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Mathematics Pre-K through 8

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Problem Posing with the Multiplication Table

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ABSTRACT Mathematical problem posing is an important skill for teachers of mathematics, and relates readily to mathematical creativity. This article gives a bit of background information on mathematical problem posing, lists further references to connect problem posing and creativity, and then provides 20 problems based on the multiplication table to be used or adapted for classroom use by teachers of mathematics.

KEYWORDS *problem posing, creativity, multiplication table, times table, mathematics, elementary mathematics*

Introduction

In *The Art of Problem Posing*, Brown and Walter (1990) provide a reader-friendly treatment on how begin with a given scenario, such as that which is provided by GeoBoards, Pythagorean Triples, or the Fibonacci sequence, and implement various strategies to generate questions and modify them to produce new and interesting mathematical problems. Subsequently, other mathematicians and mathematics educators responded in kind, which resulted in a follow-up compendium entitled *Problem Posing: Reflections and Applications* (Brown & Walter, 1993). In a similar spirit, this paper aims to focus on a particular scenario, namely, the multiplication table, and pose a handful of problems appropriate for students at a variety of ages and stages in their mathematical development. By doing so, the hope is also to adhere to the recommendation of Trivett (1980) whose paper *The Multiplication Table: To Be Memorized or Mastered?* colorfully states:

> The recommendation here is that the multiplication table should be viewed, apparently for the first time by most people, as a dynamic synergetic combination of patterns, a veritable repository of mathematical relationships waiting as it were to gush forth from kindergarten through the secondary grades (p. 21).

The specific choice of scenario is rooted in at least two fundamental beliefs. First, even curricular foci regarded as typifying mathematical learning can be explored in ways that promote meaningful mathematical thinking. Second, teachers who devote time to developing mathematical tools or concepts should try to maximize their contribution to students' learning. With regard to the former belief, perhaps no tool in the mathematics classroom epitomizes rote memorization better than the multiplication table. Trivett postulates the "stultifying" view of "rote drill" to be one reason that the table fell out of favor during the New Math era of the 1960s; incidentally, this was the same decade in which Brown and Walter began their own work on Problem Posing (Trivett, 1980, p. 21; Brown & Walter, 1993). Insofar as the latter belief is concerned, it is reasonable to assume that the reader was once faced with this very table and asked to recall number facts such as 6×7 or perhaps even 12×9. But who among our readership explored this tool more deeply? Who among our readership witnessed mathematical relationships "gush forth" once the table was committed to memory?

To be clear, the suggestion here is not to eschew memorization in favor of exploration alone: a delicate balance must be struck in this regard; one we de-emphasize for present purposes, though it might be noted that such considerations date back to the early days of mathematics education as an area of study (e.g., Thorndike, 1922). Instead, the goal at hand is twofold, yet significantly more modest. The next section hints briefly at connections between problem posing and creativity as areas of academic study, while the sequel presents, without solutions, a smattering of problems one might pose directly or strategically modify for students of mathematics already familiar on a superficial level with the multiplication table.

Mathematical Problem Posing and Creativity Studies

Mathematical problem posing has always been intimately related to mathematical problem solving, and, in that respect, dates back at least to Pólya's *How to Solve It* (1945). Among the heuristics, or strategies, for problem solving suggested by Pólya, one finds examples such as: *Do I know a simpler problem? Do I know a related problem? How do I generalize this problem?* In each case, the solver is engaging with the problem at hand by posing new ones. The connections between problem posing and solving are drawn explicitly in Chapter 6 of *The Art of Problem Posing* (Brown & Walter, 1990) and the authors comment in *Problem Posing: Reflections and Applications* that they hoped to "influence colleagues to… further explore the connections between problem posing and solving" (Brown & Walter, 1993, p. ix). More generally, Brown and Walter discuss how to pose problems and incorporate them into a curriculum. Listing attributes of a mathematical scenario, modifying them or adding new ones, and having students create and edit their own "problem journals" are some of the techniques proposed to this end. It is worth noting that the practice of encouraging students to pose problems can be transferred to subjects other than mathematics, as well; for example, Mestre (2002) discusses high-achieving undergraduates posing physics problems, though he notes that with "the exception of a few studies in the domain of mathematics education… problem posing has remained unexplored as a tool for studying cognitive processes in the sciences" (p. 15).

The role of problem posing in developing new mathematics can be seen easily in the three classical problems of Ancient Greece: doubling the cube, trisecting an arbitrary angle, and squaring the circle. In more modern times, the great mathematician Cantor (1867) wrote in his doctoral thesis: "In re mathematica ars proponendi pluris facienda est quam solvendi."[1] At the turn of the twentieth century, David Hilbert released a list of what he deemed to be twenty five of the most important problems in mathematics. The list included as its first problem one posed by Cantor, a conjecture that came to be known as the Continuum Hypothesis, which was ultimately resolved by Godel and Cohen, with the latter proof developing a new and powerful technique within set theory and mathematical logic known as *forcing*. For more on Cohen's development of forcing from the perspective of creativity studies, see Dickman (2013). The twentieth century also included several problems posed by Andre Weil that came to be known as the *Weil Conjectures*, whose resolution led to the development of étale cohomology by mathematicians such as Grothendieck and Artin. Similarly, at the turn of the twenty-first century, the Clay Mathematics Institute published a handful of problems considered to be the most important in mathematics and offered million dollar (USD) prizes for a solution to any one of them (Carlson, Jaffe, & Wiles, 2006).

In Silver's *On Mathematical Problem Posing* (1994), the author notes that problem posing has been identified as an important pedagogical focus by leaders in mathematics and mathematics education, which includes a call "for an increase in the use of problem-posing activities in the mathematics classroom" by the National Council for Teachers of Mathematics (NCTM, 1989, 1991). Silver also cites Freudenthal (1973) and Pólya (1954) in remarking that "problem posing has been identified by some distinguished leaders in mathematics and mathematics education as an important aspect of mathematics education" (p. 19). Moreover, he notes that Kilpatrick suggests the formulation of problems as both a goal *and* method of instruction in the mathematics classroom (Kilpatrick, 1987).

Relevant to the purpose of the paper at hand, Silver (1994) includes a section entitled "Problem Posing as a Feature of Creative Activity or Exceptional Mathematical Ability." This section draws parallels between rooting out important research questions (Hadamard, 1945) and the idea of *problem finding* as an important feature of

[1] "In mathematics the art of asking questions is more valuable than solving problems."

creativity (Getzels & Csikzentmihalyi, 1976). The interested reader is referred to the work of Hadamard as well as a related piece due to Poincare (1982) for more on mathematical creativity from the perspective of influential mathematicians. As Poincare remarks:

> In fact, what is mathematical creation? It does not consist in making new combinations with mathematical entities already known. Any one could do that, but the combinations so made would be infinite in number and most of them absolutely without interest. To create consists precisely in not making useless combinations and making those which are useful and which are only a small minority. Invention is discernment, choice.

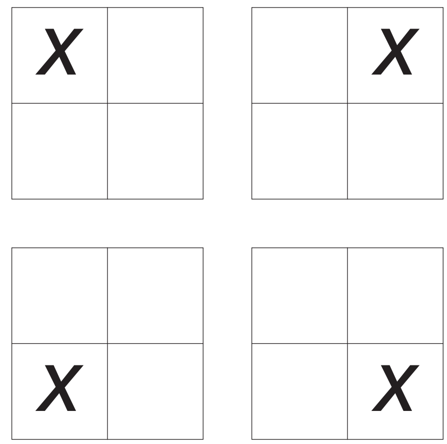
For more on connections between creativity and mathematical problem posing, in particular, see, Leung and Silver (1997), Sriraman (2004), and Van Harpen and Sriraman (2013) and the references contained therein.

Problem Posing: Multiplication Table

The following problems are posed in the spirit of Brown and Walter's *Reflections and Applications* (1993) and are not alleged to be "creative" in any precise sense; instead, such a judgment is left to the reader. The posing of problems ought not to be an unfamiliar task for the experienced teacher. From a theoretical perspective, one might use questions in a Piagetian approach to cause a sense of *disequilibrium* in the student; ideally, the resulting equilibration will lead to the learning and understanding of new concepts. Similarly, one might *scaffold* a larger problem or module by posing sub-problems; in particular, by helping a student work within his or her zone of proximal development in the sense of Vygotsky. In an everyday setting, any teacher who has written up original homework assignments or examinations will have experience in posing mathematical problems. However, it is possible that few teachers have considered using the multiplication table as a starting point for such problem posing. Thus, the reader is provided with twenty such problems below, which can be modified or distributed (with attribution) freely to students or prospective teachers of mathematics.

Twenty Problems

1. Estimate the number of distinct numbers in a 10×10 multiplication table.
2. How many of the entries in the 10×10 multiplication table are odd?
3. Starting at the 1 in a multiplication table, you can take steps to adjacent squares: right or left, up or down, but not diagonally. What is the biggest number you can arrive at in 10 steps?
4. Consider the 2×2 sub-grid cutouts from a multiplication table. For which x can you fill in the remaining three numbers?



5. Find 49 in the multiplication table. Add to it the six numbers above in its column, and the six numbers to the left in its row. What is the total?
6. What is the sum of all the entries in a 10×10 multiplication table?
7. Where do the multiples of 3 appear in the 10×10 multiplication table?
8. Which operations (Addition? Subtraction? Multiplication? Division? Other?) are related to the 10×10 times table?
9. Using a multiplication table, explain what it means for a number to be "prime."
10. Look at the numbers on the south-east diagonal starting from either 2 in the 10×10 times table: 2, 6, 12, 20, 30, 42, 56, 72, 90. What pattern or patterns do you see?
11. Starting at the 3 in row one of the multiplication table, jump like a knight (from chess) to move one square down and two to the right. Doing this repeatedly, you get the following numbers: 3, 10, 21, 36. What pattern or patterns do you see?
12. What kinds of symmetry can you find in the 10×10 multiplication table?

13. How many two digit numbers are in the 10×10 multiplication table?
14. In the 10×10 multiplication table, there exists a number x such that the number directly to the right of x is $x+8$. What is the next number over to the right (i.e., two space to the right of x)? Write your answer as an expression in terms of x.
15. How would you extend the 10×10 multiplication table to cover the negative numbers?
16. Which numbers are missing from the 10×10 multiplication table and why?
17. The numbers in the 2 row are called "even." What would you call the numbers in the 5 row?
18. Can the same number appear more than once in a single column? Why or why not?
19. Think about dividing a number in the multiplication table by the one directly to its left. When does this result in a whole number?
20. If you were trying to fill in a blank multiplication table and got stuck, what would you do?

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