

Mathematical Modeling, Sense Making, and the Common Core State Standards

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The Headlines

1. We have license to do what we've wanted to do since 1989 (modeling included!).
2. There are tools to help in the classroom.
3. There are tools to help reflect on our teaching.
4. We'll talk...

1. Where are we? (Historically speaking)

For as long as I can remember (and certainly since the 1989 *Standards*) the name of the game has been ***mathematical sense-making*** – problem solving, reasoning, connections, communication, etc.

Not Sense-Making:

How many two-foot boards can be cut from two five-foot boards?

Kurt Reusser asks 97 1st and
2nd graders:

There are 26 sheep and
10 goats on a ship.
How old is the captain?

76 students "solve" it,
using the numbers.

H. Radatz gives non-problems
such as:

Mr. Lorenz and 3 colleagues
started at Bielefeld at 9 AM and
drove the 360 km to Frankfurt, with
a rest stop of 30 minutes.

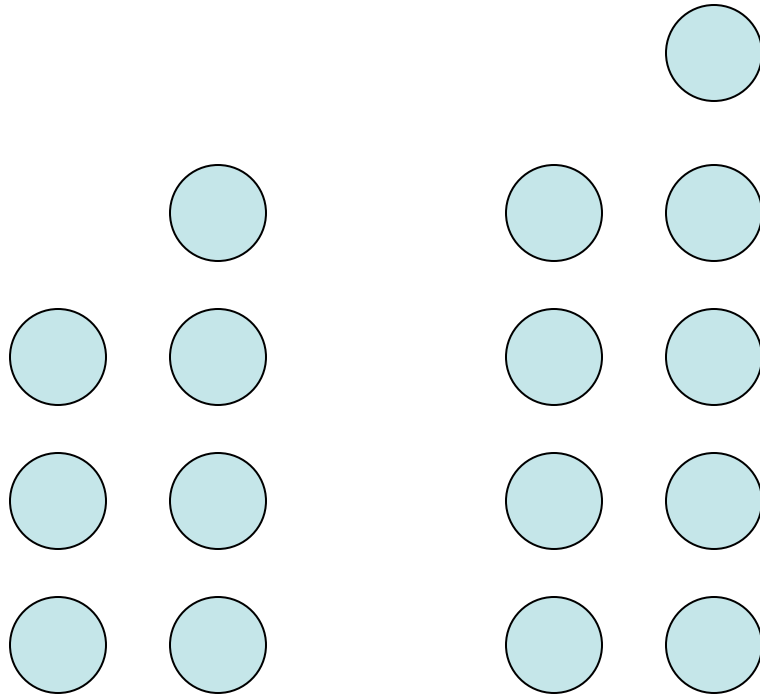
Sense-Making

What happens when you
add two odd numbers?

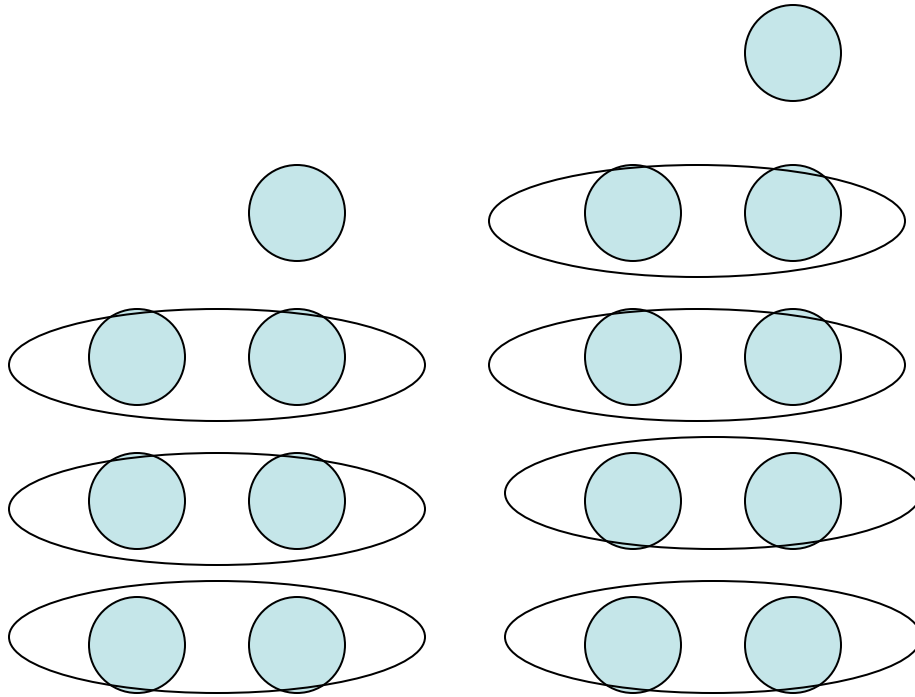
7

+

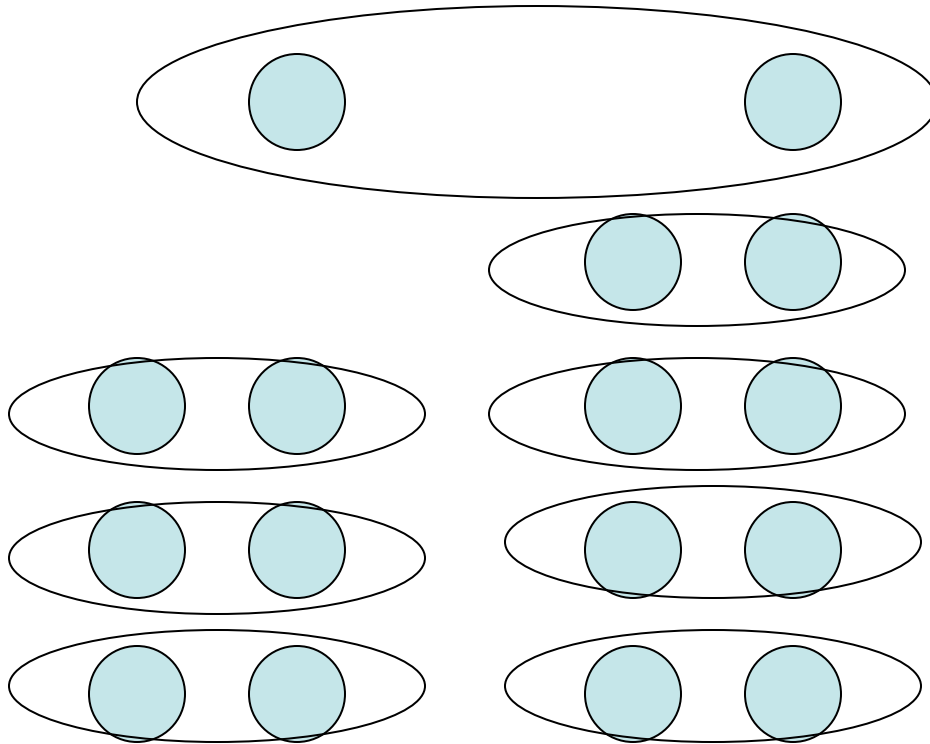
9



$$7 + 9$$



$$7 + 9$$



The Challenge:

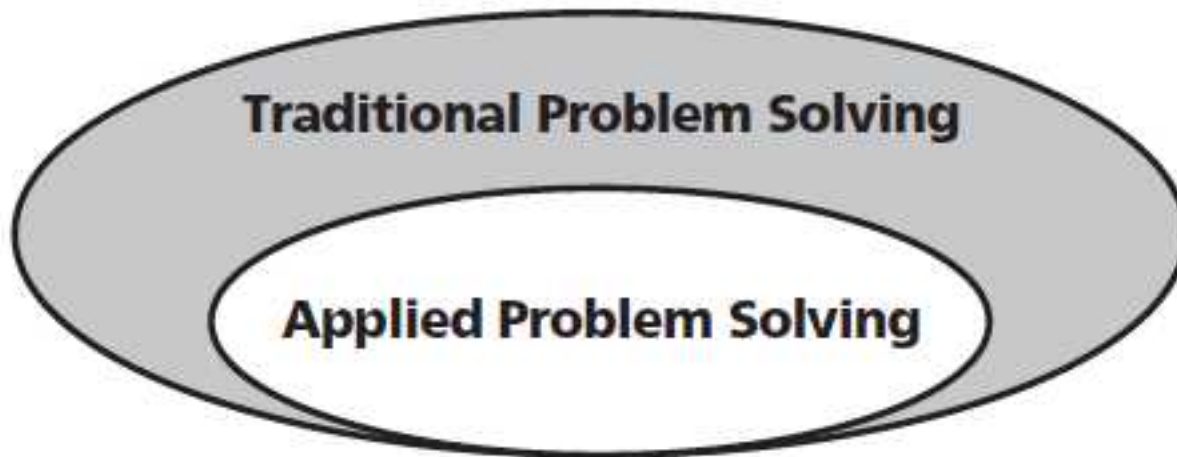
To support sense-making in our classrooms. . . making sense of the CCSSM, assessment consortia, formative assessment, professional development, and systemic alignment along the way.

Here goes... But first, a digression.

What's the relationship
between problem solving and
modeling?

Dick Lesh says it *is*:

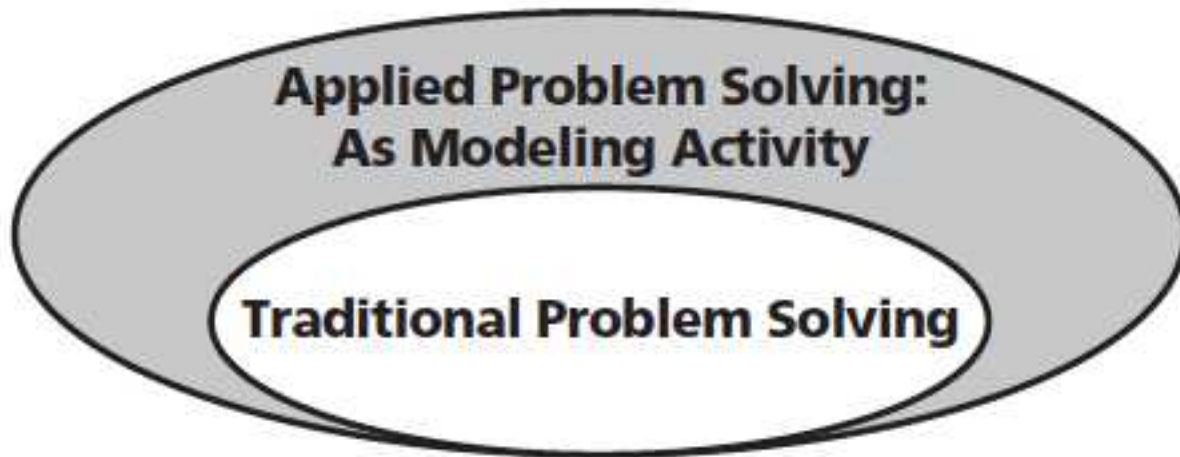
Traditional Perspective on Problem Solving
Applied problem solving is treated as a subset of traditional problem solving.



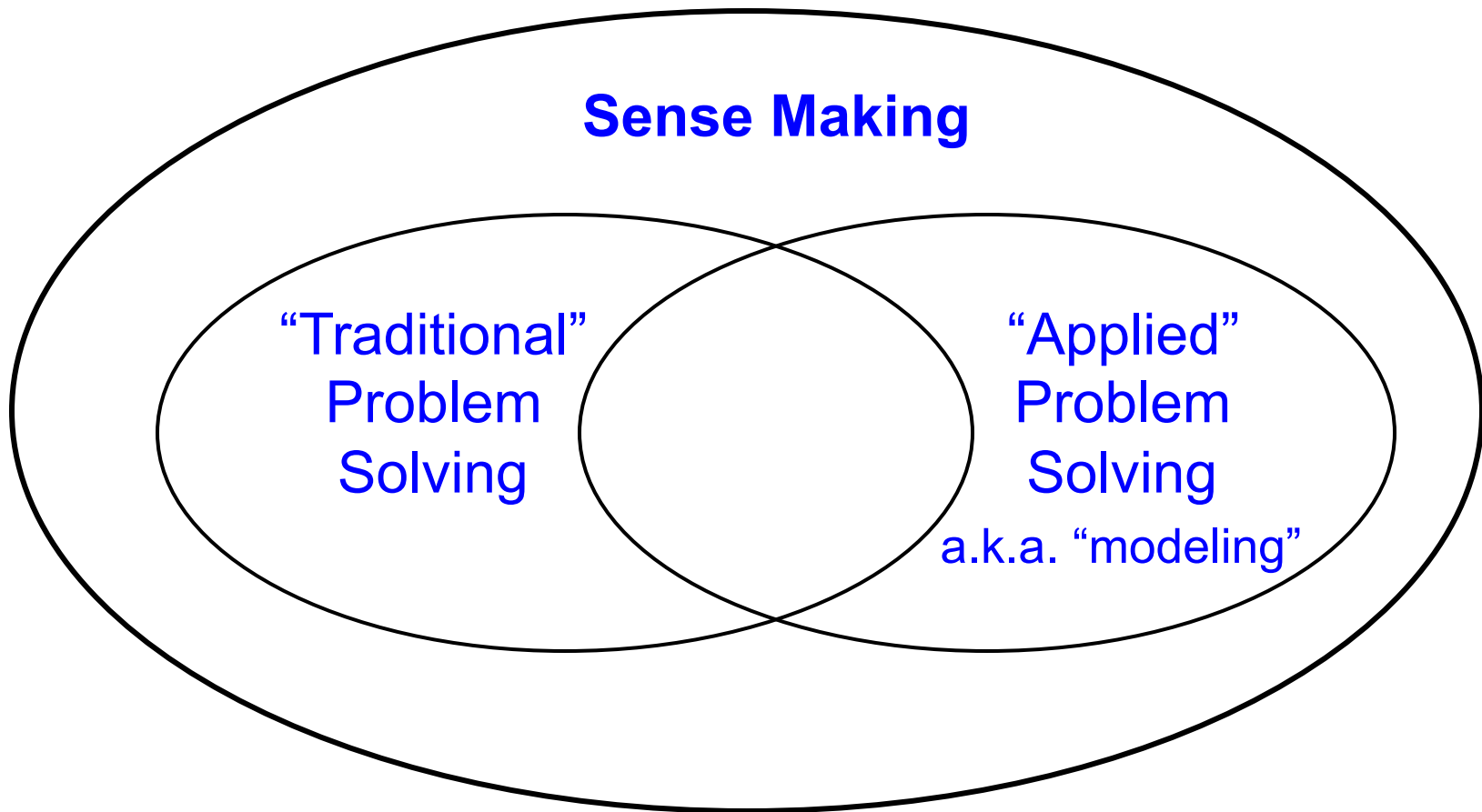
But that it should be:

Models-and-Modeling Perspective on Problem Solving

Traditional problem solving is treated as a subset of applied problem solving (i.e., model-eliciting activity).



I think it's best to think about things this way:



So, OK, back to the ranch...

Let's start with context.

The Common Core State Standards in Mathematics (CCSSM) now exist.

COMMON CORE STATE STANDARDS FOR

Mathematics



The CCSS-M have two main
foci:

Content and Practices

Content:

Key words are “focus and coherence.”

Practices:

The ways in which kids engage with mathematics, building (we hope) productive mathematical dispositions and habits of mind.

The Practices in CCSS-M:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments...
- Model with mathematics
- Use appropriate tools strategically
- Attend to Precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

But what do the words in
CCSS-M mean?

Huh?

What do I mean, what do they mean?
The words are there on the page...

Do you know the phrase

WYTIWYG

?

In a high stakes assessment context, tests drive instruction as much or more than the standards do.

This can be a positive or negative force, vis-à-vis standards and classroom practices.

Some (negative) examples . . .

23 What is the y -intercept of the graph of $4x + 2y = 12$?

A -4

B -2

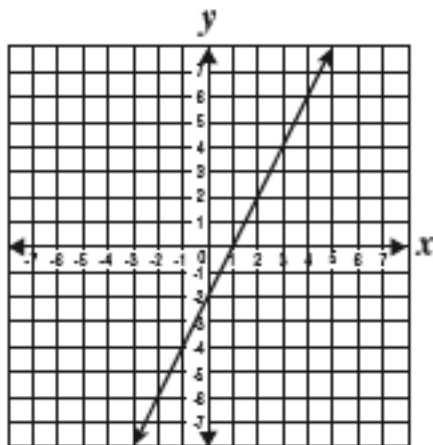
C 6

D 12

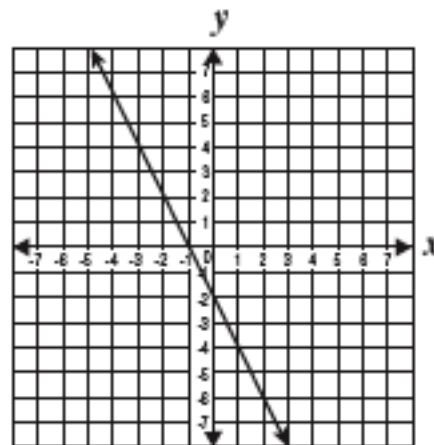
Algebra I

Released Test Questions

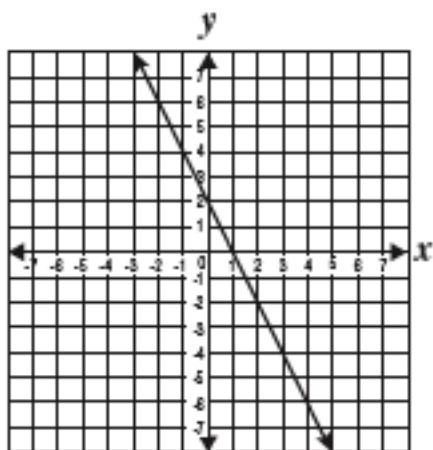
25 Which *best* represents the graph of $y = 2x - 2$?



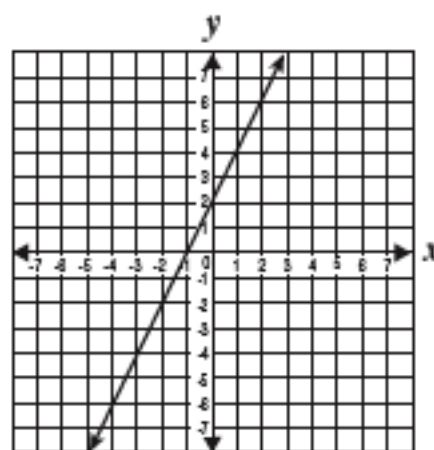
A



C

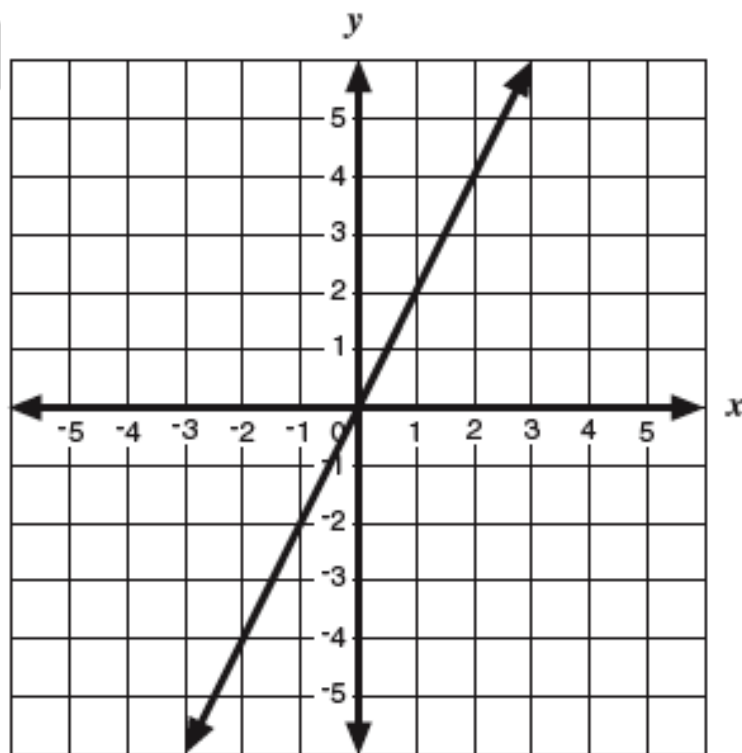


B



D

27



Which equation *best* represents the graph above?

- A $y =$
- B $y = 2x$
- C $y = x + 2$
- D $y = 2x + 2$

They're skills-oriented.
(to put it mildly)

But the CCSSM demand more.

What, and what can be done?

That's the rest of (this part of) the
conversation.

Let's get serious about what matters.

Of course content counts. (Doh!)

BUT, the real action is in the practices.

And...

You can't "check the practices off" if you do them once a month, or once per unit. That is, they have to part of our ongoing classroom activities.

Real Mathematical Substance,
as in CCSSM, hasn't been
a focus of testing in some states

...

but it will be (I hope).

The reason: The Assessments.

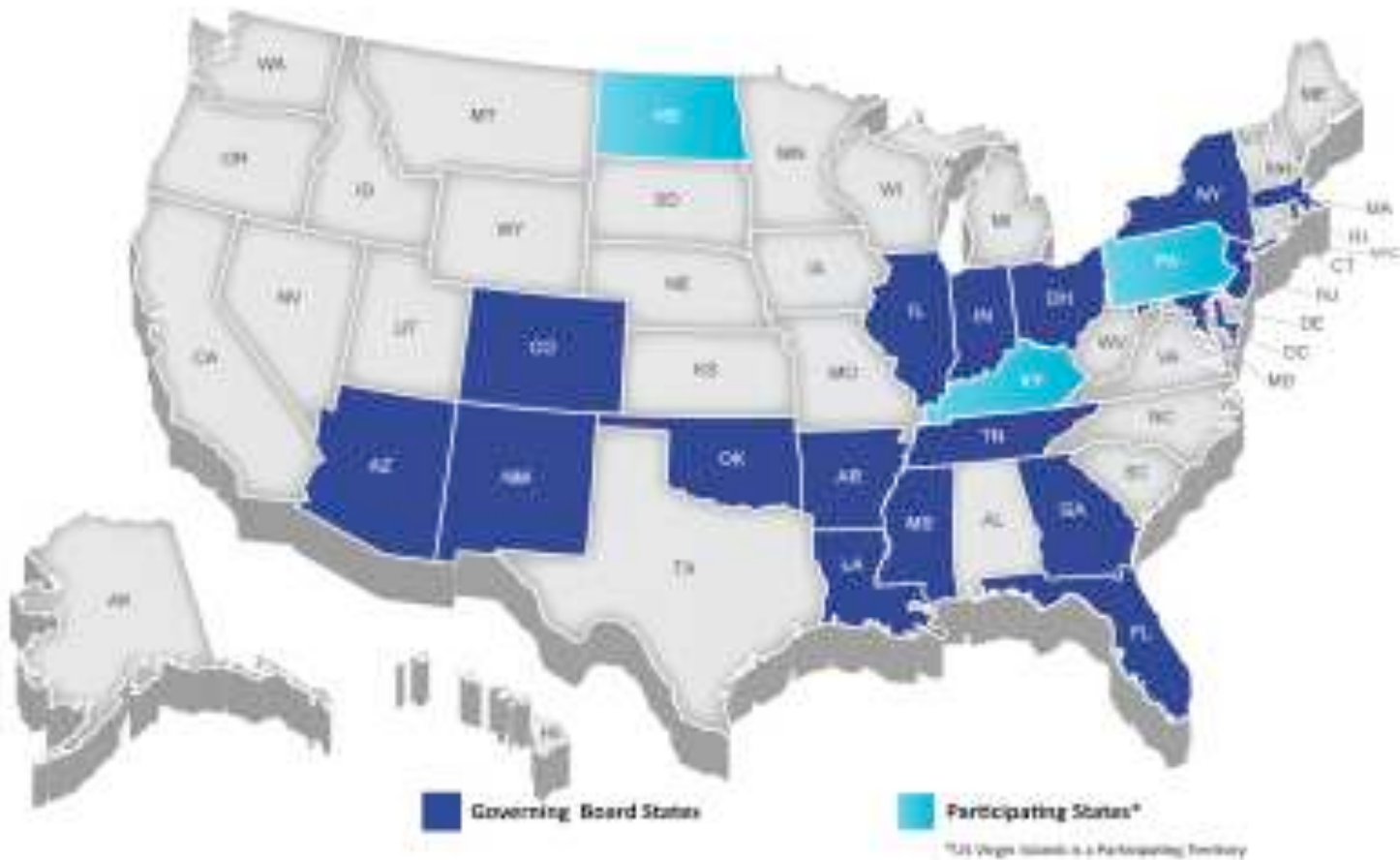
PARCC

SBAC

(Just google their names)

Most students across the US
will be taking the new tests
devised by the two consortia.

PARCC States



SMARTER

Balanced Assessment Consortium

The SMARTER Balanced Assessment Consortium is working to develop a high-quality assessment system that helps all students succeed in college and careers.

SMARTER Balanced states collectively educate about 22 million public K-12 students. These states share a commitment to develop a next generation assessment system aligned to the [Common Core State Standards](#).

The map below highlights SMARTER Balanced Governing and Advisory States. Learn more about our [Consortium Governance](#).



Here are some of the
headlines.

Both Consortia will emphasize:

- Concepts and Procedures
- Problem Solving
- Reasoning
- Modeling with mathematics

And, SBAC will give separate sub-scores for all of these.

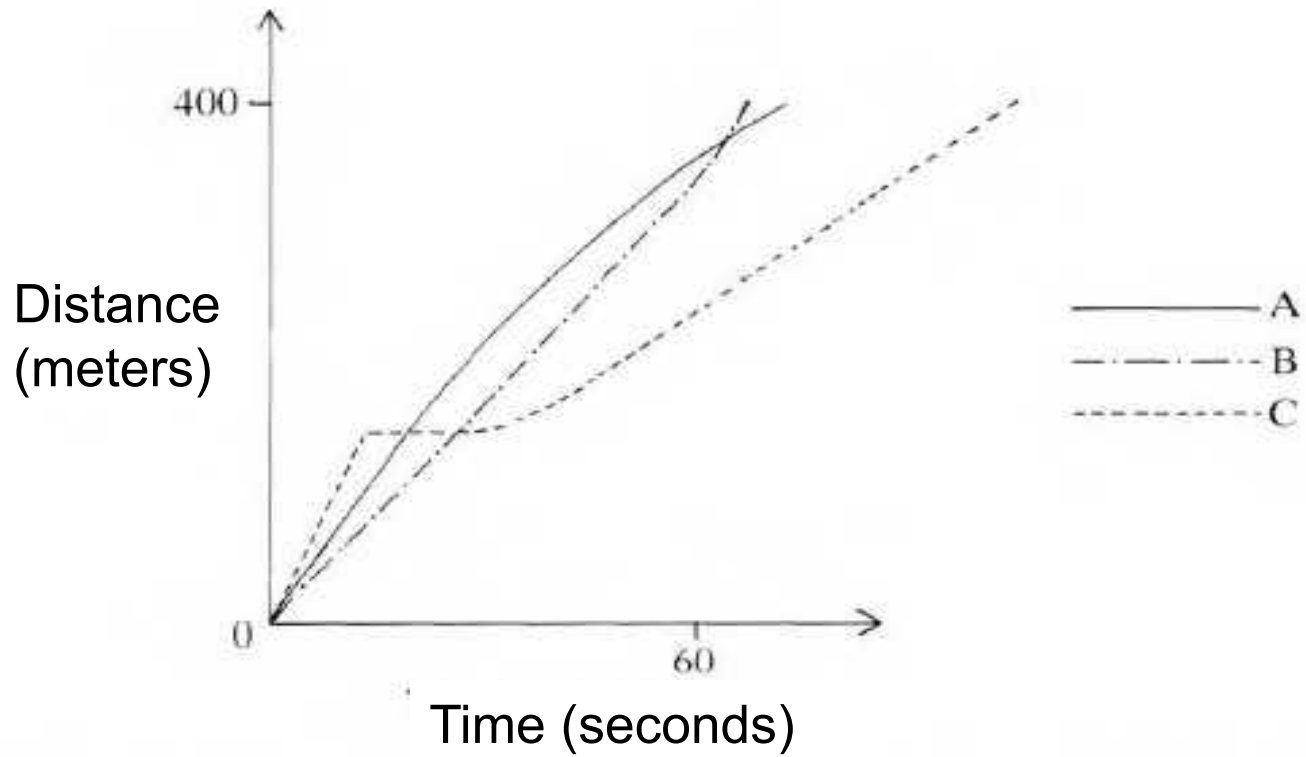
- Concepts and Procedures
- Problem Solving
- Reasoning
- Modeling with mathematics

A large part of the exams will be devoted to things we haven't tested before... And it's going to be traumatic.

(I don't have to tell you, given recent NY test results!)

But, this represents an opportunity for some neat assessment items to drive things in the right directions. Here's an example I like.

Hurdles Race



This is a rough sketch of 3 runners' progress in a 400 meter hurdle race. Imagine that you are the race commentator. Describe what's happening as carefully as you can. You do not need to measure anything accurately.

Think of the Content involved:

- Interpreting distance-time graphs in a real-world context
- Realizing “to the left” is faster
- Understanding points of intersection in that context (they’re tied at the moment)
- Interpreting the horizontal line segment
- Putting all this together in an explanation

Think of the Practices involved:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments...
- Model with mathematics...

25% Sale, Part 1

In a sale, all the prices are reduced by 25%. Julie sees a jacket that cost \$32 before the sale. How much does it cost in the sale?

25% Sale, Part 2

In the second week of the sale, the prices are reduced by 25% of the previous week's price.

In the third week of the sale, the prices are again reduced by 25% of the previous week's price.

In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

Alan says that after 4 weeks of these 25% discounts, everything will be free. Is he right? Explain your answer.

Again:

Core content, central practices.

How about...

“Ponzi” Pyramid Schemes



Max has received this email. It describes a scheme for making money.

From: A Crook
Date: Thursday 15th January 2009
To: B Careful
Subject: Get rich quick!

Dear friend,

Do you want to get rich quick? Just follow the instructions carefully below and you may never need to work again:

1. At the bottom of this email there are 8 names and addresses.
Send \$5 to the name at the top of this list.
2. Delete that name and add your own name and address at the bottom of the list.
3. Send this email to 5 new friends.

Bestsize Cans



The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm^3 .

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm^3 and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations

Want to see more?

Check out Consortium specs;
look at the

Mathematics Assessment Project
(google the name).

How do we prepare kids to do well on assessments like the these assessments?

(I thought you'd never ask!)

2.

Tools to help in the classroom

Let's talk about formative
assessment... in the service
of sense-making

The purpose of formative assessments is not simply to show what students “know and can do” after instruction,
but to reveal their current understandings so you can help them improve.

Important Background Issues

1. Formative assessment is *not* summative assessment given frequently!
2. Scoring formative assessments rather than or in addition to giving feedback destroys their utility (Black & Wiliam, 1998: “inside the black box”)
3. This is **HARD** to do. Tools help!

A Tool:

The formative assessment lesson, or FAL:

A rich “diagnostic” situation

and

Things to do when you see the results of the diagnosis.

A Challenge:

We know that students have many graphing misconceptions, e.g., confusing a picture of a story with a graph of the story in a distance-time graph.

Here's one way to address the challenge.

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

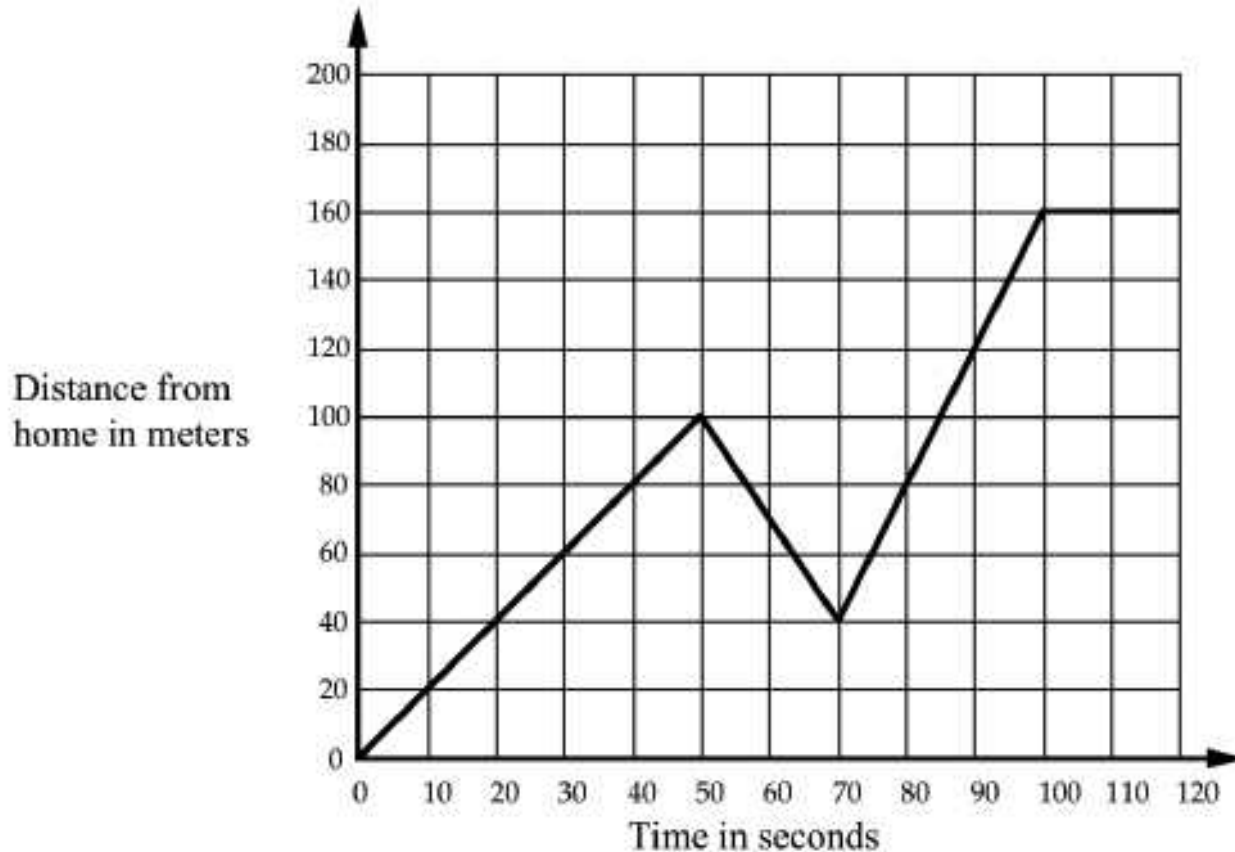
Interpreting Distance-Time Graphs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: <http://map.mathshell.org>
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Before the lesson devoted to this topic, we give a diagnostic problem as homework:

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



Describe what may have happened. Is the graph realistic? Explain.

We point to typical student misconceptions and offer suggestions about how to address them...

Common Issues

Suggested questions and prompts

Graph interpreted as a picture

E.g. The student assumes that as the graph goes up and down, that Tom's path is going up and down.

E.g. The student assumes that a straight line on a graph means that the motion is along a straight path.

E.g. The student thinks the negative gradient means Tom has taken a detour.

- *If a person walked in a circle around their home, what would the graph look like?*
- *If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?*
- *In each section of his journey, is Tom's speed steady or is it changing? How do you know?*
- *How can you work out Tom's speed in each section of the journey?*

Graph interpreted as speed v time

The student has interpreted a positive gradient as speeding up and a negative gradient as slowing down.

- *If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like?*
- *How does the distance change during the second section of Tom's journey? What does this mean?*
- *How does the distance change during the last section of Tom's journey? What does this mean?*
- *How can you tell if Tom is travelling away from or towards home?*

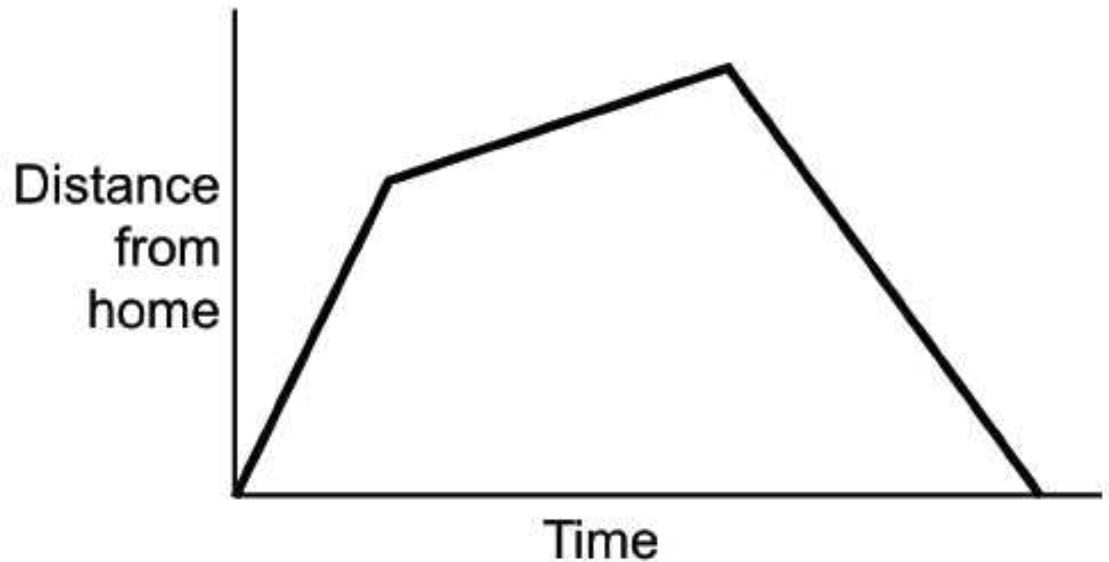
The lesson itself begins with a
diagnostic task...

Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

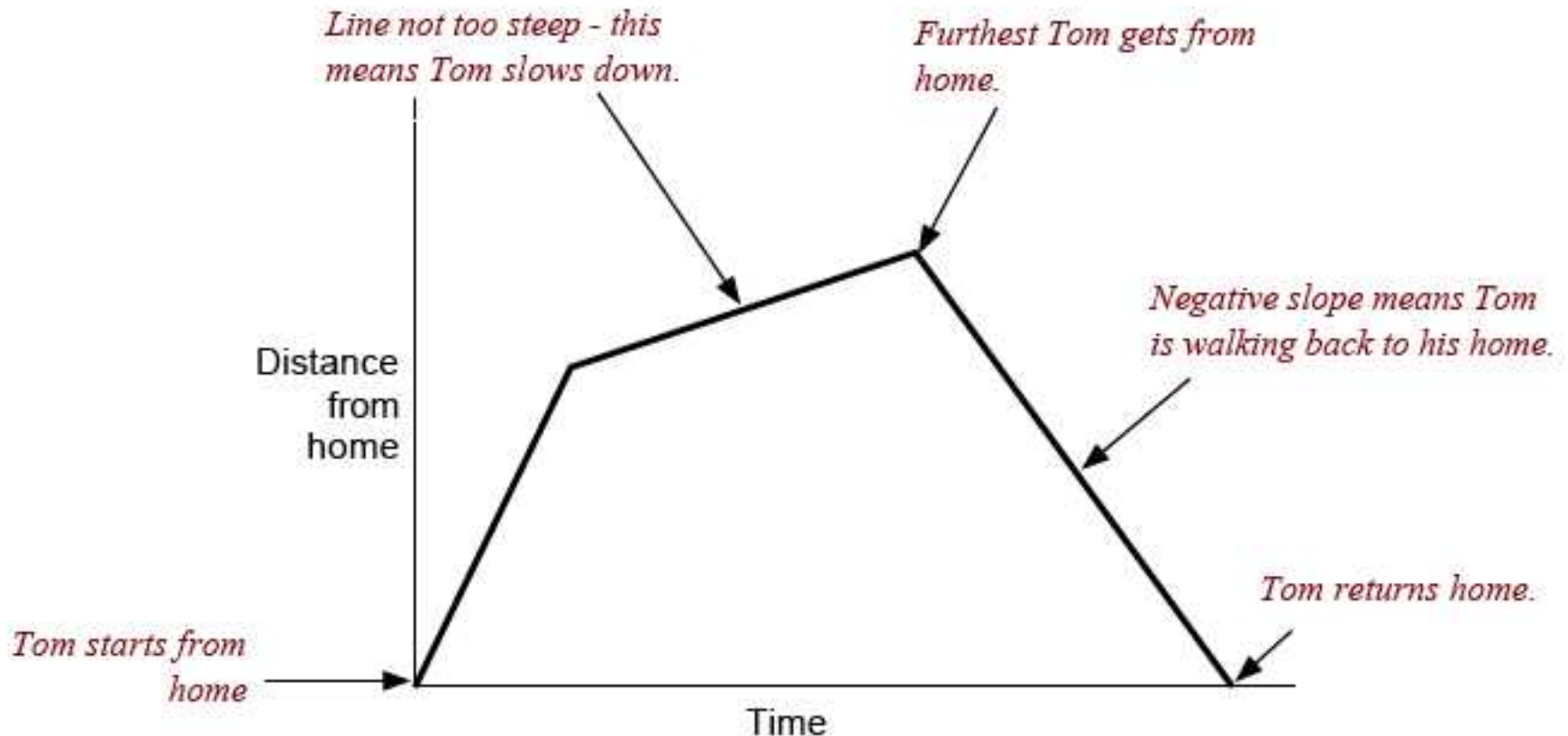
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



Students are given the chance to annotate and explain...

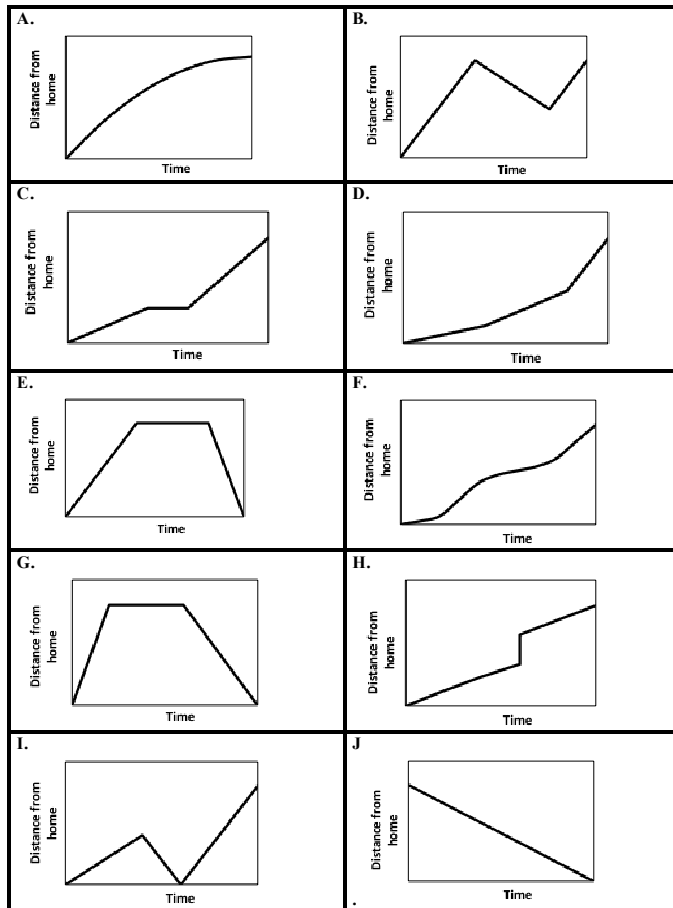
A graph may end up looking like this:



Follow-up Task: Card Sort

The students make posters.

Card Set A: Distance-Time Graphs



Card Set B: Interpretations

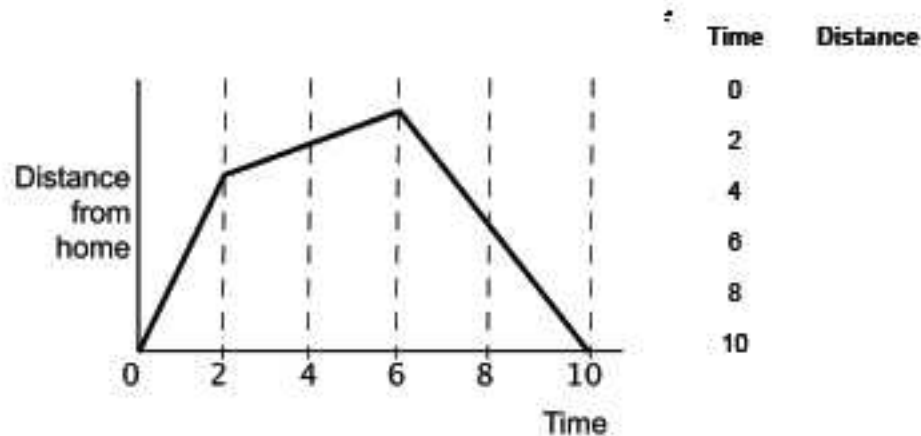
<p>1.</p> <p>Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.</p>	<p>2.</p> <p>Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top and then ran quickly down the other side.</p>
<p>3.</p> <p>Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.</p>	<p>4.</p> <p>Tom walked slowly along the road, stopped to look at his watch, realized he was late, then started running.</p>
<p>5.</p> <p>Tom left his home for a run, but he was unfit and gradually came to a stop!</p>	<p>6.</p> <p>Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.</p>
<p>7.</p> <p>Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.</p>	<p>8.</p> <p>This graph is just plain wrong. How can Tom be in two places at once?</p>
<p>9.</p> <p>After the party, Tom walked slowly all the way home.</p>	<p>10.</p> <p>Make up your own story!</p>

Students work on converting graphs to tables:

Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

Making Up Data for a Graph



On your whiteboard, create a table that shows possible times and distances for Tom's journey.

Tables are added to the card sort...

Card Set C: Tables of data

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And the class compares solutions together.

Here's another FAL:

Evaluating Statements About Length and Area

Mathematical goals

This lesson unit is intended to help you assess how well students can:

- Understand the concepts of length and area.
- Use the concept of area in proving why two areas are or are not equal.
- Construct their own examples and counterexamples to help justify or refute conjectures.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

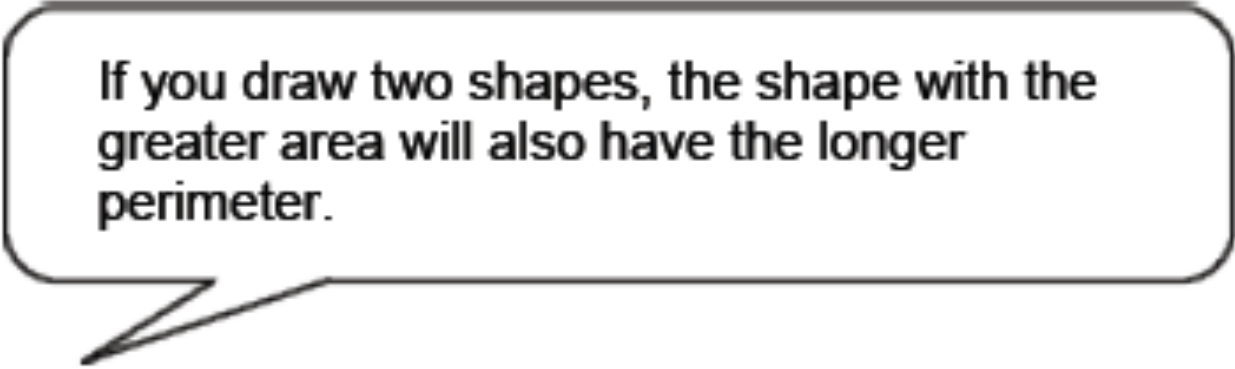
G-CO Prove geometric theorems.

This lesson involves a range of *mathematical practices*, with emphasis on:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Shape Statement 1

1. James says:



If you draw two shapes, the shape with the greater area will also have the longer perimeter.

Is James' statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.

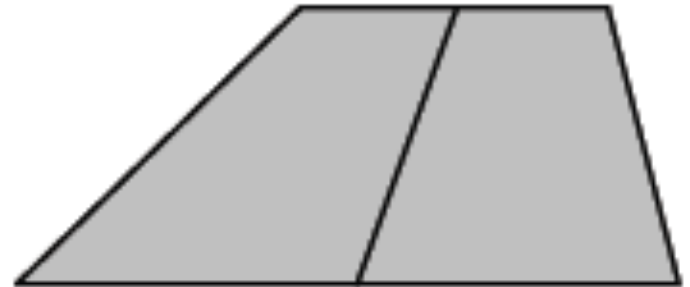
Shape Statement 2

2. Clara says:

If you join the midpoints of the opposite sides of a trapezoid, you split the trapezoid into two equal areas.

Is Clara's statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.



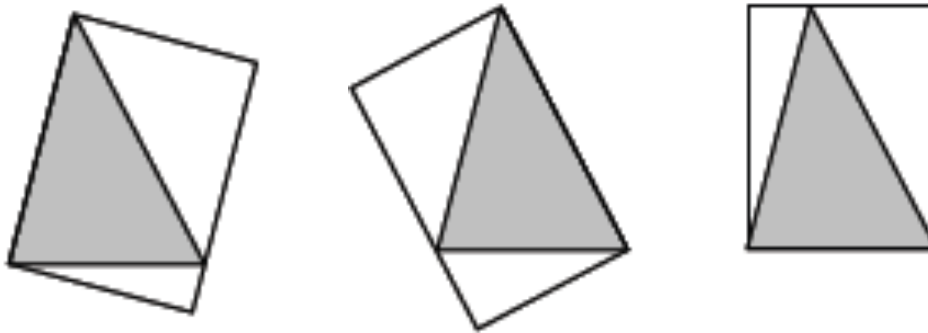
Shape Statement 3

3. Alex says:

There are three different ways of drawing a rectangle around a triangle, so that it passes through all three vertices and shares an edge. The areas of the rectangles are equal.

Is Alex's statement Always, Sometimes or Never True?

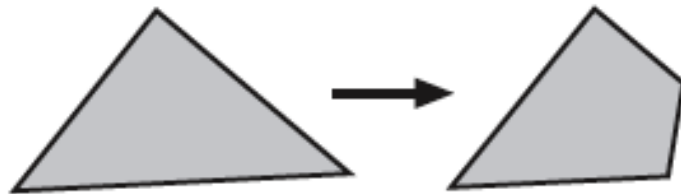
Fully explain and illustrate your answer.



There are more great tasks, e.g.,

Always, Sometimes, or Never True?

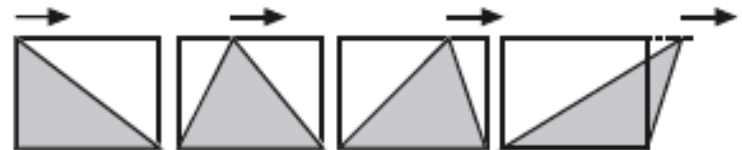
A Cutting Shapes



When you cut a piece off a shape you:

- (a) Reduce its area.
- (b) Reduce its perimeter.

B Sliding a Triangle



If you slide the top corner of a triangle from left to right:

- (a) Its area stays the same.
- (b) Its perimeter changes.

And, the students develop critiquing skills. The task:

Diagonals of a Quadrilateral

If you draw in the two diagonals of a quadrilateral, you divide the quadrilateral into four equal areas.

Is this statement always, sometimes or never true?

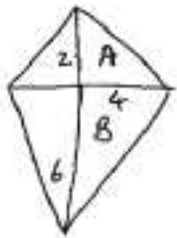
If you think the statement is always true or never true, then how would you convince someone else?

If you think the statement is sometimes true, would you be able to identify all the cases of a quadrilateral where it is true/not true?

They discuss the task, and sort out the mathematics. Then...

They're given other (hypothetical) students' work...

Student Work 1



A	B
$\frac{1}{2} \cdot 4 \cdot 2$	$\frac{1}{2} \cdot 4 \cdot 6$
$\frac{1}{2} \cdot 8$	$\frac{1}{2} \cdot 24$
$\neq 4$	12

Kite = not true

Student Work 2

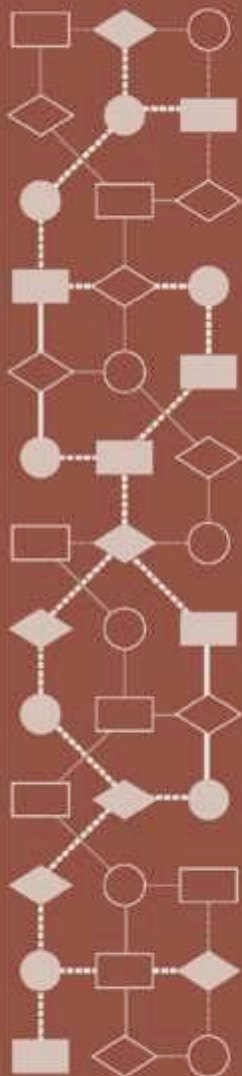


All triangles are congruent
(SSS)

Parallelogram = always true

And helped to critique it.

These are central skills called for in
CCSSM.



Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

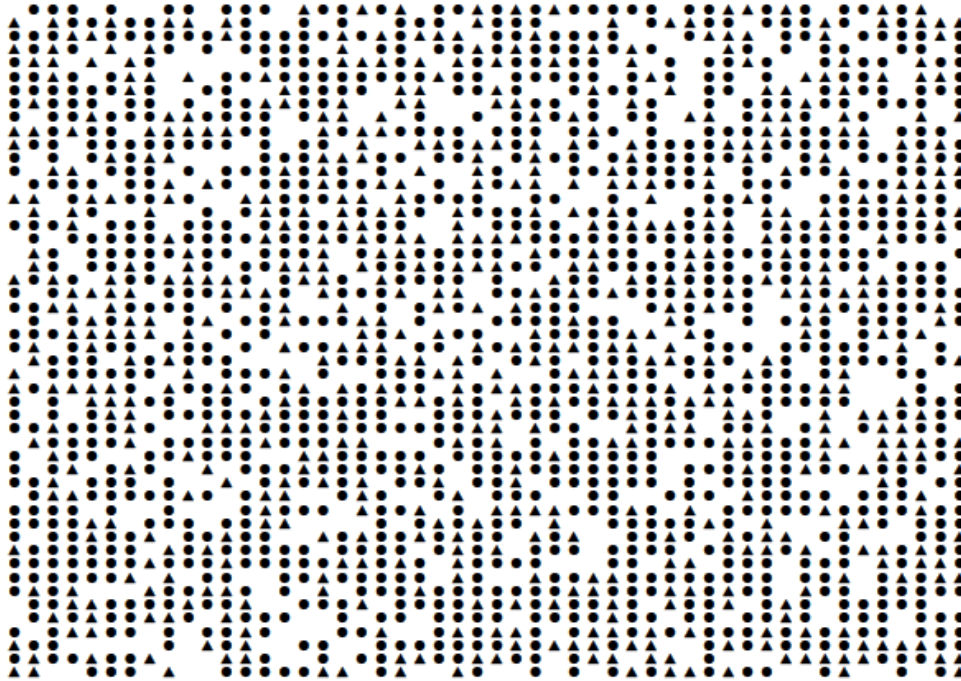
Estimating: *Counting Trees*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: <http://map.mathshell.org>
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Counting Trees

The diagram shows some trees in a tree farm.



The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one by one.

- Think of a method you could use to estimate the number of trees of each type.
- Explain the method fully.
- Use your method to estimate the number of old trees and young trees.

The Mathematics Assessment Project's goals are to:

- Help students grapple with core content and practices in CCSSM, and prepare them for the rich assessments they should (and it looks like, will) experience;
- Support formative assessment; and
- Do so in “curriculum-embeddable” ways.

We're building 20 FALs at each grade from 6 through 10.

They're **FREE**, at <http://map.mathshell.org/materials>.

Just google

Mathematics Assessment Project

To sum things up thus far:
The Common Core Standards
and their instantiation in the
Assessments offer a welcome
challenge.

We have to rise to meet that
challenge. Tools help; but how
about reflecting on our teaching?

**3. Thoughts about what to
look for in productive
mathematics classrooms
(a teaser for tonight's talk)**

What do you want to look for
in a math classroom? What
counts?

I want to return to the issue
of practices as a way of
“living” mathematics.

To illustrate this, let's look at a
video.

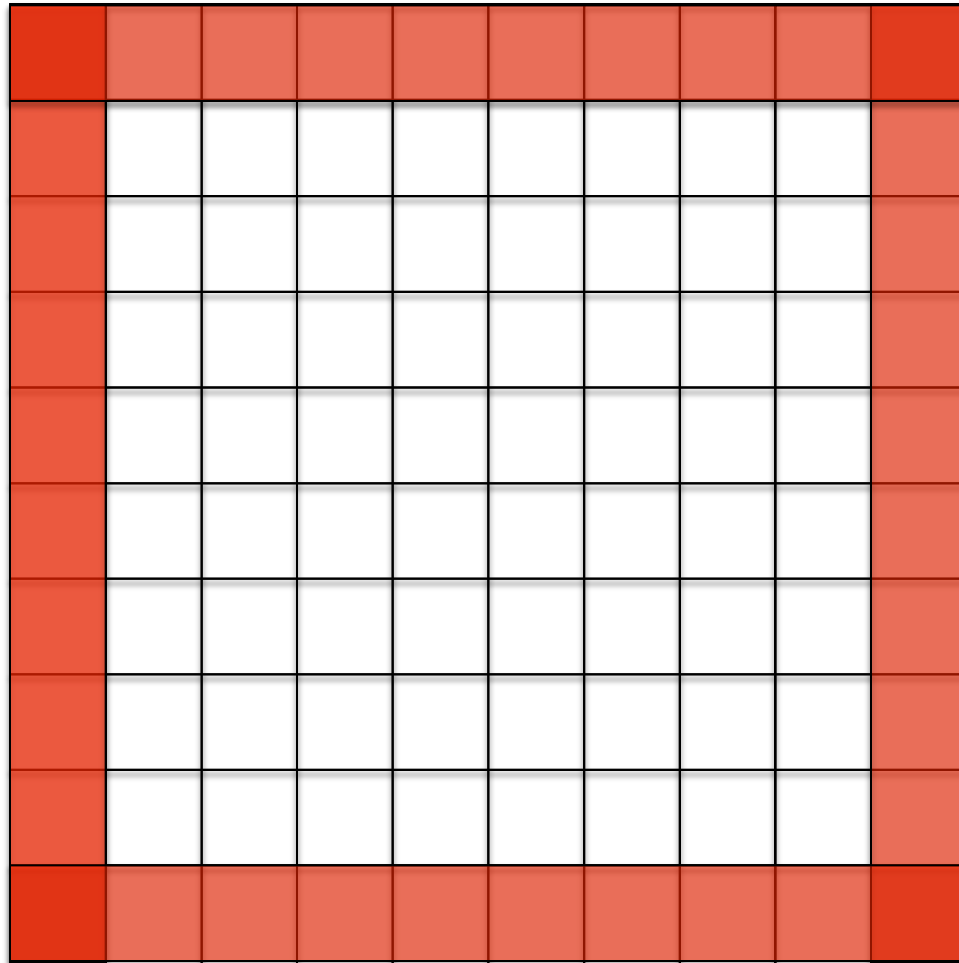
(There'll be more this evening)

The Border Problem, from

Connecting Mathematical Ideas by

Jo Boaler and Cathy Humphries

Here's a 10 x 10 grid.



How many border squares are colored in?

Key Questions for Math Classes:

- What were the big ideas, and how did they get developed?
- Did students engage in “productive struggle,” or was the math dumbed down to the point where they didn’t?
- Who had the opportunity to engage? A select few, or everyone?
- Who had a voice? Did students get to say things, develop ownership?
- Did instruction find out what students know, and build on it?

Key Questions for Math Classes:

- What were the big ideas, and how did they get developed? **Mathematical focus and coherence.**
- Did students engage in “productive struggle,” or was the math dumbed down to the point where they didn’t? **Cognitive Demand**
- Who had the opportunity to engage? A select few, or everyone? **Equity**
- Who had a voice? Did students get to say things, develop ownership? **Discourse, Student Agency**
- Did instruction find out what students know, and build on it? **Formative Assessment**

Put everything together:

Level	Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
1	Classroom activities are purely rote, OR disconnected or unfocused, OR consequential mistakes are left unaddressed.	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, OR a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The mathematics discussed is relatively clear and correct, BUT connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Students have a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The mathematics discussed is relatively clear and correct, AND connections between procedures, concepts and contexts (where appropriate) are addressed and explained.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	The teacher actively supports (and to some degree achieves) broad and meaningful participation, OR what appear to be established participation structures result in such participation.	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, AND/OR students respond to and build on each others' ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

and you have the dimensions of a framework for assessing lesson quality.

Develop rubrics tailored to different classroom activities:

- Whole Class discussions,
- Small Group work,
- Student Presentations,
- Individual work

And you get . . .

The Teaching for Robust Understanding of Math (TRU Math) Scheme

Episode Type		Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
Whole Class Activities: Launch, Teacher Exposition, Whole Class Discussion		How accurate, coherent, and well-justified is the mathematical content?	To what extent are students supported in grappling with and making sense of mathematical concepts?	To what extent does the teacher support access to meaningful participation for all students?	To what degree are students the source of ideas and discussion of them? How are student contributions framed?	To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas (when potentially valuable) or address misunderstandings when they arise?
	1					
	2					
	3					
Small Group work		How accurate, coherent, and well-justified is the mathematical content?	To what extent does teacher support students to interact with their group members to make sense of mathematical concepts, or individuals by themselves?	To what extent does teacher support/ group dynamics provide access to meaningful participation and "voice" for all students?	To what extent does teacher support/ group dynamics provide access to meaningful participation and "voice" for all students?	To what degree does the teacher monitor and help students refine their thinking within small groups?
	1					
	2					
	3					
Student presentations		How accurate, coherent, and well-justified is the mathematical content?	Cognitive Demand	Access	To what degree are students the source of presented ideas and response to presented ideas?	To what degree does the teacher use student presentations to support meaningful class engagement with core ideas?
	1					
	2					
	3					
Individual work		How accurate, coherent, and well-justified is the mathematical content?	To what extent are students supported in grappling with and making sense of mathematical concepts?	Does the teacher encourage meaningful engagement from all students?	How does the teacher frame individual student contributions? To what degree do students get to propose and defend their own ideas?	To what degree does the teacher explore student thinking about a problem (whether right or wrong) and work with the student on it?
	1					
	2					
	3					

**How can you use such a
scheme?**

As an individual, you can ask:

Before the lesson:

How can I use the five dimensions to enhance my lesson planning?

After the lesson:

How well did I do? What can I do better next time?

Better yet, think about working
with colleagues:

Imagine visiting each others'
classrooms – say having agreed to
teach the same FAL – and using
the TRU Math framework to ask:

1. What were the goals of the lesson, with respect to:
 - a. Rich content
 - b. Cognitive demand
 - c. Access
 - d. Student agency
 - e. Formative Assessment

2. What worked in moving toward these, what was problematic?

a. Rich content

b. Cognitive demand

c. Access

d. Student agency

e. Formative Assessment

3. How might we do things differently next time, w/r/t...

a. Rich content

b. Cognitive demand

c. Access

d. Student agency

e. Formative Assessment

I think (hope) that looking at
our teaching in this way can
really help.

Whew!

You've made it to part 4:

Q & A.