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**Benjamin Dickman**, bmd2118@columbia.edu, is from Brookline, MA. After graduating from Amherst College, Dickman spent the following academic year in Nanjing, China, on a Fulbright Grant to research High School Mathematics Education. He returned to his host institution of Nanjing Normal University the following year on a Chinese government grant for Mandarin studies, and spent his free time becoming fluent in the Nanjing dialect. Dickman is currently a National Science Foundation Graduate Fellow at Columbia University, pursuing his Ph.D. in Mathematics Education (and playing a lot of Boggle). Besides working as a Course Assistant for a range of classes—including Abstract Algebra, Analysis, Calculus, Problem Solving, Set Theory, and Topology—he was also able to serve as a Graduate Instructor for Teachers College doctoral candidates during a recent Summer Study Tour to Shanghai.
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