Journal of Mathematics Education at Teachers College

Spring – Summer 2010
Inaugural Issue

A Century of Leadership in Mathematics and its Teaching

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The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics and Education. Photographs are drawn from the Teachers College archives and personal collections.

This issue honors NCTM 2010 Lifetime Achievement Medalist, Dr. Henry O. Pollak, who has completed 22 years as a member of the Program in Mathematics and Education at Teachers College. Dr. Pollak has contributed so much to the mathematical preparation of the Program's graduates and to the communities of mathematics and mathematics education professionals in the United States and throughout the world.

David Eugene Smith, also pictured on the front cover, was the founding professor of the Teachers College Program in Mathematics and Education. Like Dr. Pollak, Professor Smith was widely respected by both mathematicians and educators.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics and Education. As a peer reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of Mathematics Education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2010-2011 issues are: *Teacher Education, International Education, Curriculum, Technology, and Equity*—all centered upon mathematics and its teaching. The *JMETC* will have a distinctive niche in the world of education publishing. Our readers are educators from pre K-12 and college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education.

Manuscript Submission

We seek conversational manuscripts (2500-3000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. All manuscripts may be submitted electronically at www.tc.edu/jmetc. This system will help keep the submission and review process as efficient as possible.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. All inquiries should be sent to Ms. Krystle Hecker, P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027.

More Information available online
www.tc.edu/jmetc

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *International Mathematics Education*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to international or comparative mathematics education. Articles should be submitted to www.tc.edu/jmetc by September 1, 2010. The fall issue's guest editor, Dr. Juliana Connelly, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2010, and final drafts of selected papers are to be submitted by November 1, 2010. Publication is expected in late November, 2010.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the Fall 2010 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citation checking, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form* from Ms. Hecker. Return the completed form to Ms. Krystle Hecker at JMETC@tc.columbia.edu or Teachers College, Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2011	Curriculum
Fall 2011	Technology
Spring 2012	Equity
Fall 2012	Leadership
Spring 2013	Psychology

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To obtain additional copies of *JMETC*, please visit the Journal's website www.tc.edu/jmetc.

Innovative Instruction in High School Mathematics

Cindy Cheung

New Semester, New School

Last Friday was my first day student teaching at the high school. During first period, we observed a twelfth grade mathematics class. Phil (cooperating teacher) gave out the "Math Now" as students entered the class. You could see the careful scaffolding to prepare students for the main worksheet.

In the groups I've observed, students work relatively well with each other. With clear instructions from Phil, Michelle (a second student teacher) and I were not to help students directly. Instead, we could only pose questions. It was a bit frustrating at first and I found myself breaking the rule a couple of times, but it helped me with something that I've always struggled with: questioning. In one group, there were only two people. One of the students did well with the problems but was unsure of himself. I did not tell him if his answers were correct but had the student practice self-checking and checking with his partner. Also, I had him explain to his partner how he obtained his answer.

For second and sixth periods, Phil and Jesse co-teach a 10^{th} grade class and as the math now, students wrote individual letters addressed to me and the other student teachers talking briefly about themselves and how we could help them in the next few weeks. During second period, Jesse talked about Gauss and gave the class the problem, "Add the numbers from 1 to 100." She suggested that students try to find patterns and shortcuts rather than adding all the numbers one by one.

I will end this journal by sharing what the cooperating teachers discussed with student teachers. They said there are three goals that we should work on: clarity, routine, and consistency. They didn't want to make any distinctions between cooperating teachers and the student teachers. We were all there to learn. Jesse was transparent about her desire to improve as a teacher after many years in the profession. It creates for us an environment where we are not afraid to fail, to make mistakes and to try our hardest to improve ourselves as teachers. I feel that it is very refreshing to be in a school where I see cooperating teachers trying to make connections with theory and practice in pedagogy. I'm glad that in this new placement, I will be in an environment that will enable me to make sense of the difficulties and obstacles I will face in the classroom.

Meredith Klein

Real World Applications

Although we didn't have any school this week, there are a couple of things that I have been thinking about anyway. The first is the value of "real life math problems." In seminar, we often talk about concrete (vs. abstract) problems. A couple of weeks ago, when I was tutoring the eighth grader that I work with, we came across a problem that was approximately: "The equation $x^2 + 12x + 48$ models the population of worms on an atoll in the Pacific Ocean. A piece of the atoll breaks off, carrying with it $x + 20$ worms. Find the population of worms on the remaining atoll." Naturally, the first reaction that we both had was to laugh and reread the problem. Then she asked me a great question: "What is x?" …meaning, what is the significance of x? We had a bit of a discussion about population constraints, etc, but this was a great point. The objective of the problem was subtracting polynomials, but I couldn't decide whether the person writing the problem had a sense of humor or really thought that this was a "real world application" of subtracting polynomials! Meanwhile, the idea of "x" is still very abstract, which does not fit with the objective of making the subtraction into a word problem. I think that Kimberly (a student teacher) made a great point last week about the fact that the point of math isn't to be able to do a problem on a napkin in a restaurant, but because these abstract concepts come together to define a very interesting discipline. I don't know if we should, as educators, be leading students around through these contrived problems, justifying them with the promise of "real world applications." Why not teach them matrices if we want to talk about real world applications? We could then be showing students an application for math that is actually contemporary and useful.

The second thing that has been on my mind is the value of going over homework for a large portion of a class period. I think that this strategy is not appreciated, and I understand why, but I think that it certainly can be done in a constructive way. For example, if students come to class having tried the homework with little instruction, the class discussions are more constructive for them. This is no different from doing the reading for a class prior to the lecture. Students select the problems that they want to go over and discuss approaches and solutions as a class. The teacher is only a mediator, and students direct the conversation. Because they have all tried the problems already, even if they weren't able to solve them, they can work as a class to present different approaches to a problem and even compare which ones are best.

Kitty Yang

Finding a Budding Mathematician in Every Student

At my high school placement, the ninth grade teachers reserve one morning a week for "Kid forums," during which we focus on one struggling student and identify strategies to help him or her succeed. It was with great interest that I participated in my first "Kid forum" to discuss Jake, one of the students in my morning class.

Jake is the prototypical bright but underachieving student. He takes full advantage of my cooperating teacher's policy of allowing students to make up homework during lunchtime, effectively turning homework into lunchwork. Teachers agree they like Jake in class because he is intelligent and good-natured; his failures are for lack of effort.

After the meeting, I decided to work closely with Jake. I was intrigued by his teachers' characterization of him, and I wanted to observe firsthand. In class, he speeds through exercises in his head, and sits bored for most of the period. In a lesson on solving geometric word problems, we gave the class the following problem:

> The length of a rectangle is 2 cm less than its width. The perimeter is 16 cm. What is the length?

The algebraic way to answer the question is to set up two equations and solve. Letting w denote width, length = l = w-2, and perimeter = 16 = 2w + 2l = 2w + 2(w-2). Solving, we found that w = 5, l = 3.

However, Jake solved it in a much faster way, while demonstrating a deep understanding of geometry and numbers. He said that since a rectangle has four sides, he divided 16 by 4 to get 4 (the "average" of length and width). Because the length and width differ by 2, he split the difference between the length and the width (he subtracted 1 from 4, and added 1 to 4). His answer was a rectangle with sides 3 and 5, which gives the same solution faster and more intuitively than the algebraic method.

Most notably, his process of splitting up the difference between the length and width shows he understands that to "preserve" the perimeter, whatever you subtract from the length, you must add to the width. In my opinion, his method shows that he understands the geometric properties of perimeter.

This is the type of thinking the not-so-secret mathematician in me actively seeks to highlight in students. Mathematics is inherently playful and intuitive, not to be confused with the rigid and inflexible methods we often see in mathematics. Given a less structured characterization of math, it's not difficult to find a budding mathematician in every student.

Meredith Brown

The Benefits of Working with Another Student Teacher

Both last semester and this spring, I have been fortunate enough to have been placed in a school (and with a teacher) that has another student from TC in the classroom. This has been very beneficial in many ways, and I wonder if I'll ever have the opportunity to take advantage of a set-up like this again. Every day I learn from the other teachers: sometimes they handle things differently than I would have and I see how well their approach works. I also benefit on the other side, as some mistakes can be ironed out by the time it's my turn to teach. Comments about some areas in which I could improve help me realize ways in which I could improve that otherwise my CT or I might inadvertently miss. Overall, the feedback that we receive from one another is invaluable and I appreciate the collaboration between the three of us.

The most important thing that I have learned from MK (student teacher) is to step back and be more of a facilitator than a dictator. We've heard it in class, but having students respond to one another rather than requiring feedback from the teacher (resulting in a TPTPTP . . . sort of dialogue) is vital to creating a classroom atmosphere that encourages students to develop their own understanding. I've realized that sometimes it is OK to let the students take the discussion in a different direction than I had planned; it could be quite valuable as they try to construct new knowledge from what they already know. This will be a challenge for me, but seeing MK as a "guide on the side" has been very helpful already.

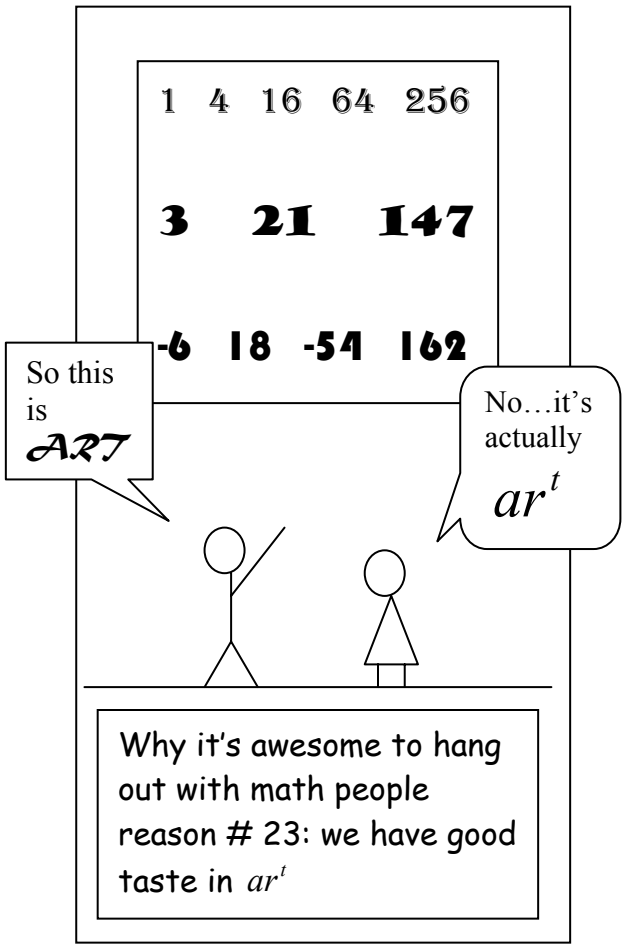
In addition to feedback on teaching, we are able to plan together. This is also useful since the ideas we have for introducing and reinforcing topics are varied and helpful. We each have a different way of approaching the topic (for example, the five fundamental loci, as we introduced last week). Since we have the combined knowledge of three different mathematical backgrounds, we can more adequately accommodate the diverse students in our classrooms.

I look forward to what the rest of the semester will bring, and the steps forward that we can each take as growing teachers collaboratively working with the geometry classes.

David Liang

Mathematics as Art

My cooperating teacher was a huge fan of hand-drawn stick figures, for it allows him to insert some humor and creativity into content. For me, it serves the same purpose – in addition to keeping the students entertained during the lesson, it's a way for me to entertain myself (as well as other teachers). I'm not quite fully sure what gave me the idea to draw a comic illustration depicting two stick figures discussing art in relation to geometric figures. If had to guess, it would most likely trace back to the prototypical image of a teacher taking a class on a field trip to the museum. Also, I find humor in irony; so what better way is there to show that than to have a student correct the teacher? The comic was produced using basic tools in Microsoft Word, such as the line, circle, triangle, and textbox tools.



Student Research in Community College Calculus

Community College mathematics often concludes with courses in differential and integral calculus. Although innovative approaches to the calculus include calculator and computer facilitated courses, the "same old—same old" approach to the calculus survives in many college and high school classrooms. Tried and true instructional methods are still common. The heterogeneous classes—heterogeneous both in terms of mathematical preparation and career objectives—found in most community college calculus classrooms present challenges to the instructors. Upon which demographic should emphasis in the calculus be placed—the marginally prepared student or the well-prepared, future economics majors or engineers? Often it is the most able student and the most likely to continue the study of mathematics in four-year colleges who is "short changed" by the instructor's efforts to reduce failure rates and avoid administrator criticism.

The Department of Mathematics at the Borough of Manhattan Community College in New York City has devised a calculus sequence that is mindful of the needs of the best prepared as well as the least prepared calculus student. Activities intended for the "best and brightest" students of the calculus emphasize opportunities for student research. Two talented BMCC Mathematics faculty members, Dr. Toni Kasper and Dr. Sofya Nayer, provided JMETC with numerous examples of original research products produced by their Community College students. JMETC's space restriction precludes many impressive examples submitted by Drs. Kasper and Nayer. However, the ones JMETC has included set a very high standard for calculus instruction in the nation's community college classrooms. Many thanks with much admiration to Toni and Sofya for sharing their research notes from the calculus classroom.

PROJECT SUMMARIES

Project #1

(Title: *The Wedding Band: An Application of Integration in Daily Life*) completed by Youssef J.

As Youseff states in his Introduction, he found himself drawn to mathematics, even in elementary school, despite the fact that no one in his family had an interest in mathematics or science. He came to the United States as a basketball player representing his country (Morocco) in the World University Games at Buffalo, but was inspired to return to the U.S. later to pursue his interest in mathematics.

This project demonstrates the applicability of mathematics in everyday life. In particular, we find the most efficient way to construct a wedding ring of fixed inner radius where the thickness (outer radius) and height of the band are constrained by a given relationship. This