JOURNAL OF
MATHEMATICS
EDUCATION
AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Mathematics Teacher Education

TABLE OF CONTENTS

PREFACE

v *Matthew DeGraaf, Teachers College Columbia University*
Beatriz S. Levin, Teachers College Columbia University

ARTICLES

1 **Resources Needed for Addressing Common Core Standards in Mathematics and Language Arts and Next Generation Science Standards**
Margaret B. Cozzens, Rutgers, The State University of New Jersey, Center for Discrete Mathematics and Theoretical Computer Science (DIMACS)

9 **The Effects of Interdisciplinary Practice on Elementary Preservice Teachers' Mathematics Teaching Self-Efficacy and Pedagogy: An Intervention Study of Music-Mathematics Integrated Education**
Song An, University of Texas at El Paso
Daniel Tillman, University of Texas at El Paso
Carlos Paez, Navajo Technical University

25 **Patterns in the Pythagorean Configuration and Some Extensions: The Power of Interactive Geometry Software**
José Contreras, Ball State University

37 **Choosing High-Yield Tasks for the Mathematical Development of Practicing Secondary Teachers**
James A. Mendoza Epperson, University of Texas at Arlington
Kathryn Rhoads, University of Texas at Arlington

45 **Classroom Experience about Cartooning as Assessment in Pre-service Mathematics Content Course**
Hoyun Cho, Capital University, Columbus, Ohio
Carolyn Osborne, Capital University, Columbus, Ohio
Tobie Sanders, Capital University in Columbus, Ohio

JOURNAL OF **MATHEMATICS EDUCATION** AT TEACHERS COLLEGE | SPRING 2015 | VOLUME 6, ISSUE 1

Resources Needed for Addressing Common Core Standards in Mathematics, Language Arts and Next Generation Science Standards

Margaret (Midge) Cozzens
Discrete Mathematics and Theoretical Computer Science Center (DIMACS)
Rutgers University

ABSTRACT There is a vital need in the mathematics and science teaching and learning community at the secondary school level for assistance for teachers in adapting curricular materials to meet the many district, state, and national demands and to facilitate high-quality learning of students and their ability to transfer this learning and apply it as they move forward to careers. This paper describes a set of resources in the form of *Teacher Guides,* which would teach educators how to keep up with the high tech, fast-paced world by allowing them to change existing teaching resources faster than new and updated curricula become commercially available. Teacher learning in the *Teacher Guides* would not be limited to a single educational practice. For example, *Teacher Guides* we anticipate creating would capitalize on the relationships and convergences found in CCSS-M practices, CCSS-LA student portraits, and NGSS science and engineering practices.

KEYWORDS *Teacher Guides, curricula adaptation, teachers, students*

Background

There is a vital need in the mathematics and science teaching and learning community at the secondary school level for assistance for teachers in adapting curricular materials to meet the many district, state, and national demands and to facilitate high-quality learning of students and their ability to transfer this learning and apply it as they move forward to careers. Some of the challenges that teachers are facing include: (1) teaching for transfer (e.g., teaching so that what is learned in one setting such as a mathematics or physics class is meaningful and available for use in another class such as Biology), (2) reading and writing across the curriculum (e.g., being able to perform well on technical reading prompts within standardized tests by employing *Claim, Evidence, and Reasoning* (CER)), (3) problem- and project-based learning (i.e., making learning more authentic and representative of advanced study and careers in STEM fields), (4) preparing students for a workplace that is collaborative and open-ended (e.g., implementing effective group work and helping students become good communicators), and (5) even familiar longstanding aspects of STEM education such as inquiry learning, which is being emphasized in new ways (e.g., the need to teach students how to ask good questions and pose meaningful problems in unstructured contexts, rather than always answering or solving someone else's questions and problems).

Science and mathematics teachers in secondary schools work in a very dynamic environment. Not only are these disciplines changing rapidly within themselves, they are merging to create new interdisciplinary niches. As a result, the knowledge, skills, and understandings that secondary school students need to pursue college majors and careers are all shifting. While many of the foundations of secondary school science and mathematics remain intact, teachers are being called upon to quickly adapt their curricula to meet many different needs, priorities, and mandates. In the pursuit of quality mathematics and science education, teachers are expected to meet multiple sets of state and national standards (i.e., the Common Core State Standards in Mathematics ((CCSSM), National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and Language Arts (CCSS-LA) and the Next Generation Science Standards (NGSS)), respond to local pedagogical initiatives, and determine the learning needs of an increasingly diverse student body. Although some STEM curricula and curricular resources address some of the emerging and shifting goals and emphases, very rarely does one text, one module, or one lesson plan address content, pedagogy, and these goals in a comprehensive way. In fact, a curriculum that did so would collapse under its own weight, given that expectations for teachers vary by state, regions, and school districts.

Good teachers are always adapting curricula to meet multiple needs and goals, for example, to meet the needs of English language learners and exceptional and special needs children (Brown & DiRanna, 2013). No one scope and sequence works for all teachers with all students everywhere. Good teaching requires improvisation and flexibility in adapting curricula to meet various goals in particular contexts defined by local circumstances and most importantly by the students in particular classes. Excellent teachers are adept at adapting curricula and integrating resources and techniques to meet multiple goals, but the best practices of teachers who are successful at adaptation have not been studied or captured for use by teachers who seek help in meeting the myriad goals and expectations placed upon them and their students. Even though a teacher can type in a content topic and search the web for existing material, there is no assurance of its quality or ability to address the teacher's specific goals and needs. There is robust research on many of the areas described above, such as writing across the curriculum, problem-based learning, etc., however, there is still a need to move from theory to teacher practice.

Project Teacher Guidance

In several recent DIMACS curriculum/module-development projects (BIOMATH, VCTAL, PS-Future) the growing need among teachers for guidance in the area of curricular adaptation manifested itself in different ways. Most notably was the consensus call from roughly 40 teachers at a meeting in Boston in July 2014 to discuss the next steps in terms of STEM teaching and learning. These teachers, some known to us for over six or more years, and others new to us at this meeting (invited by project teachers), emphatically called for help in responding to multiple pedagogical demands in a way that resulted in a cohesive effective learning experience for their students. They listed about a dozen examples, including the five listed in the opening paragraph. Further evidence for the need for guides for teachers came from field test data of modules in each of the three projects, which revealed teachers' use of DIMACS' curricular materials in unexpected ways. In some instances we found that teachers were adapting the materials (modules) they were field testing in ways that apparently affected their use in negative ways (e.g., omitting several lessons because time was needed to address some other supposedly related material), or in other cases, they used the modules in ways that enhanced learning (e.g, some teachers added writing prompts to a module that didn't include any). In essence, the curricular adaptation we saw through the evaluation of curriculum projects showed that the level of teacher adaptation existed on a spectrum from adaptations that changed the original intent (objectives) of the curriculum to adaptations that enhanced the curriculum while staying true to the original intent.

Additional motivation for the development of such resources comes from seeing in how many different ways the modules from our previous projects were being used. At project teacher meetings, teachers were often very interested to learn of innovative ways their colleagues had used them in addressing multiple curricular and pedagogical goals. It was clear that expert teachers were adept at seeing the implicit potential of these modules for meeting additional goals if a minor adaptation was made. Sometimes all it took was for an experienced teacher to give an example of an adaptation for other teachers to feel empowered to try it and achieve similar gains. Knowing that there is a quickly growing wealth of materials available for STEM teachers, we have decided to formalize what we had seen at project teacher meetings, through research and development on creating guides for the effective adaptation and use of material. In brief, we plan to develop research-based resources and tools that will enable teachers to add value to any curricular materials they might be using.

The following quotation from one of our collaborating teachers (Lea Ann Pitcher, Mathematics teacher and now Mathematics Curriculum Supervisor in Raytown

High School, Raytown, Missouri) at the Boston Project Teachers Meeting, summarizes the above rationale succinctly:

> "Current math and science standards are asking teachers to increase student interest and prepare students for the workforce while focusing deeply on content with rigor and conceptual understanding. Current materials available to teachers, including textbooks and internet resources, are attempting to meet these needs but falling short. Instead of teachers depending upon these textbooks, now is the time to provide guides to empower teachers to create personalized resources that meet the individual needs of their students with their demographics in their own locales."

It is important to understand that the reform movement related to the CCSS is very different from the recent and prior reform movements of the School Mathematics Study Group (SMSG) and the National Council of Teachers of Mathematics (NCTM) standards. Those movements were led by the emergence of new curricula based on newly stated principles. In the current CCSS movement we have the principles and standards, but not the texts. Yet teachers and schools are tasked with implementing the CCSS-M and CCSS-LA now, often in an integrated way, and in the absence of new curricula and in anticipation of extremely high-stakes tests. They have little choice but to search out, find, and adapt existing texts and curricular materials.

Teacher Guides

The *Teacher Guides* will be self-contained materials. Each *Teacher Guides* will contain text material with numerous examples and videos to illustrate adaptations used in classrooms. These guides and supporting materials will be housed on an open website available to all teachers. Excerpts from modules previously developed as part of the three DIMACS module projects will be used as examples for adaptation in the *Teacher Guides*. There are three fundamental reasons for this—first, the modules are well written and have been widely tested and used; second, the modules are all activity driven and problem-based and encourage hands-on experimentation; and third, there are no concerns about proprietary rights to the materials in the modules. An example of a module that might be used is the PS-Future *Weather Generators module*.[1] This module considers the following important science concepts: 1) the water cycle is linked to sustainability—it is not changing, but when and how water is falling is changing; 2) the differences and similarities between weather and runs of wet and dry days, which includes flow charts and tree diagrams linked to actual precipitation data provided by NCAR; and 3) natural variability and predictions. Attention to the CCSM and the NGSS involves the use of data, probability and statistics, the process of making inferences, the application of scientific reasoning, and how changes in the earth systems can cause changes in that system and other systems. This module can be adapted to include reading scientific literature, evaluating data, hypothesis and conclusions in the literature, using mathematical and computational thinking, and constructing explanations and arguments to support claims to solve complex problems—all aspects of CCS-M/LA and NGSS.

Excerpts from the *Food Web* and the *Home Range Analysis* modules[2] can be adapted to address transfer, project-based learning, and skills for the 21st century, while meeting the three sets of standards (hereafter referred to as Standards). At the outset the *Food Web* module asks students to pick a community of species and describe the resiliency of the community—they use applets to look at survival of species under certain conditions. *Home Range Analysis* explores the complex process of how data is collected and analyzed to determine the home range of a number of species, using real data, and distinguishes habitat from home range. To adapt these modules, or pieces of these modules, to meet new requirements of project-based learning (and others) one could frame the lessons around a project where students in collaborative groups pick a species and determine its community, predator-prey relationships, what could cause the species to go extinct, its home range, and provide a written report of their findings to a conservation group with recommendations for delineating the species' home range including buffer zones and corridors where needed. Many of the NGSS, CCSSM and CCSSLA Standards would be addressed from this adaptation.

Currently, no resources are available for adapting mathematics and science curriculum or materials for

---

1 http://dimacs.rutgers.edu/PS-FUTURE/availmodules
2 http://dimacs.rutgers.edu/PS-FUTURE/avilmodules

classroom use, much less to meet the new Standards, other than those designed to adapt materials for special needs students, a substantially different undertaking. These unique *Teacher Guides* that we believe are sorely needed will give autonomy to teachers to create individualized resources. They will make the teacher's increasingly demanding job manageable and more successful in improving student learning. Teachers are already attempting to write meaningful lessons to help students learn how to transfer their knowledge to be successful in college and employable after high school. Although some curricular materials address some key components such as writing, technical reading, 21st century skills (including critical thinking, collaboration, communication, and creativity), and problem-based learning, they rarely address the specific set of skills the teacher wants and needs to address. Teaching teachers to write engaging activities and adapt existing materials to meet their needs gives them the autonomy and tools to meet the needs of their students. As our teachers indicated in Boston, teachers are often asked to write curriculum within their districts without any guidance or education in how to achieve this. These cross curricular needs are not currently written in many district curricula.

Our *Teacher Guides* would differ from existing practice and available resources because they will teach educators how to keep up with the high tech, fast-paced world by allowing them to change existing teaching resources faster than new and updated curricula become commercially available. Teacher learning in the *Teacher Guides* will not be limited to a single educational practice. For example, our *Teacher Guides* will capitalize on the relationships and convergences found in CCSS-M practices, CCSS-LA student portraits, and NGSS science and engineering practices, which all depict ways to interrelate the three standards through reading, writing, and speaking. Each requires the construction of viable arguments and critique of the reasoning of others by engaging in argument from evidence. The *Teacher Guides* will offer teachers confidence and skills that allow them to adapt and use any curricular resources so they are up-to-date, aligned with state and national Standards, and tailor made to increase capacity in all students to gain STEM knowledge. Ultimately, we would expect teachers, who make multiple independent curricular adaptations, to develop new habits of mind when it comes to selecting, adapting, and implementing curricula. We would expect teachers to see curriculum as more fluid and responsive to multiple goals because they are more confident in their ability to enhance and adapt curriculum. Similarly we would expect teachers to become more adaptable and flexible in their thinking about curriculum, and some teachers to take an additional step and become developers of their own curricular components.

Teacher Guides that DIMACS hopes to create will articulate, model, and guide the teacher through the process of adapting and enhancing materials to meet a variety of needs or goals not present in the material being adapted. The adaptations can be in response to classroom-level needs, to school- or district-level initiatives, or state or national mandates. An initial introduction grounded in Teacher-Centered Systemic Reform (TCSR) (Newsome & Woodbury, 2002); addresses teacher metacognition and beliefs. TCSR takes into account the role of personal and contextual variables in changing teacher beliefs, which we believe is necessary in order to "convince" them of the value of adapting curricula to their needs and ultimately changing their practice. Following the introduction, the *Teacher Guides* will address needs and goals such as those in the following list, which is informed by prior projects' field-test results, by discussions with partner teachers, and by conversations with pedagogical, content, and Standards experts:

i. *Developing Metacognition and Transfer in Students* addresses how we develop more self-directed learners who can transfer knowledge from one context to another, facilitated by integrated reflective writing prompts and concept mapping tools; value-added is the role of metacognitive writing prompts in facilitating cognitive transfer. (Belenky & Nokes-Malach, 2012, 2013; Engle, 2006; Hammer, Elby, Scherr & Redish, 2005; Nokes, 2009; Nokes & Blenky, 2011; Nokes-Malach & Mestre, 2013)
ii. *Meeting Literacy Needs* addresses adaptations to include literacy and communication across the curriculum within the context of STEM and ELA and how teachers can implement components focused on improving students' reading, writing, and oral communication skills; value-added is integrating multiple content domains and styles of writing. (Brown & DiRanna, 2013; Love, Stiles, Mundry & DiRanna, 2007; Miller & Krumhansi, 2009; Worth, Winokur, Crissman, Heller-Winokur & Davis, 2009)
iii. *Teaching via Inquiry* addresses ways in which a teacher can add inquiry through questioning to a unit; value-added is emphasis on student question

asking and student problem posing. (Love, Stiles, Mundry & DiRanna, 2007; Worth, Winokur, Crissman, Heller-Winokur & Davis, 2009)

iv. *Teaching for 21st Century Skills* addresses how to implement effective team work and collaborative problem solving in support of CCSS and NGSS and related mandates; value-added is framing tasks so that group work and collaboration are both essential and authentic, not contrived. (DiRanna, Topps, Cerwin & Gomez Zwiep, 2009; National Research Council, 2012; National Science Foundation Board, 2010).

v. *Problem-based Learning* addresses how to add authentic problems to a module that is driven by routine exercises. Often teachers and texts ask students to solve "problems" that are in fact exercises requiring practice of techniques already demonstrated to work in solving the problem. Advice in this *Teacher Guide* will be based on *The Art of Problem Posing* (Brown & Walter, 2005) and other relevant research, as well as best practices observed of teachers adept at creating more open-ended problems or novel problems that relate to but extend to those problems the students are already familiar with; value-added is an emphasis on a problem that the students do not know how to solve at the time it is posed, and their primary task if to find a way to solve it. (Brown & Walter, 2005); Cai, Moyer, Wang, Hwang, Nie & Garber, 2013; Cai, Hwang, Jiang & Silber, in press; CCSSM, 2014)

vi. *Project-based Learning* addresses how authentic projects, which have multiple facets to them, are useful in obtaining higher student engagement from a more diverse range of students; value-added is adapting curricula to include authentic projects or enabling teachers to make existing projects authentic. (Vega, 2011)

vii. *Integrating Multiple Adaptations* addresses as a final section of the combined guides how sometimes multiple related purposes can be addressed by one multifaceted adaptation. Several of the goals implicit in (i) through (vi) are naturally related and can be integrated. For example, some inquiry-based collaborative group work can also require significant writing with peer revision, all in service of some larger project based lesson. To be effective these aspects must be coordinated seamlessly, which requires careful planning that is different from planning just one particular adaptation.

Teachers might have to contend with certain secondary adaptations as they address one or more of the above adaptations. The following are examples of related factors that will inform the advice given to teachers in the *Teacher Guides* but they will not be separate sections: a) *differentiation* addresses how to adapt or enhance curricular materials to meet local, contextual needs and build in accommodations for high and low performers; b) *curricular objectives* addresses how to assess a curriculum for the intended skills and knowledge meant to be addressed and deciding the importance of each of these to the needs of the teacher and her or his students; c) *assessment* addresses the difference between assessment and evaluation, the purpose of low and high stakes assessment, how to create new assessments aligned with curricular objectives, how to assess high, mid, and low cognitive skills, and the purposes for assessing different skill levels in different contexts; d) *curriculum delivery* addresses the delivery of curricular materials (e.g., in person, online, blended delivery, formal/informal, etc.) and changing the delivery mode from the original curriculum if necessary to achieve some other goal.

Conclusion

It can be expected that such *Teachers Guides* could have a broad impact on teachers and students across the country. Improving STEM education has inherent importance in affecting broader improvement in student scientific literacy and skills. These Guides would address the skills and means by which teachers bring integrated and engaging instruction to their students, while addressing student and teacher context that allows them to provide diversity of teaching and content, thus reaching a diverse audience and improving their engagement. A central challenge of articulating standards at national and state levels is that it can gloss over local context and, in so doing, not fully engage teachers and students. Addressing diversity needs to go beyond nominal inclusion of a diverse population/target audience and develop the means to address the relevance of local context, whether that is the context of the geography, the context of the lives and background of different groups of teachers and students, or of contemporary events/ issues. It is expected that these tools, which will include both text and videos of teachers doing adaptations, will facilitate this local development while meeting Standards in mathematics, science, and language arts for all children by explicitly addressing these concerns. In so doing, we expect to achieve greater engagement of a diverse group of teachers who, in turn, can address the diversity of their students and achieve greater engagement in STEM education and performance. Ultimately the diversity of the STEM workforce and improving STEM literacy among a diverse population begins with the early engagement of STEM teachers.

References

Belenky, D. M., & Nokes-Malach, T. J. (2013). Knowledge transfer and mastery-approach goals: An investigation into the effects of task structure and framing instructions. *Learning and Individual Differences, 25,* 21 – 34.

Belenky, D. M., & Nokes-Malach, T. J. (2012). Motivation and transfer: The role of mastery approach goals in preparation for future learning. *Journal of the Learning Sciences*, *21*(3), 399 – 432.

Brown, Z. A. & DiRanna, K. (2013). *Equal access to content instruction for english learners: an example from science.* A report from the Region IX Equity Assistance Center at WestEd. San Francisco: WestEd.

Brown, S. & Walter, M. (2005) *The Art of Problem Posing*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Cai, J., Moyer, J. C., Wang, N., Hwang, S., Nie, B., & Garber, T. (2013). Mathematical problem posing as a measure of curricular effect on students' learning. *Educational Studies in Mathematics, 83*(1), 57 – 69.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (in press). Problem posing research in mathematics: Some answered and unanswered questions. In F.M. Singer, N. Ellerton, & J. Cai (Eds.), *Problem posing: From research to effective practice.* Springer.

Cozzens, M., Crisler, N., Fleetwood, T. & Rotjan, R. (2011). *The biology and mathematics of food webs.* COMAP, Bedford, MA: COMAP.

Cozzens, M., Gaff, H. and Young, C. (2012). *Home range analysis.* Bedford, MA: COMAP.

DiRanna, K., Topps, J., Cerwin, K., Gomez Zwiep, S. (2009). Teaching learning collaborative: A process for supporting professional learning communities. In S. Mundry & K. Stiles (Eds.), *Professional learning communities for science teaching: Lessons from research and practice.* Arlington, VA: National Science Teachers Association (NSTA) Press.

Engle, R. (2006). Framing interactions to foster generative learning: A situative explanation of transfer in a community of learners classroom. *The Journal of the Learning Sciences, 15*(4), 451 – 498.

Hammer, D., Elby, A., Scherr, R. E., & Redish, E. F. (2005). Resources, framing, and transfer. In J. P. Mester (Ed.), *Transfer of Learning: Research and perspectives.* Greenwich, CT: Information Age Publishing.

Love, N., Stiles K., Mundry, S., & DiRanna, K. (2007). *The Data Coach's Guide to Improving Learning for All Students: Unleashing the Power of Collaborative Inquiry.* Thousand Oaks, CA: Corwin Press

Miller, J. S., & Krumhansl, R. (2009). Intended curriculum, enacted curriculum: learning from innovative instructional materials and making them your own. In (Ed.), *Reforming Secondary Science Instruction.* Arlington, VA: NSTA Press

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common core state standards.* Retrieved from http:www.corestandards.org

National Research Council. (2012). *Education for life and work: Developing transferable knowledge and skills in the 21st century.* Washington, D.C.: The National Academies Press.

National Science Foundation/National Science Board. (2010). *Preparing the next generation of STEM innovators: Identifying and developing our nation's human capital.* Report from National Science Foundation. Arlington, VA: National Science Foundation.

Newsome, J. & Woodbury, S. (2002). Overcoming the paradox of change without difference: A model of change in the arena of fundamental school reform. *Educational Policy, 16*(5), 763 – 782.

NGSS Lead States. (2013). *Next generation science standards: For states, by states.* Retrieved from http://www.nextgenscience.org/next-generation-science-standards

Nokes, T. J. (2009). Mechanisms of knowledge transfer. *Thinking & Reasoning, 15*(1), 1 – 36.

Nokes, T. J. & Belenky, D. M. (2011). Incorporating motivation into a theoretical framework for knowledge transfer. In J. Mestre & B. H. Ross (Eds.), *Psychology of learning and motivation (vol. 55)* (pp.110 – 131). San Diego, CA: Elsevier.

Nokes-Malach, T. J., & Mestre, J. P. (2013). Toward a model of transfer as sense-making. *Educational Psychologist, 48*(3), 184 – 207.

President's Council of Advisors on Science and Technology (2010). *Prepare and inspire: K-12 education in science, technology, engineering, and math (STEM) for America's Future*. http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-stem-ed-final.pdf

Rodgers, L. & Sain, S. (2014). *Weather generators.* http://dimacs.rutgers.edu/PS-FUTURE

Singer, F. M., Ellerton, N., & Cai, J. (2013). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics, 83*(1), 1 – 7.

Singh, K., Granville M., & Dika, S. Mathematics and science achievement: effects of motivation, interest, and academic engagement, *Journal of Educational Research, 95*(6), 323 – 332. (2002).

Vega, V. (2011). *Project-based learning.* Edutopia

Worth, K, Winokur, J., Crissman, S., Heller-Winokur, M. & Davis, M. (2009). *The essentials of science and literacy: A guide for teachers.* Portsmouth, NH: Heinemann.