JOURNAL OF
MATHEMATICS
EDUCATION
AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Making Connections in Mathematics Education

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JOURNAL OF **MATHEMATICS EDUCATION** AT TEACHERS COLLEGE | FALL 2019 | VOLUME 10, ISSUE 2

TEACHERS COLLEGE | COLUMBIA UNIVERSITY

Quantitative Literacy: Alternative Pathway for College Developmental Mathematics Students

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ABSTRACT Low passing rates in developmental mathematics have been a serious concern for community colleges for many years. A course in Quantitative Literacy (QL) offers non-STEM students an alternative option to introductory algebra as a path to a degree. This paper describes the implementation and evolution of QL at the Borough of Manhattan Community College. Students enrolled in the 17 sections of QL were compared to a matched sample of students from Elementary Algebra. The students enrolled in QL in the Spring of 2013 were 175% more likely to have passed a credit-bearing mathematics course one year later, indicating that QL represents a valuable alternative for non-STEM college students placed into algebra level remediation. Further, the implementation and preliminary results of a corequisite course combining QL with college level Quantitative Reasoning (QR) are presented.

KEYWORDS *corequisite mathematics, developmental mathematics, innovative pedagogy, quantitative literacy*

Introduction

Quantitative literacy (QL) has become an increasingly frequent topic of discussion in mathematics education over the last thirty years. Thanks in great part to the efforts of Lynn Steen, author of *Mathematics and Democracy: the Case for Quantitative Literacy* (2001) and *Achieving Quantitative Literacy: an Urgent Challenge for Higher Education* (2004), more and more educators have come to recognize the importance of preparing students for the quantitative challenges they will face in their careers and lives. However, the traditional mathematics curriculum, whether at the secondary or college level, has remained firmly aligned with the longstanding tradition of the calculus trajectory. This is particularly true for students in the standard remedial algebra course at community colleges around the country, many of whom are not bound for STEM-related majors and careers, and whose success rate in this course is egregiously low. According to a study performed by Achieving the Dream with a group of 57 participating community colleges, only a third of students placed into remedial mathematics in these colleges had progressed onto college level mathematics within three years (Bailey, Jeong, & Cho, 2010).

Borough of Manhattan Community College (BMCC) is one of twenty-four institutions comprising the City University of New York (CUNY) and serves over 23,000 students (BMCC Fact Sheet, 2019). Each year about 72% of BMCC'S new entering students are placed into developmental mathematics classes based on their performance on the placement math proficiency test. There are up to three levels of mathematics proficiency that students may need to demonstrate before enrolling in the credit bearing class. Those levels are arithmetic, elementary algebra (EA) and intermediate algebra. A student who is placed into EA or who successfully completes an arithmetic course has a choice to enroll in quantitative literacy (QL) instead of EA if their major does not require more advanced algebra-based mathematics courses (non-STEM students). Figure 1 (next page) illustrates the consistently low success rates of students enrolled in EA.

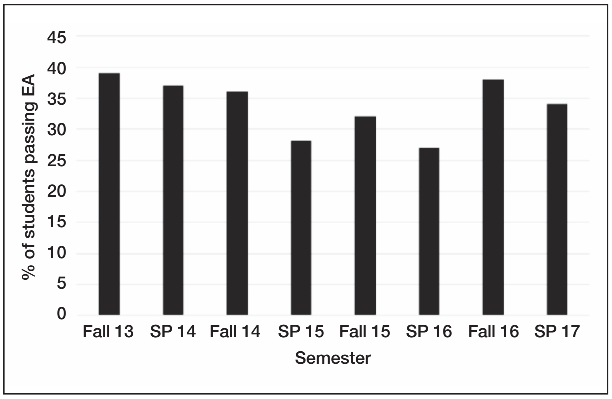

Figure 1. Success Rates in Elementary Algebra Fall 2013 – Spring 2017 semesters.

The QL pathway was introduced in 2012 as a result of BMCC's partnership with the Carnegie Foundation's Quantitative Literacy Initiative together with eight other community colleges across the country. The course was revamped in the Summer of 2015 by a team of BMCC instructors. The curriculum was rewritten to make content more relevant to BMCC students. This course eschews traditional lecture and complex algebraic computation in favor of collaborative work and open-ended problems situated in three applied contexts: citizenship, personal finance, and medical literacy. Problem-solving scenarios include population growth and density, the water footprint of major countries, the cost of an unlimited subway pass vs. single ride passes, interpreting percentages from contingency tables, cost of running a business, representational democracy in the U.S., blood alcohol content, introduction to probability, medical dosage, and compound interest. These problem situations are designed around developing specific mathematical concepts, including estimation strategies, proportional reasoning, understanding magnitude in large numbers, interpretation of probabilities, relative and absolute change, interpreting measures of central tendency, producing and interpreting graphs, calculating quantities using unit analysis, understanding variables, using formulas, solving linear equations, and linear and exponential modeling.

As noted earlier, the course employs an innovative pedagogy designed to support its curriculum. The central motif of this pedagogy is "productive struggle" (Schmidt & Bjork, 1992), wherein students grapple with ideas that are comprehendible but not yet well formed, and which has been shown to lead to greater retention in learning (Hiebert & Grouws, 2007). By facilitating learning, rather than supplying step-by-step algorithms, the QL course facilitates deep learning, develops students' tenacity in problem-solving, and builds quantitative habits of mind, supporting precisely the student-centered approach to learning recommended by Beyond Crossroads (American Association of Two-Year Colleges [AMATYC], 2006).

BMCC's Quantitative Literacy course (QL), which employed the Quantway® curriculum and pedagogical model, required a comprehensive faculty development program which was developed. Between 2012 and 2017 a total of seven faculty trainings were conducted. Over 70 part-time and full-time faculty members participated allowing gradual course expansion.

The QL course has had consistently higher passing rates compared to EA. Table 1 provides the cumulative results in passing rates between the two courses. Figure 2 illustrates semester to semester comparison.

Table 1

Students Performance in Elementary Algebra and Quantitative Literacy courses Fall 2013 – Spring 2017.

Groups	Passed
Elementary Algebra N=17088 (743 sections)	**34%** 3901
Quantitative Literacy N=3052 (122 sections)	**57%** 654

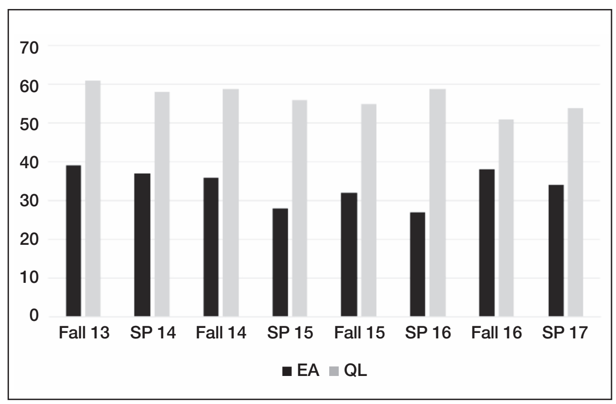

Figure 2. Students Passing Rates in QL and EA between Fall 2013 and Spring 2017 semesters.

The Quantitative Literacy group passing rates have significantly exceeded those of the corresponding developmental algebra course for all semesters the course was offered, with a statistically significant difference in passing rate ($p < 0.001$). A *Chi*-square statistics test showed a highly significant difference between the two groups' passing rates of 34 and 57 percent respectively.

However, these results were not entirely conclusive, as the exit criteria for these courses involve different assessment forms and content. To fully measure success of

QL relative to EA, students must be tracked beyond QL to assess their success in the subsequent college-level mathematics course. In particular, since students taking QL may exhibit different characteristics than the students taking algebra, the QL students' success rate of credit-bearing mathematics completion must be compared to that of a matched sample of elementary algebra students.

Methodology

Subjects & Settings

Students who were in need of remediation at the elementary algebra level and who registered for QL or EA in the Spring 2013 were the target population of the study. Both courses are zero credits and meet two days per week for one hour and 40 minutes each. All basic skills mathematics courses have an enrollment cap of 25 per class at BMCC. All EA sections used the same textbook supplemented by an online homework system. In order to pass EA, students must score a 60 or better on a standardized computer-based final exam and achieve a 74 overall average in the course. At the time of this study, all QL sections used the Quantway® curriculum, supplemented by an online homework system designed for the course and supplied by the Carnegie Foundation. In order to pass the QL course, students must score a 60 or better on the standardized paper final exam and achieve an average of 70 or better in the course.

The Spring 2013 QL cohort of 418 students was deemed the first one large enough to provide statistically persuasive results and an opportunity to assesses the performance beyond the completion of the course.

The students who enrolled in QL in Spring 2013 were distributed across most of the major degree programs offered at BMCC. The largest program majors represented proportionally in the QL group were as follows: Liberal Arts (46%), Criminal Justice (19%), Health/Nursing (14%), and Human Services (10%). Likewise, the EA students were distributed across BMCC's 23 major programs.

Research Design

Students who had enrolled voluntarily in QL differed from the EA students on several key characteristics at the beginning of the Spring 2013 semester:

- The QL group was slightly less likely to be first-time freshmen
- The QL group was more likely to be female
- The QL students had lower arithmetic placement scores (COMPASS 1); COMPASS is a standardized mathematics placement exam
- The QL group had higher average cumulative GPA
- The QL group had higher average total credits accumulated

Since several of these differences could explain higher performance on the part of the Quantitative Literacy group, a propensity matching algorithm was employed to account for potential confounding variables between the two cohorts (Rosenbaum & Rubin, 1983). In lieu of a randomized study, use of propensity matching creates a synthetic balance between the two student groups, and inferences will be valid if there are no residual confounding variables due to unobserved covariates that could substantially bias the results. A sub-sample of 418 propensity-matched algebra students was selected from the overall group of algebra students using *Thoemmes'* propensity matching algorithm for SPSS (Thoemmes, 2012). The algorithm employed the following set of covariates: age, reading score, high school grade point average, first time freshman status, COMPASS 1 score, COMPASS 2 score, gender, underrepresented minority status, cumulative GPA coming into the semester, and total credits coming into the semester. The algebra students were matched to the QL students in equal numbers. Selection was determined by identifying the students that were most closely matched for the above list of covariates. After the matching process, there were no significant differences between the QL and EA sample groups on the demographic and prior performance indices.

At the City University of New York, a student must complete a college level mathematics course with grade C or better in order to transfer to a four-year CUNY campus. Both the QL course group and its EA matched group were assessed on the basis of the completion of the next sequential mathematics course. For students in QL course, the next sequential mathematics course was a credit-bearing mathematics course. For this group, success is defined as achieving a grade of C or better. For students in the EA course, the next sequential mathematics course was either a credit-bearing mathematics course or, in the case of 20 students (4.8% of the sampled population), an Intermediate Algebra course. For these 20 students, success in the next sequential mathematics course was defined as a passing grade in Intermediate Algebra.

Data and Results

Comparisons of the course pass rates were conducted for QL and matched EA students. Table 3 shows the pass rates for the QL and EA groups, both after and before the propensity matching process.

Table 2

Quantitative Literacy and Elementary Algebra, Spring 2013 Pass Rates

	QL (N = 418)	EA Matched (N = 418)	All EA Students (N = 2433)
Passed	**53%**	29%*	33%*

* Fisher's exact test shows these differences between QL and EA pass rates to be significant at a level of $p = .000$.

Further comparisons were made among students who did not pass the course.

At BMCC students who do not pass a course can be categorized as the following:

- Student stayed in the course the entire semester yet failed to meet the standards for passing the course (F grade).
- Student unofficially withdrew from the class, meaning they stopped coming to class and did not take the final exam (WU grade).
- Student officially withdrew from the class (W grade).
- Student never showed up for class.

Comparisons were conducted in terms of each category for QL and matched EA students. The results are shown in Table 3.

Table 3

Quantitative Literacy and Elementary Algebra, Spring 2013 Pass Rates with Categorization of Unsuccessful Students

	Quantitative Literacy (N = 418)	Elementary Algebra (Matched) (N = 2433)
Pass:	**53%**	**29%**
Fail:	**29%**	**51%**
Withdrew Officially	8%	9%
Withdrew Unofficially	8%	9%
Never attended	2%	2%

* Fisher's exact test shows these differences between QL and EA failed rates to be significant at a level of $p = .000$.

Those students in each cohort who passed their developmental course in the Spring were followed through the Summer and Fall semesters. In Summer 2013, 48 of the QL students and 62 of the EA students enrolled in classes (not necessarily mathematics). This difference was not considered statistically significant in determining the difference in overall passing rates by the end of Fall 2013 semester. At the end of Fall 2013, students from the original cohorts in QL and EA were assessed in terms of whether they had completed their next level mathematics course. Table 4 shows the QL and matched EA groups' mathematics course enrollment and respective course pass rates by the end of Fall semester 2013.

Table 4

QL and EA Matched Cohort Groups Passing Rates by the end of Fall 2013

	QL Group	EA Matched Group
Enrolled in Next Sequential Math Course	159	87
Passed	110	44
% Passed	69%	51%

Discussion

Students enrolled in QL in the Spring 2013 semester were 2.5 times as likely (110/44) to have completed their next sequential mathematics course one year later, compared to a matched sample of EA students. This result is largely the consequence of the greater passing rate in QL compared to EA. Yet students successfully completing QL were more likely than their algebra counterpart to pass their next level course by 69% to 51%. As noted earlier, twenty of the EA students who passed EA went on to take Intermediate Algebra as their next course. Four of these students passed Intermediate Algebra by the end of 2013. If the sample of successful EA students is restricted to exclude these students, hence including only those who went on to a credit bearing mathematics course as their next enrolled course (i.e, 67 instead of 87 enrolled students, and 40 instead of 44 passing students), the percentage of successful QL students who satisfied their college mathematics requirement by the end of 2013 (69%) is greater than the percentage of suc-cessful EA students who satisfied their college mathematics requirement by the end of 2013 (60%). This result suggests that QL may prepare students at least as well as does EA for a credit-bearing mathematics course. This may be explained, in part, by the fact that little of the content of the credit-bearing mathematics courses offered at the 100-level (Liberal Arts Mathematics, Statistics, Quantitative Reasoning, and Nursing Math) involves the intensive symbol-manipulation characteristic of algebra and many of its specific forms (expressions and equations involving polynomial, rational, or radical expressions). In some cases, the content of the QL course seems more applicable to these credit-bearing courses (dimensional analysis and proportional reasoning in the case of Nursing Math; percentages, two-way tables, and probability in the case of Statistics). Furthermore, it could be argued that the pedagogical methodologies employed in QL, in particular the concept of productive struggle, provide students with a stronger foundation in the kinds of general problem-solving that await them in the subsequent courses.

To explain the significantly higher passing rates in QL, compared to those in EA, both content and pedagogy should be considered. It can be argued that students are more motivated to learn mathematics when they perceive that mathematics to be useful and relevant to their lives. In the case of QL, students are grouped with peers within the class, affording the opportunity to engage in a collaborative problem-solving process enriched by the involvement of multiple perspectives and mutual assistance. Students arguably feel more connected with their peers and their class, strengthening their sense of involvement, which has been argued a crucial aspect of motivation and retention in developmental students (Tinto, 1997; Tinto, Russo, & Kadel, 1994).

Limitations

The study is limited by the accuracy of the selected covariates in determining the propensity matched sample of algebra students. Because students were not randomized, there is still the possibility that the two student groups were not balanced with respect to unobserved factors related to the outcome of interest, even with propensity matching.

Furthermore, as stated earlier, the category of "next sequential course" differed slightly between the respective cohorts. For the entire QL cohort, this category was defined as a credit-bearing, college-level mathematics course, whereas for the EA cohort, this category also included twenty students who continued on in the STEM pathway, which requires a second level of algebra remediation, offered as Intermediate Algebra. Of these twenty students, four passed Intermediate Algebra. Given the relatively small scale of this subgroup of students (4.8% of the sampled population of 418 EA students, and 23.0% of the successful EA students), this may be seen as a small limitation. Note that when the EA sample is restricted to exclude these students, QL students are 2.75 times as likely as EA students to pass their subsequent mathematics course (compared to 2.5 times in the case of the unrestricted sample), and successful QL students are still more likely to pass their subsequent mathematics course than successful EA students. However, in future studies, the respective populations of student might be initially restricted to include only students who are not enrolled in STEM majors before applying the matching algorithm.

Conclusion and Recommendations

The experience of Quantway's implementation at BMCC suggests that Quantitative Literacy offers a promising avenue in the effort to address low rates of successful mathematics remediation in college students. Despite the two distinct pathways that a BMCC student placed into developmental mathematics can choose, a significant number of students do not enroll into the credit bearing mathematics course immediately after successful completion of the developmental pathway. As a result, students delay the completion of their degrees. One recent promising strategy in developmental education that addresses this challenge is a corequisite model (Logue, Watanabe-Rose, & Douglas, 2017). Students are placed into a credit bearing course with just-in-time remedial support. One of the corequisite courses that was developed in Fall 2017 combines the developmental 4-hour, 0-credit QL course with a 3-hour, 3-credit QR course into a new 6-hour, 3-credit course titled, "Quantitative Literacy and Reasoning. The curriculum was written by expanding the available QL materials and introducing college level QR topics such as probability, statistics and financial literacy. The developmental curriculum was carefully embedded so the new curriculum is cohesive, providing students adequate support to be successful. Students are provided with a workbook and access to the online homework platform, both free of charge. Two sections of the course were offered in each of Spring 2018, Fall 2018 and Spring 2019 semesters. A total of 120 students were enrolled in these sections and 59% of them passed the course. The statistics are promising, but further analysis is needed to compare these success rates with students enrolled in a traditional two semester sequence.

Based on the experience and outcome of QL at BMCC, it is recommended that community college mathematics departments consider offering a Quantitative Literacy based course as an alternative to an elementary algebra-based course for non-STEM students. This recommendation is made with awareness of the many obstacles and challenges facing the large-scale implementation of QL. One such consideration involves the uncertainty some students may feel about their future career trajectory, and correspondingly their major. If a student decides at a later point to change to a STEM major, this student would be required to return to the developmental mathematics level and take the EA course.

It is likely that the above rationale may have played some role in preventing, before now, the implementation of an alternative curricular pathway such as Quantway®, and of obstructing such implementation in the future. However, it should also be noted that there are numerous curricular implications of a student's potential to change majors, in a variety of subjects, and more likely than not, such a decision will add to the number of courses that student is required to take. In any case, a one-size-fits-all curriculum does not support the standards advocated by the American Association of Two-Year Colleges (AMATYC). In Beyond Crossroads (2006), AMATYC posits that the central curricular challenge to be addressed is that of designing "curricula that address the needs of as many academic paths and disciplines as possible" (p. 38). The three goals of developmental mathematics curricula and program development, as outlined in Beyond Crossroads (AMATYC, 2006) are given below:

- Develop mathematical knowledge and skills so students can successfully pursue their career goals, consider other career goals, and function as successful citizens.
- Develop students' study skills and workplace skills to enable them to be successful in other courses and in their careers.
- Help students progress through their chosen curriculum as quickly as possible.

The QL curriculum is designed precisely around the objective of helping students to function as successful citizens. Its classroom format, involving productive struggle and collaborative learning, models a workplace environment more closely than does the traditional lecture format. The third objective above argues for a curriculum that will facilitate a more efficient progress through one's *chosen* curriculum (emphasis ours), which, based on the success of QL students in the above study, a QL course would seem to support.

Finally, consider the assertion, made in Beyond Crossroads (AMATYC, 2006), that in the case of developmental mathematics curricula, "faculty need to do more than teach the same mathematics again" (p. 41). One might speculate whether the resistance to an alternative pathway such as Quantway® may have, at its origins, motivations that are not directly pertinent to the academic futures of students. Mathematics is a traditional subject by nature, and many instructors can be expected to be resistant to teaching a course so different in content and pedagogical character than that which they are accustomed to teaching (and which characterized their own mathematics education). Here, as is often the case, change will not be easy, but the effort offers the potential for significant rewards to students, colleges, and faculty.

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