

ON STUDYING OLD TEXTBOOKS

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The five papers presented to the reader's attention in this volume are the fruits of a course in the history of mathematics education given in spring 2024. Its participants focused on the analysis of old textbooks, and this analysis is represented in the papers published here. The five papers have much in common, including their similar aims and methodologies, and I will try to address these briefly below.

Why should old textbooks be studied and how can this be done? It is sometimes said that in old textbooks one can find certain techniques that could be made use of in today's classrooms. This assertion may be justified to a certain extent, but direct transfer from a different time and a different culture usually proves impossible, so to hope that the answers to today's questions lie in the past is naive.

Schubring (2022) devotes a whole chapter to discussing the questions formulated above, noting that "Textbooks constitute a major source for investigating the interfaces between research and teaching" (p. 4). Indeed, textbooks contain routine knowledge, as it were, and what such knowledge is considered to consist of specifically is telling in and of itself. It is also important to note, however, that very often textbooks turn out to be the main, and sometimes almost the only, source of our knowledge about what went on in classes in school. Of course, there are also various accounts, memoirs, letters, newspaper articles, students' notebooks, recommendations written by all kinds of administrators and supervisors, including official curricula and much else; but, first, these documents do not by any means always survive into our time, and second, all such sources usually give only a very fragmentary picture. Clements and Ellington (2012), for example, recommended students' notebooks (cyphering books) as a main source of knowledge about the school mathematics of the past, arguing that it was precisely they that preserve what was actually studied in school, while textbooks reflect only the so-called "intended curriculum." One cannot but agree that textbooks are by no means always read in school in their entirety. Nonetheless, it is specifically textbooks that lie on the teacher's desk, and since the time when they started being widely distributed, on the desk of every student as well, while students' notebooks (unquestionably an important source) nonetheless inevitably reach us only in part—usually, we do not even know how many notebooks students had. It is a different matter that, as we will discuss below, one must not confine oneself to the analysis of textbooks alone. In order to understand how mathematics was taught in the past and why it was taught the way it was, one must make use of the most varied sources.

But at this point, we might ask: what's the point of knowing how people taught previously? This question is a rhetorical one, since everyone always makes references to the past—it is hardly possible not to—but nonetheless, we should say that without understanding the past, it is impossible to understand the present. A person who sincerely thinks that it is only now, for instance, that people have started talking about the need to study geometric transformations turns away from all experience, successful and unsuccessful, accumulated by humanity in studying this topic; such a person considers what happens today as something isolated and separate, not as part of a long and complicated process, and this is hardly a way to accomplish anything.

But once we have agreed that researching the mathematics education of the past in general and that focusing on textbooks as a means and object of such research in particular is a useful task, we must address the methodology of such research, especially since, although the study of old textbooks began very long ago (the first dissertation in mathematics education defended in the United States, Stamper, 1906, was at least partly devoted to the analysis of textbooks), the methodology of such research is still in its formative stages.

It should be said that mathematics textbooks have lately been recognized as such an important aspect of education that special conferences are devoted to them (the first was held in 2014, see Jones et al., 2014). Studies of textbooks may focus on the most varied topics, from the relative numbers of pages allocated to different sections, to characterizations of the figures that appear in a textbook. The study of historical textbooks becomes substantive when it is connected with temporal changes and social history.

Schubring (1987) proposed analyzing textbooks in terms of three dimensions, the first of which “consists in analyzing the changes within the various editions of one textbook,” the second in “finding corresponding changes in other textbooks belonging to the same” group, and the third of which “relates the changes in the textbooks to changes in the context (changes in the syllabus, ministerial decrees, ...etc.)” (p. 45). In all three dimensions, the factor of time—of processes that took place in the past—is emphasized.

The second dimension, it seems to us, would also include the simple comparison of textbooks of different generations from one angle or another. As examples of such studies, I would cite my own papers Karp (2015; 2023), which analyze problem sets from Russian and American textbooks of different periods, making it possible to observe transformations in the understanding of what the actual outcome of education was supposed to be, and consequently, of the very process of teaching, the recognition of which in turn allows us to ask what it was that brought these transformations about.

We should point out here that the historical method used in historical research, including research in mathematics or mathematics education, is based on juxtaposing and comparing documents, not statistical data. Of course, if it is discovered that, for example, until a certain time there were tens of times fewer computation problems in geometry textbooks than there are in textbooks today, such numerical findings are useful and meaningful. But usually, studies of this kind involve less computing than determining which solutions to problems students were intended to find, comparing problems with the theoretical texts in the textbook, and so on. Such methods of analysis, in our view, can be quite rigorous and well-grounded.

Studying a single textbook and even only a single edition of it, although inevitably poorer than studying a group of textbooks, nonetheless is also possible and useful. It is only important, as we have already said, not to remain confined to what is printed in the textbook. The textbook was written for a specific group of learners, under specific political circumstances, in light of a specific labor market, in the context of a specific philosophy and tradition of education, a specific technical basis for publication, and finally, by specific individuals with their specific characteristics. All of this forms what Schubring calls “context,” and it is precisely the textbook’s relation to this context that must constitute a part of its study (given that the objective is something broader than simply a short resume of a forgotten volume—which might be helpful to researchers, but can hardly be considered a study on its own).

How, then, should the study of the context and the textbook itself be related? Once again, Schubring (1987), noting the complexity and multidimensionality of the processes connected with the writing and use of textbooks, urged researchers to study “units” in which various aspects or dimensions interact. One such “unit” might be the life of the author of a textbook, his book, and its distribution. Such an approach can reveal the author’s interaction with the textbook market, with those who make use of the textbook, and with those who approve and finance the textbook. But other unifying ideas are also possible, and indeed, the text of a textbook itself suggests a specific “unit”—how and why it was written the way it was, and what this was connected to.

The first thing one must determine in studying a textbook is what exactly one wishes to find out and understand. Then, one must establish the circumstances under which it was written. This requires, on the one hand, a general historical study of the time when it was written, and on the other hand, if possible, of the author’s biography—for which it turns out to be expedient to work with various dictionaries and encyclopedias, search for other publications from the author’s time, and analyze archival materials and various currently offered databases. But the main part of the investigation consists in the critical reading of the textbook itself and the selection of topics, sections, and other features that are in one way or another connected with the questions that have been posed.

It should be said at once that such a connection may be anything but simple. In one of the papers published below, Christine Gao set herself the task of studying how Nazi ideology was expressed in textbooks of the time. Evidently, it had to manifest itself in “general” sections – sections on the history of mathematics and its significance, or in the descriptions of a textbook’s aims. Naturally, it could also manifest itself in word problems. Siegmund-Schultze (n.d.) cites a monstrous problem in the process of solving which schoolchildren had to reach the conclusion that a great deal of money is spent to make the lives of criminals and the mentally ill more comfortable, which could be used to improve the lives of those who are healthy and law-abiding. But even word problems do not tell the whole story: changes in the curriculum may also reflect changes in ideology. In this respect, a mathematics textbook, of course, is in somewhat lesser danger than, say, a history textbook, from which one can simply exclude certain sections, while, on the other hand, adding chapters that are politically expedient to the government. A course in mathematics is relatively stable. And simply to deny the facts, as this is done in history textbooks, is also not possible here. But to expand one section or another, to add material where it seems to be useful to the regime, is possible, and such changes must also be taken into account.

Let us repeat that reading with close attention, not only to the content of what is presented in the textbook, but also to the language of the presentation, to the structure of the text, to how the theoretical material is laid out and to what kind of problems are offered to the students, to the intended organization of the textbook’s use, and to many other parameters—all carried out against the background of the study of the textbook’s context—is precisely what constitutes the main method for studying historical textbooks. It should be remarked that such a method might appear imprecise to someone trained in doing statistical research, but it is just such a methodology or one like it that is used to conduct research in numerous disciplines.

Perhaps the most natural thing to recall in this connection is research in literary criticism, in which what is studied is some specific work of literature. As an example of successful analysis, let me cite Vygotsky (1971), who demonstrates that the order of events as they are presented in a short story that he analyzes by no means coincides with the order in which these events are actually supposed to have happened, and that by this means a certain effect is achieved. There exists a vast literature, for example, about the use of specific aspects of language, about what is achieved through the use of “special” language, for example, common colloquial language, or on the contrary, elevated literary language. Without adding any more examples, let us say that the text of a mathematics textbook, although it does not claim to be a work of literature, nonetheless also reveals a great deal to an attentive reading.

Text and context—these are the main objects of analysis in the study of historical textbooks. Let us say a few words about how these general principles are put into practice in the papers published here. Christine Gao’s paper has already been mentioned; we would just add that mathematics textbooks from Nazi Germany have not been sufficiently studied, and literature about them in English is altogether scarce.

Three other papers are also devoted to non-American textbooks; or more precisely, two, since Wilson’s paper is devoted to a periodical—although this periodical, too, was published in Romania, at the end of the nineteenth century, in order to carry out certain functions that were not sufficiently met by ordinary education, in the opinion of the publishers. Nonetheless, from a methodological point of view, although the study of this periodical has features in common with the study of textbooks, it still differs in many respects, and the author discusses these differences.

The paper by Daria Chudnovsky offers parallel analyses of the text of a textbook and its context, namely, the biography of its author, the famous Russian mathematician Nikolai Bugayev, and his overall philosophical views. Omar Faruque’s paper, more descriptive in character, is devoted to a popular Bengali arithmetic textbook written at the end of the nineteenth century. In view of the fact that not enough is known about the teaching of mathematics in European colonies in general and in India in particular, the relatively detailed analysis of this textbook appears quite useful.

One more paper is devoted to American textbooks. Moroney analyzes a textbook in arithmetic, one of whose authors was David Eugene Smith, the founder of the Program in Mathematics at Teachers College. This analysis is supplemented by archival findings: the surviving correspondence between Smith and his coauthor. But the very description of this textbook by itself helps us to understand much about the mathematics education of that time.

This last remark, as it seems to us, applies to the other papers as well. Of course, they do not exhaust the problems they pose, and the studies initiated here may be continued, expanded, and deepened, but a definite step forward has been taken, and people who are interested in the history of mathematics education—and this subject should be of interest to all mathematics educators—will be interested to find out about what is new and little known.

And here yet another, perhaps most important, observation must be made. The study of historical textbooks must be continued. I know of only one book (the already cited Schubring, 2022) that is entirely devoted to historical mathematics textbooks. Meanwhile, an enormous number of textbooks remain unanalyzed, and while in the United States and certain other countries lists of mathematics textbooks (of different degrees of completeness) have at least been compiled, in many countries even this is lacking, and one might conclude that in the “old days” people did not study at all, or studied very badly. By no means proposing to borrow anything from the past, let us say once again that by studying old textbooks, we can better understand what role mathematics played in the life of people in the society of their time, how that life and the attitude toward mathematics developed and changed, how the conception of what should be taught and how it should be taught became transformed, and how methodological and mathematical ideas spread through the world, influencing other countries. There is much room here for new and useful studies. We would like to hope that such studies will be continued.

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