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Mathematics Pre-K through 8

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Preservice Teachers Connecting Mathematics and Drumming

Anne Marie Marshall
Lehman College

ABSTRACT Increasingly, elementary classroom teachers are being called to teach a myriad of subjects, including visual art, dance, and music. Preservice teachers must be prepared to teach and integrate multiple subjects. To that end, preservice teachers will need experiences in their preparation that help them to see connections across content areas and equip them with the knowledge and skills necessary to plan and teach meaningfully integrated lessons. This paper describes an exploratory experience that afforded preservice teachers an opportunity to explore the connections between music and mathematics during a hands-on drumming lesson.

KEYWORDS *elementary mathematics, elementary mathematics methods course, integrated curriculum, integrating mathematics and music*

Academics and practitioners have advocated for integration of content in K–12 classrooms for many years (Brewer, 2002; Ellis and Fouts, 2002). Research indicates that using an interdisciplinary curriculum provides opportunities for more relevant, less fragmented, and more stimulating experiences for learners (Frykholm & Glasson, 2005; Koirala & Bowman, 2003). Further, studies suggest that arts integration increases student achievement (Catterall, Dumais, & Hampden-Thompson, 2012; Walker, McFadden, Tabone, & Finkelstein, 2011). Proponents of arts integration suggest that the traditional STEM acronym (short for science, technology, engineering, and mathematics) should become STEAM—to include an A for the arts to make the integration more powerful (Piro, 2010; Robelen, 2011).

As an elementary mathematics methods instructor, I have included curriculum integration in my courses for preservice teachers (PSTs). Integration of content is discussed and studied in the courses, and a major assignment requires PSTs to create an integrated mathematics unit highlighting mathematics outside of the school mathematics curriculum. In the past, PSTs have created mathematics units in which another content area was

added arbitrarily or artificially. For example, PSTs would create an “integrated” mathematics and science lesson that would take place at a zoo. The lessons called for elementary students to count animals or create story problems about the animals that they would see in the exhibits. While the setting might be fun and exciting for young students, the experience did not provide a rich opportunity to use mathematics to understand science, nor did it create a mathematics lesson that could not otherwise be conducted in the classroom. In an attempt to help PSTs better understand the process of creating high-quality integrated lessons, I worked with an ethnomusicologist to plan an introductory integrated lesson focusing on mathematics and music. During one class session of a mathematics methods course, two instructors—one mathematics educator (myself) and one ethnomusicologist—taught a lesson that explored the connections between mathematics and drumming. The goal was for PSTs to examine specific connections between mathematics and music and to understand meaningful content integration. This paper discusses the fundamental connection between mathematics and music and relays the experiences of PSTs in an elemen-

tary mathematics course lesson designed to illuminate these connections.

Integrating Mathematics and Music

Mathematics has long been used to understand music. As composer Reginald Smith Brindle (1987) concludes, nature itself “is amazingly mathematical” (p. 42 – 43). According to Brindle (1987), “mathematics is the basis of sound” and sound itself “in its musical aspects...exhibits a remarkable array of number properties” (p. 42 – 43). As mathematician David Wright (2009) points out, “It is not surprising that the symbiosis of the two disciplines is an age-old story,” reminding us that Pythagoras identified the “integral relationships between frequencies of musical tones in a consonant interval” and that Johann Sebastian Bach studied the “mathematical problem of finding a practical way to tune keyboard instruments” (p. ix).

The National Council of Teachers of Mathematics’ (NCTM) *Principles and Standards for School Mathematics* (2000) and the *Common Core State Standards for Mathematics* (CCSSM) (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA Center & CCSSO], 2010) all promote processes and practices that call for encouraging students to connect mathematics across content areas and to the real world, to see mathematics as useful and relevant, and to focus on real-world problems. Integrating curriculum provides rich opportunities for students to focus on relevant applications to the real world and make meaningful connections within mathematics as well as across different disciplines.

NCTM’s *Principles and Standards for School Mathematics* (2000) states that effective teaching also involves “approaching the same problem from different mathematical perspectives” (p. 3 – 4). NCTM deemed curriculum connections so important that “Connections” was named and described as a unique process standard. The Connections standard states, “Instructional programs from prekindergarten through grade 12 should enable all students to recognize and apply mathematics in contexts outside of mathematics” (p. 66). Further, the standard describes an important link between mathematics and science. This link is “not only through content but also through process” as “the processes and content of science can inspire an approach to solving problems that applies to the study of mathematics” (p. 66). Similarly, mathematics and music share both content (fractions, intervals, harmonics, etc.) and processes (creativity, problem solving, perseverance, precision, etc.).

It is important for students to be afforded opportunities to experience mathematics in a broader context. NCTM argues that students in kindergarten through grade 2 learn mathematics primarily through connections with the real world and students in grades 3 – 5 should learn to apply mathematical ideas in other disciplines. These ideas should expand into middle and high school as students use mathematics to explain complex applications in the outside world. NCTM’s Curriculum Focal Points (2006) also includes expectations for students to be provided with opportunities to apply mathematical ideas to other disciplines and to problems in everyday life.

Mathematics and music have many connections. Drumming, in particular, naturally complements mathematics. Most basic to drumming is the concept of learning to count repeating patterns (1, 2, 3, 1, 2, 3, etc.). Clapping or tapping two half beats during one whole beat space can help students begin to understand the meaning of fractions. Students can sense fractions as they learn the beat of particular time signatures (half time, three-quarter time, etc.). Later, the mathematics of ratios, symmetry, translations, frequencies, intervals, and harmonics can be made evident. Rhythms, specifically polyrhythms, are a primary component of the traditional music of many cultures, as well as jazz and popular music. Performing a musical measure according to two different musical meters simultaneously produces polyrhythms. In classrooms, polyrhythms can be demonstrated by dividing a musical measure in multiple ways, connecting fractions and the concepts of factors and multiples. For example, if one drum plays two equal beats in the same space of time that another drum plays three equal beats, the measure is divided into six beats (or a multiple of six). Figure 1 demonstrates a common West African polyrhythm in which six equal beats are divided in two different ways. In the top line, six beats are divided by three, thus splitting the six-beat measure into two groups of three beats. Counting this pattern, the drummer says 1-2-3-4-5-6, or 1-2-3-1-2-3. In the bottom line, six beats are divided by two, thus splitting the six-beat measure into three groups of two beats. Counting this pattern, the drummer says 1-2-3-4-5-6, or 1-2-1-2-1-2.

6 beats divided by 3 (results in 2 groups of 3 beats)	X (1	2	3)	X (1	2	3)
6 beats divided by 2 (results in 3 groups of 2 beats)	X (1	2)	X (1	2)	X (1	2)

Figure 1. West African polyrhythm example.

Exploring Multiples: An Elementary Connection

The concepts of factors and multiples are important in the elementary school mathematics curriculum. The CCSSM states that students must gain familiarity with factors and multiples starting in fourth grade. Specifically, students should be able to “Recognize that a whole number is a multiple of each of its factors” and “Determine whether a given whole number in the range 1 – 100 is a multiple of a given one-digit number” (NGA Center & CCSSO, 2010, p. 29). Students are expected to identify and work with factors and multiples in arithmetic and again when they explore fractions, using factors and multiples to find least common denominators to compare, add, or subtract fractions. Students also use these concepts when finding common multiples, least common multiples, and greatest common factors of two or more numbers. These concepts are often taught procedurally. Activities that allow students to explore factors and multiples by having experiences “discovering” these numbers can help students develop conceptual understanding of these topics as well.



Figure 2. Drumming notation example (Barshad, 2011).

The Lesson

The participants in this lesson were 18 graduate students, all PSTs, enrolled in an elementary mathematics methods course at a large urban public institution during the Spring 2012 semester. The mathematics and drumming lesson was designed to engage PSTs in a hands-on experience that afforded an opportunity to discover the connections between music and mathematics. Three unique drumming excerpts were played for the class, each deriving from a distinct region and world culture. PSTs were asked to listen and take notes on what they heard, how the music made them feel, and make conjectures about the cultural connection of each piece.

Then, PSTs were given an example of drumming notation (Figure 2) without any instructions on what the symbols represented. They were asked to work with a partner to explore the notation and make some hypotheses about what the symbols denoted.

The ethnomusicologist revealed the meaning of the symbols in the notation and played the string of notes on a drum. He taught the PSTs how to play the notation on their desks using their hands and invited several PSTs to the front of the room to play on the West African djembe drum. The ethnomusicologist described and showed several variations of drumming notations.

As PSTs reflected on the drumming excerpts they had listened to and the drumming pieces they played, they were able to tell us that they had heard different instruments at the same time and sometimes the instruments ‘landed’ on the same beat and sometimes they did not. The instructors were able to connect this to polyrhythms. Focusing on the Time Unit Box System notation, a popular system for notating rhythms in music, the instructors showed PSTs a new string of notes in a simple polyrhythm (Figure 3). The polyrhythm example showed how three distinct patterns played at the same time could create a unique sound. In this case,

Bell Pattern	X		X		X	X		X		X	X		X		X		X	X				
	Kon		kon		ko	lo		kon		ko	lo		Kon		kon		ko	lo				
Counting by 3	X			X			X			X			X			X			X			
	(1	2	3)	(1	2	3)	(1	2	3)	(1	2	3)	(1	2	3)	(1	2	3)	(1	2	3)	
Counting by 2	X		X		X		X		X		X		X		X		X		X		X	
	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)	(1	2)

Figure 3. Polyrhythm.

the overall pattern, worth 12 total beats, included three patterns—one based on two beats, one on three, and one on six. Figure 3 shows this 12-beat pattern repeated.

In order for PSTs to learn more about polyrhythms, the class was divided into two groups. Initially, one group clapped the “Counting by 3” pattern and one group clapped the “Counting by 2” pattern, while the ethnomusicologist played the Bell pattern (“Kon kon kolo kon kolo”) on a cowbell. Later, groups of PSTs played the pattern themselves. After playing the polyrhythm, PSTs discussed the mathematics involved in what they had just done. In this lesson, we created a rhythm on one drum that is mathematically equivalent to skip counting by 3’s (the “Counting by 3” pattern), and created a rhythm on another drum that is mathematically equivalent to skip counting by 2’s (the “Counting by 2” pattern). PSTs observed that every six beats, both drums landed on the same beat. The overlapping beats were visible in the polyrhythm notation (figure 3).

We used this as an opportunity to make connections to a previous mathematics lesson on skip counting and common multiples in which the numbers found by skip counting by 2’s were highlighted in one color and skip counting by 3’s were highlighted in a different color on a 100 chart. In this activity, a pattern emerges—every sixth number on the chart contains two colors and is a common multiple of two and three (which are both factors of 6). PSTs identified the same pattern emerge through their music. The drumming activity provided a real-world context to help students explore these multiplication concepts. The PSTs experienced how integrating mathematics and music could be used to show the same concept of finding multiples that was learned earlier in the semester. For homework, PSTs were required to write a lesson plan that integrated mathematics and music and reflected an appropriate CCSSM standard.

What Students Learned

Preservice teachers were able to see connections between mathematics and the world of music as a result of this experience. Discussion notes, written reflections, and homework were used as evidence to explore the impact of the experience on PSTs’ understandings of mathematics, music, and integrating content. Several ideas emerged from their work.

This activity aimed to build on PSTs’ prior experiences to deepen understanding of important mathematical concepts. The lesson discussion focused on the division

of twelve beats across the three patterns and the connection to the mathematical idea of common multiples. Furthermore, two additional elementary mathematics topics that could be connected to drumming were also discussed. PSTs described how the same activity could be used in an early elementary lesson on patterns in music in which students would identify, recreate, notate, and extend different drumming patterns. PSTs also proposed that the drumming activity could be used for older elementary students in a lesson on fractions. Students could explore the division of whole notes into half notes, quarter notes, eighth notes, etc. and use both rhythms and music notation to explore the concept of equivalent fractions.

Opportunities to make mathematical connections and see how mathematics is connected to the real world help students to develop deep understanding and illustrate that mathematics is not just a subject to be learned at school. Written reflections on the experience revealed that it helped several PSTs to understand that mathematics “lives everywhere.” One student reported, “After the lesson, I realized that math is really a subject in life, it is related to everything.” One PST shared, “I was surprised to see that math and music have so much in common.” Many PSTs reported learning about specific aspects of music that were related to mathematics. For example, PSTs wrote, “I learned that segments and counting is what actually gives us music,” “I learned that math and music have patterns,...symbols, and notation,” and “Music uses fractions, parts, and wholes.” The lesson appeared to help students appreciate the complexity and ‘mathematicalness’ of music.

In addition to reporting on what they had learned from the experience, many PSTs reported on how they believed integrated lessons would benefit their future students. This is a particularly important insight for PSTs as research suggests that mathematics and music are related in the brain from very early in life (Burack, 2005) and that music can help children learn mathematics (Geist, Geist, & Kuszniak, 2012). PSTs commented on the affordance of integrating subjects, as it allows for “practical approaches to learning and using mathematics” and “makes students think about math in the real world.” One PST pointed out the importance of encouraging students to speak about what they already know and then working with that as a foundation. Many PSTs reported that integrating subjects would help students enjoy mathematics. Notably, one student commented that, “Integrating subjects helps students to answer the notorious question ‘When are we ever going to use this?’”

Discussion

According to the Mathematical Sciences Education Board (1993), “Students learn important mathematics when they use mathematics in relevant contexts in ways that require them to apply what they know and extend their thinking” (p. 3). By integrating mathematics with other subjects, teachers can help promote curiosity for the subject. In addition, integration helps make mathematics more relevant to students’ lives. Real-world problems are multi-disciplinary in nature; therefore, school mathematics should help prepare students by presenting mathematical problems in context.

Traditionally, PSTs take a series of distinct teaching methods courses such as *Methods of Reading*, *Methods of Science*, *Methods of Mathematics*, etc., in their preparation. Consequently, PSTs learn how to teach mathematics in isolation from other subjects and could potentially exit a program without ever learning how to integrate subjects. This experience provided a relevant and engaging context—music—and demonstrated connections to previously learned ideas about multiples. After this lesson, some PSTs reported that the experience helped them understand how to “integrate math in fun and meaningful ways.” This experience gave PSTs an opportunity to understand how to integrate mathematics and music and also provided them with some skills to try curriculum integration themselves.

Standards for science, mathematics, technology, and English language arts each address the importance of making connections across disciplines (International Society for Technology in Education, 2007; National Council of Teachers of English/International Reading Association, 1996; NCTM, 2000; NGA Center & CCSSO, 2010; NRC 1996). The National Standards for Music Education (2014) Content Standard #8 specifically focuses on “Understanding relationships between music, the other arts, and disciplines outside the arts” (National Association for Music Education, 2014). Consequently, elementary educators responsible for delivering instruction in all content areas should learn how to create new, or modify existing, curricula to meaningfully integrate multiple subject areas, including the arts.

In many school districts, music, art, and other non-tested subjects are being removed from the curriculum as teachers are required to focus time and resources on teaching mathematics and English language arts (Darling-Hammond, 1999; Oreck, 2006) due to pressure from policies such as No Child Left Behind (NCLB, 2001). Further, budget cuts are typically made first in

non-tested subjects such as science, social studies, and arts programs (Pederson, 2007; Schneider, 2005). With the elimination of specialists such as art and music teachers, general education classroom teachers will need to find ways to integrate the arts into the curriculum (Oreck, 2006). Hence, teacher preparation programs must find ways to include curriculum integration into their programs.

Notably, in order for PSTs to fully understand meaningful integration of mathematics and other subjects, and to be able to transfer these ideas to practice, they may need multiple opportunities to experience such learning themselves. Indeed, further research on the effectiveness of these sorts of opportunities is needed. Specifically, future studies could aim to understand how PSTs apply and extend such experiential learning in their practice.

Conclusion

“Mathematics in K – 12 should provide opportunities for students to recognize and use connections among mathematical ideas, understand how mathematical ideas interconnect, and recognize and apply mathematics in contexts outside of mathematics;...[all students should] experience mathematics by working on problems arising in contexts outside of mathematics” (NCTM, 2000, p. 65). This exploratory experience provided PSTs an opportunity to explore the connections between music and mathematics during a hands-on drumming lesson. In the age of high-stakes testing and teacher accountability, it is critical that PSTs are prepared to implement an integrated approach to teaching multiple subjects. PSTs will need experiences in their own preparation that will not only help them to see connections across content, but also provide opportunities to learn how to plan and teach meaningful integrated lessons.

References

- Barshad, A. (2011). Bangin’. *New York Magazine*, October 30. Retrieved from <http://nymag.com/news/intelligencer/topic/drum-circles-2011-11>
- Brewer, T. M. (2002). Integrated curriculum: What benefit? *Arts Education Policy Review*, 103(4), 31 – 36.
- Brindle, R. S. (1987). *The new music: The avant-garde since 1945*. New York, NY: Oxford University Press.
- Burack, J. (2005). *Uniting mind and music: Shaw’s vision continues*. *American Music Teacher*, 55(1), 84 – 87.

- Catterall, J. S., Dumais, S. A., & Hampden-Thompson, G. (2012). *The arts and achievement in at-risk youth: Findings from four longitudinal studies*. Retrieved from <http://www.arteducators.org/research/research>
- Darling-Hammond, L. (1999). *Teaching as the learning profession: Handbook of policy and practice*. San Francisco, CA: Jossey Bass.
- Ellis, A. K. & Fouts, J. T. (2002). Interdisciplinary curriculum: The research base. *Music Educators Journal*, 87(5), 22 – 26.
- Frykholm, J., & Glasson, G. (2005). Connecting science and mathematics instruction: Pedagogical context knowledge for teachers. *School Science and Mathematics*, 105(3), 127 – 42.
- Geist, K., Geist, E. A., & Kuszniak, K. (2012). The patterns of music: Young children learning mathematics through beat, rhythm, and melody. *Young Children*, 67(1), 74 – 79.
- International Society for Technology in Education. (2007). *National educational technology standards and performance indicators for students*. Eugene, OR: International Society for Technology in Education.
- Koirala, H. P. & Bowman, J. K. (2003). Preparing middle level preservice teachers to integrate mathematics and science: Problems and possibilities. *School Science and Mathematics*, 103(3), 145 – 154.
- Mathematical Sciences Education Board. (1993). *Measuring what counts: A conceptual guide for mathematics assessment*. Washington, DC: National Academy Press.
- National Association for Music Education. (2014). National Standards for Music Education. Retrieved from <http://musiced.nafme.org/resources/national-standards-for-music-education>
- National Council of Teachers of English/International Reading Association. (1996). *Standards for the English language arts*. Retrieved from <http://www.ncte.org/library/NCTEFiles/Resources/Books/Sample/StandardsDoc.pdf>
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: NCTM.
- National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: NGA Center & CCSSO.
- National Research Council. (1996). *National science education standards*. Washington, DC: National Academy Press.
- No Child Left Behind (NCLB) Act of 2001, Pub. L. No. 107 – 110, § 115, Stat. 1425 (2002).
- Oreck, B. (2006). Artistic choices: A study of teachers who use the arts in the classroom. *International Journal of Education & the Arts*, 7(8), 1 – 27.
- Pederson, P. V. (2007). What is measured is treasured: The impact of the No Child Left Behind Act on nonassessed subjects. *The Clearing House*, 80(6), 287 – 291.
- Piro, J. (2010). Going from STEM to STEAM: The arts have a role in America's future, too. *Education Week*, 29(24), 28 – 29.
- Robelen, E. W. (2011). STEAM: Experts make the case for adding arts to STEM. *Education Week*, 31(13), 8.
- Schneider, C. (2005). Measuring student achievement in the future based on lessons from the past: The NAEP arts assessment. *Music Educators Journal*, 92(2), 56 – 61.
- TERC. (2006). *Investigations in numbers, data, and space*. Menlo Park, CA: Dale Seymour.
- Walker, E. M., McFadden, L. B., Tabone, C., & Finkelstein, M. (2011). Contribution of drama-based strategies. *Youth Theatre Journal*, 25(1) 3 – 15.
- Wright, D. (2009). *Mathematics and music*. Providence, RI: American Mathematical Society.