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NOTES FROM THE FIELD

Comparing and Contrasting Secondary and Undergraduate Precalculus Teaching

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ABSTRACT Precalculus is a course that bridges the divide between early secondary mathematics and early tertiary mathematics. As such, it exists in both high schools and colleges, taking different forms depending on the curricular and pedagogical requirements of each institution and student body. Each precalculus class, secondary and tertiary, can benefit from the other's instructional methods. To begin this exchange of ideas, two perspectives of precalculus teaching are presented and contrasted: one from a high school teacher and one from a college instructor. Teaching contexts, key issues, and active learning pedagogical techniques are described, and potential pedagogical takeaways for each teacher's specific concerns are suggested.

Introduction

Precalculus is designed to reinforce previous algebra knowledge so that students can transition to higher mathematics. It is therefore often encountered at two levels: secondary and tertiary education.While the content is similar, often the pedagogies involved are tailored as answers to specific challenges in each context, pedagogies that could prove useful when shared between levels of education. We'd like to catalyze this sharing of ideas with two reflective accounts of precalculus teachers in public New York City educational institutions: one secondary and one tertiary, both of whom believe in using active, exploratory learning to teach mathematics. We dive into each teacher's pedagogical choices, the reasoning behind them, and their consequences, culminating in a discussion about how some of the habits in secondary precalculus could be beneficial to college precalculus, and vice versa.

Secondary Precalculus

The public high school instructor's precalculus course, taught during the 2023-24 academic year, met five days a week for 40-minute class periods. The curriculum covered functions, the unit circle, trigonometry, polar coordinates, linear algebra, sequences and series, parametric equations of conic sections, rational functions, and an introduction to calculus. The school emphasized college preparation through rigorous academics, varied student clubs and productions, and counseling services to assist with college applications and logistical questions. The school was an NYC Specialized High School, so students needed to test into it through the SHSAT.

This teacher's class sessions started with an independent or group work Do Now activity for approximately 5 minutes, designed to activate prior knowledge. The instructor supported their instruction by making printed worksheets and notes, keeping everything students needed for the day's lesson in one place. The printed materials often contained setups for exploratory activities, new vocabulary words, and example problems. Exploratory activities were emphasized because they allowed students to generate and discuss their own ideas and because the school's mathematics department pushed for more widespread use of active learning in the 2023-24 school year. Throughout each unit, key concepts were introduced and reinforced frequently so students could internalize and apply them in STEM contexts both within and beyond precalculus.

The instructor held office hours every weekday and was available by email seven days a week.

Pedagogically, an important capstone of this teacher's teaching was mathematical projects. Students were assigned projects twice during the school year: once during the laws of trigonometry unit, and once during the rational functions unit. Projects were designed to provide a novel classroom experience for students that allowed for more peer-to-peer interaction and deeper engagement with the material. However, the instructor felt only one of the projects achieved this. The failure: a project assigned in the laws of trigonometry unit where students formed groups of four and developed a lesson plan based on one of the unit's topics. The major problem related to workload distribution, as not all students contributed to their group equally, resulting in uneven motivation and learning during the project (and frustratingly, lower scores on the midterm's trigonometry questions!). On the other hand, the project in the rational functions unit had pairs of students create an exam with solutions, which was more successful as it corrected the workload distribution issue and truly allowed students to learn about common misconceptions through discovery learning.

Undergraduate Precalculus

The undergraduate instructor's precalculus course was also taught in the 2023-24 academic year, but at a public college serving a significant population of students from historically underrepresented groups (71%) and first-generation (56%) students. The curriculum was department mandated, cutting topics such as conic sections, sequences and series, trigonometry, polar coordinates, and parametric equations, while stressing linear and quadratic functions, limits, the definition of the derivative, and the derivative laws. The real-world applications of these topics were narrowly focused on business and financial use cases. Classes were 75 minutes and met four times per week, with one of the four meetings reserved for practice with a teaching assistant. This TA session was mandatory and run by an undergraduate teaching assistant who had taken the course previously.

Many of the undergraduate instructor's students had taken precalculus in high school, allowing the instructor the flexibility to position certain students as "experts" and peer supports throughout the course. Students ranged from first to third year undergraduates, so the instructor felt it prudent to focus on establishing class norms early in the semester. The course final exam was high stakes, as students who scored below a certain threshold would automatically fail the course, preventing them from advancing further into their degree. This was especially notable as students were only allowed to attempt a course three times before they could no longer take the course again. Given this fact and the compressed nature of college math courses, the pedagogical choices the instructor made were primarily influenced by these factors. Specifically, the instructor was selective with the topics given exploratory learning time during class, giving priority to topics that would be more difficult and have a larger presence on the final exam. Some students' prior experience with the material did allow the instructor to implement some peer learning, with students teaching other students as a regular feature of the classroom. Near the end of the semester, the instructor also scheduled review days as a final "boost" prior to the final exam.

Allow us to use an example undergraduate precalculus class about limits to illustrate these considerations and practices. The class started with a discussion question to activate prior knowledge: what's a limit? Students listed examples and properties of limits, so the instructor filled in the gaps by providing the formal, mathematical definition of a limit, but also made sure to describe limits in more colloquial language to bridge students' intrinsic understanding and mathematical formalism. From here, limit theorems (and informal proofs of said theorems) were presented as extensions of the limit definition, with practice focused on how to use these theorems due to the relevance this had to questions on the final exam. Practice problems had numeric or algebraic answers, with students required to explain their reasoning to hopefully develop both procedural and conceptual understanding. Lastly, an exit ticket was given out to students as a combined form of attendance and formative assessment.

What Can We Learn from Each Other: Starting the Conversation

As research has noted, precalculus is a mutable course that can cover a variety of topics and utilize a wide spectrum of pedagogical strategies (Voigt et al., 2020). We see this in these two reflections of secondary and precalculus teaching: while both teachers sought to use exploratory learning, the ways they manifested this were different. The main points of this comparison can be seen in Table 1:

Table 1

Summary of Secondary and Undergraduate Precalculus Reflection Differences

Secondary Precalculus	Undergraduate Precalculus
Curriculum: functions, the unit circle, trigonometry, polar coordinates, linear algebra, sequences and series, parametric equations of conic sections, rational functions, and an introduction to calculus.	*Curriculum:* linear equations, quadratic functions, functions, the circle, linear regression, economic functions, limits, continuity and differentiability, asymptotes, derivative rules, related rates, marginal functions.
Class Structure and Key Concerns: 40-minutes classes with printed worksheets containing lesson materials. Lessons spiraled key concepts throughout a unit to ensure students could internalize and apply them to other areas of mathematics. The math department also pushed for more active learning in the classroom.	*Class Structure and Key Concerns:* 75 minutes with projected slides as notes for each class session. The final exam determined if students could pass the course, and was a primary concern for the instructor. Time was allotted for practice and exploration of specific topics contingent on the topic's relevance to the final exam.
Exploratory Pedagogy Example: math projects for out-of-classroom exploration and group work that encouraged students to extend their understanding to a novel task.	*Exploratory Pedagogy Example:* peer-led instruction to facilitate student-to-student interactions and provide variety in classroom activities.

Looking specifically at the exploratory pedagogy described, the secondary teacher's use of out-of-class explorations was an interesting pedagogical technique for incorporating exploratory learning without having to use class time. Such a time-efficient method could have been useful in the college precalculus class, and we can use the experience described here to create a preliminary structure for such a project. A college mathematics project would likely benefit from:

- pairs of students working together;
- defined group roles or project requirements that clearly demarcate tasks; and
- using project tasks that students have experience doing (e.g., exams, quizzes, and lessons).

In addition to this project outline solving the conundrum faced by the college instructor, mathematics projects also provide the additional benefit of supporting students' conceptions of mathematical representations, connections, and problem solving by exposing students to multiple ways a single topic can be approached mathematically (National Council of Teachers of Mathematics [NCTM], 2000; NGA Center for Best Practices & CCSSO, 2010" with "Common Core State Standards Initiative, 2010).

On the flipside, the secondary teacher can draw from the college instructor's experience with peer learning and teaching to increase students' motivation and (equal) engagement in mathematics learning. Specifically, peer learning has been shown to improve students' motivation, with some research showing that near-peer learning can significantly improve students' views and attitudes towards mathematics (Wilson & Grigorian, 2019). Taking the college instructor's experience and mapping it onto the secondary teacher's parameters, the following could be a light framework for thinking about implementing peer learning:

- identify students who have prior knowledge or mastery of a given topic;
- check the maturity level of the class and the relationships between students to ensure a peer leader would not cause a classroom management issue;
- provide clear materials and guidance for what the peer leader is expected to do (e.g., work with students on practice problems, teach a small topic, etc.); and
- have consistent time each week for peer learning and teaching to build the habit for the class.

Beyond the pedagogical, the specialized nature of the undergraduate instructor's precalculus course is a trove of real-world applications that the secondary teacher could draw from to motivate students by answering the age-old question, "when are we going to use this?" This is especially relevant to the pair of educators in these reflections, as NYC is a financial hub where being able to understand and use mathematics through a financial lens could be an important tool for opening career paths.

In summary, while secondary and tertiary mathematics education sometimes feel like disconnected worlds, important lessons can be learned from each other's successes and tribulations. These lessons can then be adapted, implemented, and evaluated again to create a synergistic feedback loop of pedagogical improvement that helps both secondary and tertiary mathematics teaching. And of course, more broadly, stronger interconnections between secondary and tertiary mathematics educators can only serve to strengthen the math teaching community, opening future opportunities for collaboration.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* https://corestandards.org/wp-content/uploads/2023/09/Math_Standards1.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.*

Voigt, M., Apkarian, N., Rasmussen, C., & the Progress Through Calculus Team. (2020). Undergraduate course variations in precalculus through Calculus 2. *International Journal of Mathematical Education in Science and Technology, 51*(6), 858–875. https://doi.org/10.1080/0020739X.2019.1636148

Wilson, A. T. & Grigorian, S. (2019). The near-peer mathematical mentoring cycle: Studying the impact of outreach on high school students' attitudes toward mathematics. *International Journal of Mathematical Education in Science and Technology, 50*(1), 46–64. https://doi.org/10.1080/0020739X.2018.1467508