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Mathematical Modeling—The Teacher's Responsibility

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Mathematical Modeling has recently become a compulsory part of the Common Core State Standards (CCSS) in the United States. Future teachers must have a strong background about different aspects of modeling and also about appropriate methods for how modeling can be taught. In which ways modeling is the teacher's responsibility is presented in this paper on the basis of theoretical reflections as well as an in depth focus on more practical experiences and results of empirical studies of pre- and in-service teacher education. Finally a model of necessary teaching competencies for modeling is described in the four dimensions and a conclusion is drawn.

Keywords: teaching competencies for modeling, teacher education, Common Core State Standards

Introduction

This paper is structured as follows: In the first part, some comments are given concerning Mathematical Modeling as an educational standard and what this means for the Common Core State Standards (CCSS) in the United States from the author's perspective and on the basis of what has been learned in Germany since the Education Standards in Mathematics came up in 2003. In the second part, research activities and results, especially of teacher education within modeling, are pointed out. From this research we derive the question that drives the third section: Which competencies are required for teachers to teach modeling? (Werner Blum's paper in these proceedings focuses on the other perspective (see page 54); that is, the learners, and—perhaps most essential along with their needs—how they learn modeling effectively, especially through quality teaching.) The fourth part, focusing on the need for persons who are able to qualify the college teachers or teacher trainers to teach modeling, is especially important. Several implications are formulated before concluding with the discussion.

Mathematical Modeling as an Educational Standard

The starting question of this section is: Why are there education standards in mathematics and how has Germany dealt with this? Germany's poor results on the PISA in 2003, especially at the lower end of the achievement scale, caused the Ministry of Education in Germany to establish the Education Standards in Mathematics. Since 2003 (see Blum, Drüke-Noe, Hartung, & Köller, 2006) the Mathematics Education Standards have been implemented for the end of primary school (grade 4), for the end of "Hauptschule" (grade 9), and for the end of normal secondary school (grade 10).

When you have a look at our German mathematics education standards, you find that they describe competencies that are related to a special field and what pupils have to achieve for a particular grade, which occurs at the end of grade four or at the end of Hauptschule. The Common Core State Standards of the United States on the other hand define "standards" as what students should understand and be able to do.

This definition is a little bit different between the two countries; in Germany, the key word in the education standards in mathematics is *competencies*. Based on the definition by the German psychologist Weinert (2001), competencies refer to ". . . a roughly specialised system of abilities, proficiencies, or skills that are necessary to reach a specific goal. This can be applied to individual dispositions or to the distribution of such dispositions within a social group or an institution." The design of the Education Standards in Mathematics in Germany is a pragmatic three-dimensional model, that which consists of: three levels of performance; five mathematical content areas or guiding ideas, which are curriculum-wise valid; and six mathematical competencies, also called the process dimension. One of the central competencies—and this is why I characterized competencies before—is mathematical modeling. Mathematical modeling, shortly described as the transition process between reality and mathematics, is also a compulsory part of the standards for all mathematics teachers in Germany.

The Common Core State Standards in Mathematics (CCSSM) of the United States have strong goals stated. For example, one reads: "It is time for states to work together to build on lessons learned from two decades of standards based reforms." Looking at the earlier NCTM Standards, one can see that Mathematical Modeling is not explicitly mentioned like in the CCSSM. Modeling, when thinking about the short definition from before, was more or less implicitly included in the NCTM Standards within the expression 'connections.' Under the heading of *connections*, one finds that they see "mathematical connections in the rich interplay among mathematical topics in contexts that relate mathematics to other subjects." Of course, this description is not satisfying for me as a researcher in mathematical modeling, but the CCSSM shows that modeling is a central standard now. Under its description we read: "Modeling: …students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace." This citation makes clear that the extra-mathematical contexts, situations, and real problems have to be solved with the help of mathematics. Transition processes back-and-forth between reality and mathematics will take place—otherwise we are not speaking of true mathematical modeling. Mathematical Modeling as a compulsory standard is a challenge for mathematics teachers in the USA—as it is still a challenge for German mathematics teachers since the inclusion of our modeling standard in 2003. If we are to take the Common Core State Standards seriously, it says: "It is time to recognize that these standards are not just promises to our children, but promises we intend to keep," then teachers have the responsibility to teach highly qualitative lessons in mathematical modeling. In this way, "Modeling helps to keep the kids together" (Pollak, 1979).

Research in Mathematical Modeling Teacher Education

What do we know—especially from empirical studies—about the teacher's responsibility while engaging in modeling activities? In the last 10 to 15 years, a lot of empirical research was carried out with the goal of investigating the teaching and learning processes of mathematical modeling: first, in secondary and tertiary institutions, but also, in part, in primary school. Learning and teaching go hand in hand, and so they often were and are investigated together, because it makes sense in order to get a fuller picture of the interplaying processes. Looking at the research on mathematical modeling, one can distinguish between two groups of studies which strongly maintain the teacher's perspective. I label the groups here as studies with a focus on answering questions about: (a) effective or adequate teaching of modeling in classroom or laboratory settings and (b) how teacher education in universities can be developed, optimized, and evaluated. In some cases, research studies can be classified in both groups, though this depends on one's subjective interpretation.

The number of research studies is much larger in group (a) than in group (b). I point out below some central studies of group (a) and their results in view of characterizing the responsibility of teachers. First, I must mention the extensive research carried out by Richard Lesh (Lesh & Doerr, 2003). The *Model-Eliciting-Activity (MEA) Approach* can be seen as an important outcome from the debate on modeling from the United States and from Lesh and his MEA-group. In the opinion of Lesh, MEAs should start in kindergarten, so that learners can adopt positions typical of various professions, like engineering or economics, for the purpose of understanding how mathematics is needed in real life. Beyond the individual learner's perspective, Lesh keeps the important role played by teachers during modeling activities in mind. To name a few examples, his research focuses on: teachers' MEAs; teachers' understandings and interpretations of real-life contexts; and, their reflections about this, which again have effects for the learners (Schorr & Lesh, 2003). An important research question was: Which kind of teacher interventions are adequate while modeling? The results of the DISUM project (Didactical intervention modes for mathematics teaching oriented towards self-regulation and directed by tasks; see Blum & Leiß, 2008) showed that in everyday classrooms, nearly no strategic interventions take place, and most interventions are not adaptive.

The diagnosis of a learner's problems while modeling comes of course one step before providing teacher intervention or feedback. But diagnoses are only possible if the teacher has enough content knowledge and pedagogical content knowledge. Thus in this case teachers need to have a strong knowledge of modeling and understand the specific modeling task he or she has given the students. The results of the CoCA project make clear that, firstly, most teachers are *not* able to diagnose the needs of their learners and to give feedback; and, secondly, teachers who received these diagnoses and feedback themselves performed better with their learners (Leiß, 2007; Blum, 2011).

There are several best practise reports about modeling courses and reflections at the university level (e.g., Blomhoj & Kjeldsen, 2007; Schwarz & Kaiser 2007; Maaß & Gurlitt, 2010). However, the results show a change in mathematical beliefs of the university students or in-service teachers only because of dealing with modeling problems. Furthermore, the difficulty associated with modeling competencies can be sub-categorized into three essential barriers: *material*, *time*, and *assessment*, though for teachers experienced with modeling, time was not as strong a barrier as it was for the inexperienced teachers (Schmidt, 2009; Borromeo Ferri & Blum, in press).

Another interesting and important aspect to evaluate is the development of workshops or university courses for teaching and learning modeling. This was the central point of several design-based research studies conducted by Lesh and others, and is also a research field within the "Modelling Group" at the University of Kassel. The approach of Lesh's multitier professional development is that teachers learn to understand how their students think while modeling. "Multitier professional development is designed to focus teachers' attention on students' modeling behavior" (e.g., Lesh & Doerr, 2003). We have similar approaches when designing courses that use video sequences of students' modeling processes or their materials, as well as course evaluation questionnaires, interviews, and learning diaries filled out over the semester. We thus arrive at the following conclusion: "We would like to emphasize once again the necessity of becoming mathematics teachers having vast opportunities to deal with mathematical modelling both on a theoretical and on a practical level, including experiences with modelling at school" (Borromeo Ferri & Blum, 2010).

As I mentioned at the beginning, more research is needed in this whole field of teacher education within mathematical modeling: for example, longitudinal studies, which investigate the effects of in-service teachers' knowledge about teaching modeling that they learned at university on the modeling competency of their students. This could be compared with teachers who had not had the opportunity to learn about how to teach modeling even at a basic level.

Competencies for Teaching Modeling

We begin by looking into two results from large-scale studies that investigated mathematics teachers' pedagogical content knowledge (PCK) and content knowledge (CK). First, the data based on the COACTIV study (Kunter, M., Klusmann, U., Dubberke, T., Baumert, J., Blum, W., Brunner, M., et al, 2007) in Germany, which used a representative sample of secondary teachers and their students. This study used classes from the PISA study in Germany in 2003 and 2004, and had the central result show that teachers' PCK is a significant predictor for his or her students' achievement gains. The second study, the TEDS (Teacher Education Development Study), carried out internationally, came to similar results (Blömeke, Kaiser, Lehmann), which means concretely that *teachers matter most*. So the learning outcome and the effects of teachers' competencies in general is represented in this model, in which quality teaching has an important impact on students' learning. Looking to psychometric analyses on the basis of a regression model, one sees that the PCK of teachers, in particular, classroom management and the cognitive level of tasks, have a strong influence on mathematics achievement on the individuals. Beneath a strong mathematical background, PCK is needed—as I have mentioned—for teaching modeling and applications. Together with Werner Blum, we developed a model that includes required competencies to teach modeling in a qualitative way, and the theoretical background to support our conclusions.

Up until now this model has not had the empirical evidence to the effect that several dimensions and sub-dimensions were not tested. Currently we are working on test instruments and it is—speaking from a psychometrical perspective—not easy to do so. However, our model arose as the outcome of several modeling seminars at our university, which are evaluated and have been modified many times in response to the variegated needs of the university or of the in-service teachers while teacher training (Figure 1). Now we have a deeper look into the four dimensions in which I will only highlight a select few aspects in the interest of time.

The first dimension, the theoretical dimension, focuses on the question, 'what is meant by mathematical modeling,' and how it is interpreted internationally. This is summed up under (b) aims and perspectives. Bearing in mind that mathematical modeling is a complex process, while also one that can be represented through different modeling cycles showing the transition processes between reality and mathematics, it is the *teacher's responsibility* to know at least some of the modeling cycle and to reflect on his or her own modeling activities. This means that pre- and in-service teachers have to work on a modeling problem, in particular at the beginning of the course, as well

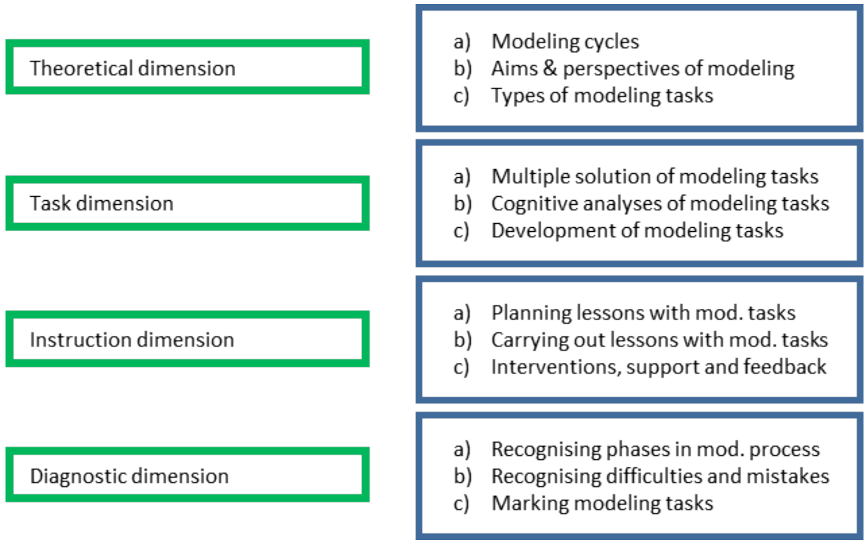

Figure 1.

as later on. In the words of Alan Schoenfeld: Understanding that modeling is not a spectator sport should be clear to teachers as soon as possible. It does not matter if the cycles have four or seven steps, but it is important to know which model of the cycle can be used for different purposes. I will show you this aspect later on. Starting with this theoretical part is often not easy for my university students, but in the upcoming, more practical-oriented lessons they have recognized that this is a fundamental need for getting the whole picture.

Below I include several quotations from a recent series of workshops I held at Teachers College. Because my students write learning diaries during their process of understanding the teaching and learning of mathematical modeling, the importance of the first dimension becomes apparent: "After teaching our modeling task, I know why we learned theoretical background about the perspectives and cycles: you need it to get the whole picture." (Chris)

The goal of the task dimension is to work and discuss with teachers the criteria for modeling tasks. What is a good modeling task? Furthermore, they learn to carry out cognitive analyses of modeling tasks, which means, concretely, to classify the solution steps in the different phases of the modeling process. This is non-trivial. Another important aspect is the development of a modeling task within a group. This often takes a long time, but it is very helpful for the students: "It was good that we created our own modeling task in our group. However, we recognized that this was a difficult undertaking." (Tina)

Closely related to the question of what describes a good or a bad modeling problem is to think about qualitative instruction. Within the instruction dimension, aspects of lesson planning with modeling problems are in the foreground. As was the case during my workshop at Teachers College, the students taught their modeling problem at school and we reflected on it afterwards. In doing this, the balance between theory and practice is reinforced. With their theoretical knowledge about teacher interventions and feedback, teachers are able to act strongly on a metacognitive level: "Teaching the modeling task in grade 9 was important and helpful for my understanding of modeling and the practical transformation in school. [. . .] It was good to have a chance to implement the modeling problems at school." (Birgit)

Finally, the diagnostic dimension suggests that teachers are able to recognize difficulties and mistakes in different phases of the modeling process, and thereby have the knowledge to develop tests and to mark them: "Diagnostic competencies and knowledge about interventions are important for teachers in general, but now I used this knowledge when teaching mathematical modeling specifically and I was able to help my students in class very adaptively while solving the task." (Tim)

This model for teaching competencies shows a lot of the teachers' responsibilities and after a full course for pre- and in-service teachers the participants are sure that "teaching modeling at school means that you must have a broad knowledge about it." (Carolyn)

Network-Strategies for Qualifying Teacher Trainers

When talking about the implementation of the upcoming Common Core State Standards, surely a lot of work must be done to convince teachers and parents. For some of them the Standards seem to be new, different from previous NCTM publications, and hence a challenge. This was and is still a normal reaction of teachers in Germany, and since our own implementation in 2003 there have been plenty of critics and critiques. Also in the United States, a lot of workshops will be organized so as to soften the critics and to make clear that the Standards are not bad, but instead that they give the opportunity to think about the teaching culture of mathematics and the ways in which we deal with mathematical content.

A legitimate question to ask is: Who will qualify as teacher trainers or university teachers so that the Standards, and Mathematical Modeling in particular as a "new" standard, will be successfully implemented into the lessons? Logically we might assume that curriculum developers are able to do this with regard to the development of standards in general, but in this case a more specialized expertise in mathematical modeling is needed from university teachers and researchers in the field of mathematical modeling. This is surely also true for practicing teachers looking to gain the broad knowledge necessary for the teaching and learning of mathematical modeling. If we want to implement modeling in our day-to-day teaching, then we have to start at the university level with mandatory courses. This means, on the one hand, modeling from an applied perspective, and on the other hand, the teaching and learning of modeling. Furthermore, teacher training for in-service teachers is important, but a good way to spread these teaching competencies for modeling is to educate so-called "multipliers." These multipliers from different school districts are then able to give courses on the teaching and learning of mathematical modeling for other teachers. It is important that there is horizontal and vertical collaboration and good network-strategies as well, in order to provide high-quality instruction.

Conclusion

The goal of my talk was to highlight the responsibility of teachers as it relates to mathematical modeling. Mathematical modeling, and this we know from empirical studies, cannot be expected to transfer naturally from learning mathematics, but rather must be learned specifically. We have seen that the model of teaching competencies contains theoretical and practical aspects so teachers, if they are trained in these competencies, would have, for starters, a good foundation with which they can carry out their responsibilities.

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