JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Growth through Reflection in Mathematics Education

TABLE OF CONTENTS

JOURNAL OF **MATHEMATICS EDUCATION** AT TEACHERS COLLEGE | SPRING 2019 | VOLUME 10, ISSUE 1

TEACHERS COLLEGE | COLUMBIA UNIVERSITY

The Life of Maryam Mirzakhani

Alanna Gibbons
Teachers College,
Columbia University

ABSTRACT Maryam Mirzakhani is the first and the only female winner of the Fields Medal since its establishment in 1936. She is arguably one of the greatest mathematicians of our generation. This biographical paper outlines her life and work. Her mathematical theorems and noteworthy accomplishments are just as impressive as her determination, imagination, and optimistic outlook on life. Mirzakhani's success came from her passion, creativity, and playful approach to mathematics. She felt the most rewarding part of mathematics was the enjoyment of understanding or discovering something. Mirzakhani's work was visual and focused on patterns and ideas rather than on numbers and algorithms. She constructed mathematics in an artistic way, and her goal was to always find truth and beauty in the world. Mirzakhani will forever be an inspiration to anyone who has studied her work and read her story.

KEYWORDS *curves, geodesics, hyperbolic geometry, moduli spaces, topology, biography, history of mathematics*

> *"The beauty of mathematics only shows itself to more patient followers." ~ Maryam Mirzakhani*

Iranian mathematician and Fields Medal winner Maryam Mirzakhani dedicated her life to developing theoretical mathematics. She contributed especially to the fields of Teichmüller and Ergodic theory, hyperbolic and symplectic geometry, and moduli spaces (Myers & Carey, 2017). She also blended mathematical areas of diverse fields, from algebraic geometry to topology and probability theory (Riddle, 2017). Mirzakhani's work has been labeled as "the beginning of a new era" because of her major contributions to the beautiful blend of dynamical systems and geometry that we are able to enjoy today (Myers & Carey, 2017). Although she is most known for being a prominent female mathematician, Maryam Mirzakhani was also a devoted colleague, friend, teacher, learner, mother, and wife. It is important to document the life of arguably one of the greatest mathematicians of our generation. There have been studies of Mirzakhani's work done in the past (Klarreich, 2018; O'Connor & Robertson, 2017; Myers & Carey, 2017; Rafi, 2017; Riddle, 2017; Series, 2017). This biographical paper examines the life of Maryam Mirzakhani through the lens of a mathematics educator.

Early Life

Maryam Mirzakhani was born on May 3rd, 1977 in Tehran, Iran to Ahmad Mirzakhani and Zahra Haghighi. Mirzakhani's parents always encouraged her to have a meaningful profession and supported her in everything she did (O'Connor & Robertson, 2017). She grew up in Tehran at the time of the Iran-Iraq war. However, she felt fortunate to come of age after the war had ended and the political, social, and economic environment had stabilized (Myers & Carey, 2017). She believed this allowed her great opportunities.

Mirzakhani had a sister and two brothers. She first became interested in the sciences when her older brother came home from school one day and began telling her about what he had learned (O'Connor & Robertson, 2017). Her first memory of mathematics was when her brother told her about summing numbers 1 to 100 (Mirzakhani, 2014). He shared how Gauss was alleged to have solved the problem. The solution fascinated her even though she was unable to find it on her own at the time.

Growing up, Mirzakhani had no intention of being a mathematician. At eight years old she used to tell herself stories about a girl who achieved great things, such as becoming mayor or traveling the world (Klarreich, 2018). It was clear at a young age that Mirzakhani had a wonderful imagination. As a child, her goal was to read every book that she could find. Mirzakhani's passion was reading novels, and her dream was to become an established writer (Mehta, Mishra, & Henriksen, 2016). Furthermore, Mirzakhani enjoyed watching biographies of famous women, such as Helen Keller and Marie Curie, on television (O'Connor & Robertson, 2017). Their stories sparked her ambition to do something great with her life (Klarreich, 2018).

Around the time the Iran-Iraq war ended, Mirzakhani was completing her studies at elementary school. She sat for and passed an admissions examination to Farzanegan, the National Organization for Development of Exceptional Talents, or NODET (Zorich, 2015). NODET was a middle and high school in Iran for girls with exceptional talents. Mirzakhani was accepted and decided to attend. During her first year at NODET, Mirzakhani did not do well in mathematics. Her teacher at the time told her that she was not talented in the subject (O'Connor & Robertson, 2017). Reflecting on this experience, Mirzakhani said, "at that age, it's so important what others see in you. I lost my interest in math" (Klarreich, 2018). Luckily, Mirzakhani had an encouraging mathematics teacher the following year. This greatly bolstered her confidence. She quickly began to once again show her talent in the subject.

In middle school (in the early 1990's), Mirzakhani met someone who would become a lifelong friend and share her motivation and academic interests. Her name was Roya Beheshti (Mirzakhani, 2014). One afternoon, Mirzakhani and Beheshti came across some mathematics questions from the International Mathematical Olympiad (Klarreich, 2018). This was an annual competition for high school students, and the girls were excited to be able to solve some of these problems on their own. Mirzakhani and Beheshti knew that they could do even better if they received mathematics problem-solving instruction like the classes that were already offered to boys at other high schools. Mirzakhani and Beheshti convinced the principal of NODET to organize preparation courses for the International Mathematical Olympiad (Zorich, 2015). Mirzakhani has noted that the principal's positivity and support influenced her life tremendously. The principal responded by saying, "You can do it, even though you'll be the first one" (Klarreich, 2018).

Before this moment, the Iranian Mathematical Olympiad team had never included females (Zorich, 2015). Mirzakhani and Beheshti attended their preparation courses and, through their determination, achieved excellent results. At the 1994 Hong Kong International Mathematical Olympiad, Mirzakhani won a gold medal with a score of 41 out of 42. She was the first female Iranian student to ever win a gold medal at such a competition (Obasi, 2016). Mirzakhani competed again at the International Mathematical Olympiad in Canada in 1995, where she became the first Iranian student to achieve a perfect score (42 out of 42) and to win two gold medals (Obasi, 2016). By this point Mirzakhani's outstanding mathematical ability was clear to those around her, and she was being hailed as a genius. Although she did not like the pressure that the title "genius" brought, Mirzakhani began to consider pursuing a career in mathematics in her senior year of high school. Even though mathematics at that time was not an easy career choice for an Iranian woman, her early success made it possible (Rafi, 2017). The more time that Mirzakhani spent on mathematics, the more excited she became. As she got older Mirzakhani began to enjoy solving harder problems in mathematics, especially those involving geometric structures (Mehta, Mishra, & Henriksen, 2016).

Education and Professional Life

Mirzakhani attended the prestigious Sharif University of Technology in Tehran for her undergraduate studies. At Sharif University, Mirzakhani and her classmates attended problem-solving sessions and informal reading groups (O'Connor & Robertson, 2017). She attributed much of her mathematical interests to the friendships and support that she gained from attending Sharif. Mirzakhani published several academic papers as an undergraduate, including "Decomposition of Complete Tripartite graphs into 5-Cycles" (published in 1995 with E. S. Mahmoodian), "A Small Non-4-Choosable Planar Graph" (1996), and "A Simple Proof of a Theorem of Schur" (1998) (O'Connor & Robertson, 2017). In 1999, Mirzakhani obtained her Bachelor of Science in Mathematics from Sharif University (Obasi, 2016). Mirzakhani moved to the United States to pursue a Doctorate of Philosophy in Mathematics at Harvard University. As a student at Harvard, Mirzakhani was best known for her relentless questioning. The language barrier was a definite challenge—Mirzakhani asked her professors questions in English while taking notes in Farsi (Myers & Carey, 2017). Mirzakhani's mathematical background prior to her studies at Harvard was mostly in combinatorics and algebra (Mirzakhani, 2014). Although she always enjoyed complex analysis, it was not one of her areas of expertise. This sparked Mirzakhani to attend a seminar taught by 1998 Fields Medalist Curt McMullen. She was fascinated by how McMullen could explain concepts simply and elegantly (Mirzakhani, 2014).

Mirzakhani went on to earn her Ph.D. from Harvard in 2004 under the supervision of McMullen as her academic advisor (Series, 2017). Curtis McMullen described Mirzakhani as a student who "was filled with fearless ambition" (Myers & Carey, 2017). He also recalled that she "had a sort of daring imagination. She would formulate in her mind an imaginary picture of what must be going on, then come to my office and describe it. At the end, she would turn to me and say, 'Is it right?' I was always very flattered that she thought I would know" (Klarreich, 2018). At the same time, Mirzakhani felt that she learned an endless amount from McMullen, and because of him, she developed a long list of initial ideas that she wanted to explore (Mirzakhani, 2014).

Mirzakhani's dissertation (2004) was titled, "Simple Geodesics on Hyperbolic Surfaces and Volume of the Moduli Space of Curves." In her dissertation, Mirzakhani solved several deep problems about hyperbolic surfaces. She created a formula that shows how the number of simple geodesics of length L grows as L gets larger (Klarreich, 2018). Another brilliant result of her dissertation was a new proof of the Witten conjecture (Mirzakhani, 2004). Mirzakhani's impressive connection between two major mathematical ideas brought her widespread recognition as a mathematician. Solving just one of these major research questions would have been impressive for any mathematician, but to solve both was simply phenomenal. Alex Eskin, a mathematician at the University of Chicago, put it succinctly— "It's the kind of mathematics you immediately recognize belongs in a textbook" (Klarreich, 2018). Her dissertation work resulted in multiple papers, including three that were published in top mathematics journals: "Weil-Petersson Volumes and Intersection Theory on the Moduli Space of Curves" (2007); "Simple Geodesics and Weil-Petersson Volumes of Moduli Spaces of Bordered Riemann Surfaces" (2007); and "Growth of the Number of Simple Closed Geodesics on Hyperbolic Surfaces" (2008) (Riddle, 2017). After turning down a junior fellowship at Harvard, Mirzakhani accepted a 2004 Clay Mathematics Institute Research Fellowship at Princeton University (Rafi, 2017). This fellowship provided Mirzakhani with a generous salary, paid research expenses, and freedom to choose where she wanted to perform research.

Mirzakhani worked for several years at Princeton University where she rose to assistant professor in a short time (Obasi, 2016). In 2008, at the age of 31, Mirzakhani became a Full Professor of Mathematics at Stanford University in California (Zorich, 2015). One of Mirzakhani's colleagues at Stanford, Ralph L. Cohen, described her as a wonderful colleague (Myers & Carey, 2017). He explained, "She not only was a brilliant and fearless researcher, but she was also a great teacher and terrific Ph.D. adviser. Maryam embodied what being a mathematician or scientist is all about: the attempt to solve a problem that hadn't been solved before, or to understand something that hadn't been understood before. This is driven by a deep intellectual curiosity, and there is great joy and satisfaction with every bit of success" (Myers & Carey, 2017). At that time, Mirzakhani was considered a leader in the fields of hyperbolic geometry, topology, and dynamical systems (Rafi, 2017).

Mathematical Contributions

It is not difficult to draw parallels between how Mirzakhani created stories as an eight year old and how she approached life as an adult (O'Connor & Robertson, 2017). The only difference was that her made up stories were based on mathematical research about hyperbolic surfaces, moduli spaces, and dynamical systems. Mirzakhani did not have a specific method for developing new proofs. She related this obstacle to "being lost in a jungle and trying to use all the knowledge that you can gather to come up with some new tricks, and with some luck you might find a way out" (Myers & Carey, 2017).

Mirzakhani's major contribution in mathematics was the theory of moduli spaces of Riemann surfaces. Mirzakhani mainly worked on problems related to geometric structures on surfaces and their deformations (Mirzakhani, 2014). She was interested in understanding hyperbolic surfaces and found it fascinating to discover the many ways one could look at the same problem and approach it with different methods and perspectives. She studied the geometries behind moduli spaces and their applications to differential, hyperbolic, and algebraic geometry. In her early work, Mirzakhani found the volume of a moduli space with a given genus as a polynomial in the number of boundary components (Obasi, 2016). As mentioned, this resulted in Mirzakhani discovering a newfound proof of Witten's conjecture. Edward Witten and Maxim Kontsevich's formula was on the intersection numbers of tautological classes on moduli space (Obasi, 2016). Mirzakhani's formula now included characteristic classes for the moduli spaces of Riemann surfaces with marked points (Zorich, 2015). In 2006, Mirzakhani took on the problem of what happens to a hyperbolic surface when its geometry is deformed using a mechanism akin to a strike-slip earthquake (Klarreich, 2018). Most mathematicians felt that this problem was completely unapproachable before Mirzakhani solved it with only a one-line proof.

Mirzakhani's work was influential in topology, prime numbers, and cryptography (Myers & Carey, 2017). Her findings were highly theoretical in nature and had surprising impacts on ideas in theoretical physics of how the universe came to exist. Since her proofs inform quantum field theory, they are applicable to engineering and material science (Myers & Carey, 2017). They also provide connections with theoretical physics, topology, and combinatorics (Mirzakhani, 2014). Mirzakhani collaborated with Alex Eskin, Professor at the University of Chicago, and Amir Mohammadi, Associate Professor at the University of California, San Diego, to answer a mathematical challenge that physicists had struggled with for a century: the trajectory of a billiard ball around a polygonal table. This finding on the dynamic of abstract surfaces was described as "probably the theorem of the decade" (Klarreich, 2018). In 2011, Mirzakhani and Eskin published a paper on this work: "Counting Closed Geodesics in Moduli Space" (O'Connor & Robertson, 2017).

Fields Medal and Accomplishments

In 2006, Mirzakhani was named one of Popular Science's "Brilliant 10" extraordinary scientists (Riddle, 2017). Mirzakhani was awarded the International Mathematical Union's premier prize, the Fields Medal, on August 13, 2014 for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces" (O'Connor & Robertson, 2017). Every four years since 1936, the Fields Medal is awarded to mathematicians under the age of 40 to honor their existing outstanding mathematical work and for the promise of their future achievement (Sury, 2014). This is considered the highest mathematical achievement and has often been described as the "mathematician's Nobel Prize" (Mehta, Mishra, & Henriksen, 2016).

Mirzakhani was both the first woman and first Iranian to ever win the Fields Medal (Rafi, 2017). Hassan Rouhani, the Iranian president at the time, acknowledged her achievement by saying the "unprecedented brilliance of this creative scientist and modest human being, who made Iran's name resonate in the world's scientific forums, was a turning point in showing the great will of Iranian women and young people on the path towards reaching the peaks of glory…in various international arenas" (Myers & Carey, 2017). At 38 years old, Mirzakhani became an icon and made international headlines. Mirzakhani was highly influential in a male-dominated field. She also represented Iran's tradition of intellectualism.

Mirzakhani received many academic awards and acknowledgments during her lifetime. In addition to the Fields Medal, she won the 2009 American Mathematical Society (AMS) Blumenthal Award, the 2013 AMS Ruth Lyttle Satter Prize in Mathematics, and a 2014 Clay Research Award, to name a few (Riddle, 2017). She was a noteworthy guest speaker at many academic conferences and was elected to the 2015 Paris Academy of Sciences, the 2015 American Philosophical Society, the 2016 National Academy of Sciences, and the 2017 American Academy of Arts and Sciences (O'Connor & Robertson, 2017). Despite all this attention, Mirzakhani stayed modest, focused, and humble. In fact, when she was awarded the Fields Medal, she admitted that she thought notification letter was a mistake! "To be honest, I don't think I've had a very huge contribution," she replied (Klarreich, 2018).

Personal Life

Maryam Mirzakhani described herself as the "slow mathematician." She enjoyed pure mathematics because of the longevity of the questions it offered (Myers & Carey, 2017). Mirzakhani said that this sometimes felt as if she was torturing herself along the way, but her positive outlook would shine through with a follow-up statement of "but life isn't supposed to be easy" (Mehta, Mishra, & Henriksen, 2016).

On March 25th, 2005, Mirzakhani married her partner Jan Vondrák, a Czech man with a Ph.D. in Computer Science from the Charles University of Prague and a Ph.D. in Applied Mathematics from the Massachusetts Institute of Technology (O'Connor & Robertson, 2017). Vondrák was also a postdoctoral teaching fellow at Princeton University from 2006-2009. Following this, Vondrák worked as a computer scientist at IBM Almaden Research Center in San Jose, California. In 2016, Vondrák became an Associate Professor at Stanford University. Although Mirzakhani is remembered most for her mathematical mind, Vondrák described her as being athletic (Myers & Carey, 2017). She was a fast swimmer, and she had aspired to run a marathon someday. He also said that she "loved stories about people who were different" (Myers & Carey, 2017).

In 2011, Mirzakhani and Vondrák had a daughter named Anahita (O'Connor & Robertson, 2017). According to her sister Leila, Mirzakhani always set boundaries for her family's privacy (Myers & Carey, 2017). After winning the Fields Medal, Mirzakhani avoided the attention and focused on what she found most important —her family and mathematics. Mirzakhani was always working on her mathematics around the house, often doodling notes and making connections with papers laid out on the floor. Mirzakhani said that doodling helped her focus and that the process of drawing something helped her to stay connected to her research (Mehta, Mishra, & Henriksen, 2016). When Anahita was three years old, she would often shout, "Oh, Mommy is painting again!" when she would see Mirzakhani drawing (Klarreich, 2018).

Mirzakhani felt that the most rewarding part of mathematics was the enjoyment of understanding something new and the excitement of discovery. She described this as the "aha" moment (Mehta, Mishra, & Henriksen, 2016). She believed seeing the beauty of mathematics requires energy and effort, and that discussing mathematics with others with different perspectives is one of the most productive ways of making progress (Mirzakhani, 2014).

Mirzakhani continued to remain humble and grounded. At academic conferences, she could always be found talking with graduate students. She would generously share her ideas with the community, and she would always listen to the work of other mathematicians with excitement (Rafi, 2017). Mirzakhani's lectures were enthusiastic, optimistic, and inspiring. Students felt motivated to persevere through difficult problems and to appreciate the beauty of mathematics upon leaving her talks (Zorich, 2015).

Death and Remembrance

Mirzakhani was diagnosed with breast cancer in 2013 (Rafi, 2017). Mirzakhani refused to take a long-term leave from her work at Stanford University for her illness. She continued her obligations as an editor of the Journal of the American Mathematical Society for as long as she could. Despite the troubles that Mirzakhani faced, she still spoke about mathematics and offered helpful insights to those who asked for it (Rafi, 2017). By 2016, the dreadful disease had spread to her liver and bones. At 40 years old, Mirzakhani died at Stanford Hospital on July 14, 2017 (Series, 2017). The mathematical community lost one of its most brilliant minds much too early. Mirzakhani was a calming force that "rose above the pressures of academia" (Rafi, 2017). Mirzakhani's death at such an early age provoked the mathematical community to wonder what she could have achieved if she had had more time to continue working. Mirzakhani is deeply mourned and survived by her husband and daughter (Series, 2017). However, even with cancer, Mirzakhani described herself as lucky. She felt that she was born with a good mind and into a loving family, and "there's too much trouble in the world for people to be crying for her." She asked for people to "cry for those who are close to you and who you can help" (Myers & Carey, 2017).

Marc Tessier-Lavigne, President of Stanford University, said, "Maryam is gone far too soon, but her impact will live on for the thousands of women she inspired to pursue math and science. Maryam was a brilliant mathematical theorist, and a humble person who accepted honors only with the hope that it might encourage others to follow her path. Her contributions as both a scholar and a role model are significant and enduring, and she will be dearly missed here at Stanford and around the world" (Myers & Carey, 2017).

Classroom Connections and Closing Remarks

There are many benefits to including history, or biographies of historical mathematicians, in mathematics education. Using history to teach mathematics can improve students' motivation, develop their mathematical thinking, and reveal the humanistic aspects of mathematical knowledge (Liu, 2003). Mathematics lessons in secondary school can be taught using Maryam Mirzakhani's story and philosophies. She was passionate about mathematics and creativity. Her work was visual and focused on patterns and ideas, rather than on numbers and algorithms. In addition, she applied a slow and steady approach to her mathematical work. If students are taught to follow her lead, they would likely feel comfortable taking time to work as mathematicians and may find it easier to find solutions to difficult problems. Teaching about Mirzakhani's positive attitude and her mathematical journey has the ability to inspire students to embrace new mathematical ideas and to approach mathematics the same way she did.

Maryam Mirzakhani truly saw mathematics everywhere. Her mathematical discoveries were influenced by her interests, from running and swimming to doodling in her mathematics notebooks. She had a mindset that was always open to new ideas and a willingness to try new and innovative things (Mehta, Mishra, & Henriksen, 2016). Mirzakhani was driven by wonder and creativity. Her raw talent was rare, as was her drive to tackle the most challenging problems (Rafi, 2017). Mirzakhani's mathematical theorems are just as impressive as her determination, imagination, and optimistic outlook on life. She will continue to inspire all of those who believe in seeing the beauty of mathematics and solving problems in different ways. Maryam Mirzakhani's mathematical legacy will be remembered and celebrated for many years to come.

References

Klarreich, E. (2018, May 09). Meet the first woman to win math's most prestigious prize. Retrieved from https://www.wired.com/2014/08/maryam-mirzakhani-fields-medal/

Liu, P. (2003). Do teachers need to incorporate the history of mathematics in their teaching? *Mathematics Teacher, 96*(6), 416–421.

O'Connor, J. & Robertson, E., (2017, August). Maryam Mirzakhani. Retrieved from http://www-history.mcs.st-andrews.ac.uk/Biographies/Mirzakhani.html

Mehta, R., Mishra, P., & Henriksen, D., (2016). Creativity in mathematics and beyond – Learning from Fields medal winners. *TechTrends, 60*(1), 14–18. doi:10.1007/s11528-015-0011-6

Mirzakhani, M. (2004). *Simple geodesics on hyperbolic surfaces and the volume of the moduli space of curves.* ProQuest LLC. Retrieved from https://books.google.ca/books?id=vz4kHQAACAAJ

Mirzakhani, M. (2014, August 12). Maryam Mirzakhani: 'The more I spent time on maths, the more excited I got.' Retrieved from https://www.theguardian.com/science/2014/aug/13/interview-maryam-mirzakhani-fields-medal-winner-mathematician

Myers, A., & Carey, B. (2017, July 24). Maryam Mirzakhani, mathematician and Fields Medal winner, dies at Stanford. Retrieved from https://news.stanford.edu/2017/07/15/maryam-mirzakhani-stanford-mathematician-and-fields-medal-winner-dies/

Obasi, C. (2016). Investigating mathematics teachers' utilization of women mathematicians as role models in motivating female students to study mathematics using logistic mathematical model: A prerequisite for sustainable mathematics development. *Nigerian Academy of Education (NAE)*, 31.

Rafi, K. (2017). Maryam Mirzakhani (1977–2017). Nature, 549(7670), 32–32. doi:10.1038/549032a

Riddle, L. (2017, December 7) Maryam Mirzakhani: May 3, 1977–July 15, 2017. Retrieved from https://www.agnesscott.edu/lriddle/women/mirzakhani.htm

Series, C. (2017, October 1). Maryam Mirzakhani and her work (1977–2017). *Mathematics Today*, pp. 192–194.

Sury, B. (2014). Mathematics prizes awarded at ICM 2014. *Resonance, 19*(9), 866–872. doi:10.1007/s12045-014-0096-9

Zorich, A. (2015). The magic wand theorem of A. Eskin and M. Mirzakhani. *The Mathematical Gazette, 101*(552), 545–546.