic, Invoice Example.

THE BRONSON IMPORTING COMPANY
2487 Boston Avenue
Chicago

May 17, 1924

Sold to J. M. Bryan Co.,
La Salle, Ill.

Terms: 2/10,N/30

May	17	7 bx. Castile soap	4	20	29	40		
		750# Java coffee		30	225	00		
		16 hf.cht. Oolong tea	56	25	900	00		
							1154	40

The exercises include a focus on real-life, practical application, financial literacy, and problem-solving skills through a variety of topics, avoiding the abstract mathematical concept in favor of a practical, application-centered understanding. Some of these problems appear in a limited drill-like practice, such as Figure 2 (Wentworth & Smith, 1919, p. 5), while others are designed to build a deeper understanding of a given principle, definition, or process through their problem-solving, such as Figure 3 (Wentworth & Smith, 1919, p. 53), Figure 3 comes right after seeing $\frac{2}{3}\times\frac{3}{4}$ as an example of how to take the product of two fractions, using cancellation.

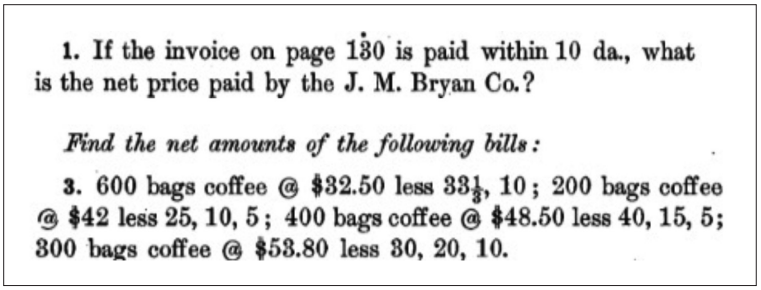
As discussed, much of this book is intended to help those who would work in the commercial and industrial world. The problems thereby reflect this in industry and sales-related financial needs, such as bills, invoices, and statements. For example, Wentworth and Smith included what the documents looked like and how they would be used before giving exercises related to the understanding of such documents. See Figure 4 (Wentworth & Smith, 1919, p. 126), Figure 5 (Wentworth & Smith, 1919, p. 130), and Figure 6 (Wentworth & Smith, 1919, p. 131).

One significant contextual event affecting this textbook is World War I, referred to in its pages as the European War. Although WWI is incorporated somewhat subtly, there are several references throughout the text to the post-war economy and the needs of the common man. For example, the large focus on financial literacy is driven by these personal needs as well as the economic and industry needs. Figure 7 is an excerpt from the beginning of Chapter VII on Thrift and Investment (Wentworth & Smith, 1919, p. 98). The topic is approached as the first of many problems that the authors explain are guaranteed to be faced by everyone and therefore must be learned and understood for their security.

In other areas, references to the war, and life during it, are embedded into problems, relating them to very real experiences for American people, as seen in Figure 8 (Wentworth & Smith, 1919, p. 111), or otherwise used as

Figure 6

Chapter VIII- Mercantile Arithmetic, Exercises 1 and 3 in Invoices and Statements.



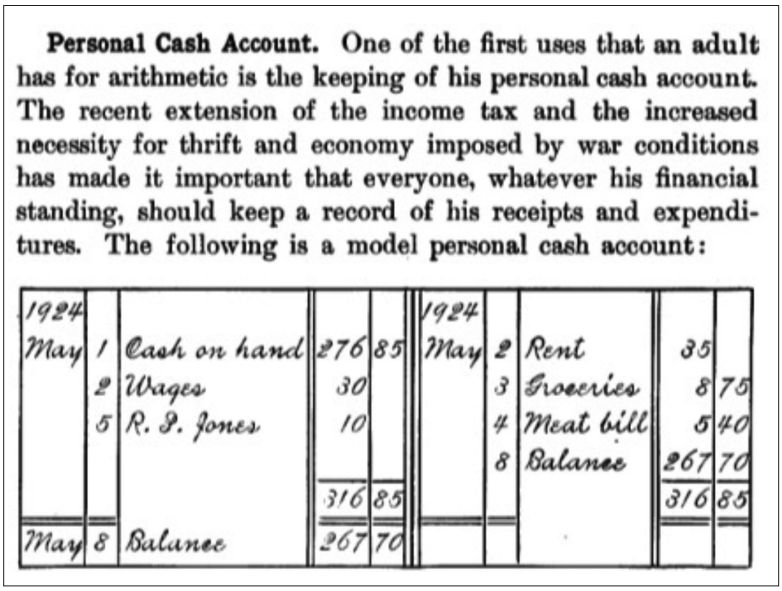
1. If the invoice on page 130 is paid within 10 da., what is the net price paid by the J. M. Bryan Co.?

Find the net amounts of the following bills:

3. 600 bags coffee @ $32.50 less 33⅓, 10; 200 bags coffee @ $42 less 25, 10, 5; 400 bags coffee @ $48.50 less 40, 15, 5; 300 bags coffee @ $53.80 less 30, 20, 10.

Figure 7

Chapter VII– Thrift and Investment, discussion on personal cash accounts.

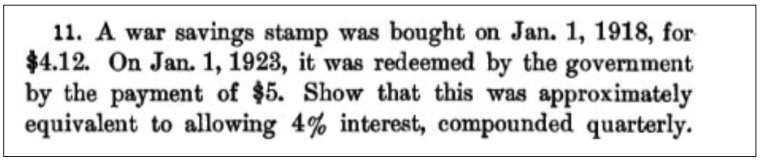


Personal Cash Account. One of the first uses that an adult has for arithmetic is the keeping of his personal cash account. The recent extension of the income tax and the increased necessity for thrift and economy imposed by war conditions has made it important that everyone, whatever his financial standing, should keep a record of his receipts and expenditures. The following is a model personal cash account:

1924					1924				
May	1	Cash on hand	276	85	May	2	Rent	35	
	2	Wages	30			3	Groceries	8	75
	5	R. S. Jones	10			4	Meat bill	5	40
						8	Balance	267	70
			316	85				316	85
May	8	Balance	267	70					

Figure 8

Chapter VII- Thrift and Investment, Exercise 11 in Compound Interest



11. A war savings stamp was bought on Jan. 1, 1918, for $4.12. On Jan. 1, 1923, it was redeemed by the government by the payment of $5. Show that this was approximately equivalent to allowing 4% interest, compounded quarterly.

driving forces behind specific subject matter, including the increased need for understanding the metric system, given a vast increase in post-war foreign trade.

Another important aspect to consider is how concepts are presented in ways that help prepare teachers. Consider Chapter III: Subtraction. The purpose of the chapter is to think about subtraction with different kinds of numbers, particularly as may be seen in the business world. Pedagogical and methodological considerations are often built into the written portions of the chapter to help guide pre-service teachers to recognize good student practices and their implications for teacher knowledge. For example, at the start of this chapter, before any content information, the authors explain that students with knowledge of any of five popular methods of subtraction in America can use that method to move forward with their elementary arithmetic, but that a teacher or *Higher Arithmetic* student will need working knowledge of all five. (Wentworth & Smith, 1919, p. 39) They then move on to language and etymology, including minuend, subtrahend, remainder, difference, rest, balance, and minus. For the average student, this may not be the most interesting, but for the teacher-in-training, it refines their language so that they speak about topics with intent and knowledgeable direction.

Before moving on to detail the five methods, Smith and Wentworth first offer a relationship, both numerically and algebraically, to explain how difference remains the same when numbers are increased by the same values (7-2=17-12=27-22=…) (Wentworth & Smith, 1919, p. 40) They also offer the method of partitioning numbers. These supply a deeper understanding of principles between numbers in subtraction which leads to deeper conceptual understanding. They are relationships that a teacher should be able to see and readily explain, and that a *Higher Arithmetic* student should be able to understand and recognize algebraically. Four of the five methods are then displayed through examples that remain simple, not broaching how to handle values of ten or higher.

The page of exercises that follows this introduction first offers five practice problems related to the methods (four practice and one pedagogical defense on a method), before offering four more problems that use the four methods but expand their use to include other numbers and units such as fractions, length metrics, money, time, subtraction of numbers with different amounts of digits, and numbers which are generally larger. This is completed with a single question on the algebraic variable notation justification/understanding (Wentworth & Smith, 1919, p. 41).

The chapter continues by introducing the fifth method, the "complementary method" which is more involved and requires new vocabulary and a full, detailed example. The subsequent page of exercises focuses on identifying a "complement" before including practice using the complementary method. These again remain simple initially but expand to include one question with numbers without the same number of digits, and one question using decimals (Wentworth & Smith, 1919, p. 42-43).

The third section of the chapter briefly introduces three concepts: checks in subtraction, a special case in which a short method can be used and what it looks like, and the first look at solving truly algebraic subtraction problems (expressions with a's and b's). The majority of the 23 practice problems are drill exercises for checks and short methods, eventually expanded to include units of length and degree, decimals, and two problems to practice algebraic subtraction (Wentworth & Smith, 1919, p. 44-45).

The chapter concludes with a page of exercises that use these concepts and expand upon them by including problems with more than two subtracted values, or by applying the topic to subtracting two Roman numeral values (which is included to show how the principles of subtraction can be applied to this particular case, and therefore likely to others as well) (Wentworth & Smith, 1919, p.46).

In general, none of the topics included in the short chapter are exceedingly difficult, and most can be used at the elementary level. However, they are presented in a way that highlights what is simple and what complicates the concepts for students. In this case, different numbers of digits, different numbers of numbers, remainders over 10 that must carry a digit, units, the sudden appearance of variables (used sparingly so as not to overwhelm), and the actual algebraic relationships that govern subtraction. While students need to learn to navigate all these things, the take-aways for teachers are the importance of the order in which you may introduce the various concepts, the language that you use, the small additions that complicate things for children and how you might slowly or limitedly introduce them, and deeper teacher understanding, so that they know how to use all the various pieces at the same time to reach all of their students.

Conclusion

Higher Arithmetic, written by Wentworth and Smith in their textbook collaborations would be one of the last textbooks written by the two authors while Wentworth was still alive. In Smith's ongoing aim for better-prepared secondary mathematics teachers, he and Wentworth set about after World War I to write a textbook that would apply to the real-world needs of the common man in a fast-developing time of industry and machinery. As more school-aged children found their future working-skills needed developing, Smith and Wentworth endeavored to create a secondary system of arithmetic that would provide the review and advancing tools they would need, while simultaneously helping their teachers to be better educators with more options and choices at hand. By all accounts, they were successful in this. They created a textbook that was flexible and adaptable for students, while informative for future teachers, rooting the content in the post-war needs of the time, with hands-on practice and a natural progression of student challenges and mathematical understanding.

So, the question remains as to why, although it was likely very successful given the popularity of all *Wentworth-Smith Series* texts, little attention seems to have been given to *Higher Arithmetic*. One possible answer lies in the prolific nature of their textbook creation. In the same year as *Higher Arithmetic*, Wentworth and Smith published School Arithmetic, which would have a secondary publishing the following year in 1920. It is possible that this publication, the last in Wentworth's lifetime, overshadowed *Higher Arithmetic*. Another possible factor is that only ten years after Wentworth's death, the United States would experience a massive economic decline, leading to the Great Depression, and calling for very different needs of the populace for some time. It is possible that *Higher Arithmetic* simply became lost in transition.

Despite this, it remains clear that Wentworth and Smith were, together, a powerful and collaborative team with many surviving texts that are, to this day, highly thought of and remembered for their progressive

intentions for teacher training and modernization of material. These goals are as important today as they were in Smith and Wentworth's time. Considering the turbulent climate we find ourselves in today, with ongoing wars and possible economic decline, a text that was well-placed for similar events is quite relevant. *Higher Arithmetic* suggests that teacher preparedness and modern relevance are critically important to both educational success and effective textbook creation.

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