

# Journal of Mathematics Education at Teachers College

Spring – Summer 2010  
Inaugural Issue

A CENTURY OF LEADERSHIP IN  
MATHEMATICS AND ITS TEACHING

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# TABLE OF CONTENTS

## Foreword

- v **Honoring the Past—Anticipating the Future**  
*Bruce R. Vogeli, J. Philip Smith, Erica Walker*

## Preface

- vi **Addressing Critical Issues in the Preparation of Teachers of Mathematics**  
*Stuart Weinberg, Director of Field Experience, Teachers College, Columbia University*

## Articles

- 1 **Examining What Teachers Do When They Display Their Best Practice: Teaching Mathematics for Understanding**  
*Edward Silver, University of Michigan*
- 7 **Current and Needed Research on Alternative Certification Programs**  
*Edward Ham, Ph.D. Candidate, Teachers College, Columbia University*
- 12 **Inside the UTeach Program: Implications for Research in Mathematics Teacher Education**  
*Nicholas H. Wasserman, Ph.D. Candidate, Teachers College, Columbia University*
- 17 **Improving Preservice Field Placements in Secondary Mathematics: A Residency Model for Student Teaching Through Lesson Study**  
*Theresa Gurl, Queens College of the City University of New York*
- 21 **A Study of the Relationship Between Student Teachers' Expectations of Pupil Success and the Management of Classroom Discourse**  
*Stuart Weinberg, Teachers College, Columbia University*
- 25 **Increasing Perceived Efficacy for Teaching Mathematics: An Exploratory Study**  
*Deborah Rosenfeld, Teachers College, Columbia University*
- 36 **An Analysis of a Misconception of Probability among Future Mathematics Teachers**  
*Patricia Jendraszek, Mercy College*
- 46 **Slideware Strategies for Mathematics Educators**  
*Christian Stryker, United Arab Emirates University*

## **TABLE OF CONTENTS, continued**

### **Notes from the Classroom**

- 51**     **Ideas for Middle School Mathematics**  
*Amanda Giambruno*  
*Heidi Li*
- 52**     **Innovative Instruction in High School Mathematics**  
*Cindy Cheung*  
*Meredith Klein*  
*Kitty Yang*  
*Meredith Brown*  
*David Liang*
- 54**     **Student Research in Community College Calculus**  
*Sofya Nayer and Toni Kasper, Borough of Manhattan*  
*Community College*

### **Other**

- 56**     **ABOUT THE AUTHORS**

The *Journal of Mathematics Education at Teachers College* is a publication of the  
Program in Mathematics and Education at Teachers College  
Columbia University in the City of New York.

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This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics and Education. Photographs are drawn from the Teachers College archives and personal collections.

This issue honors NCTM 2010 Lifetime Achievement Medalist, Dr. Henry O. Pollak, who has completed 22 years as a member of the Program in Mathematics and Education at Teachers College. Dr. Pollak has contributed so much to the mathematical preparation of the Program's graduates and to the communities of mathematics and mathematics education professionals in the United States and throughout the world.

David Eugene Smith, also pictured on the front cover, was the founding professor of the Teachers College Program in Mathematics and Education. Like Dr. Pollak, Professor Smith was widely respected by both mathematicians and educators.

**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics and Education. As a peer reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of Mathematics Education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2010-2011 issues are: *Teacher Education, International Education, Curriculum, Technology, and Equity*—all centered upon mathematics and its teaching. The *JMETC* will have a distinctive niche in the world of education publishing. Our readers are educators from pre K-12 and college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education.

**Manuscript Submission**

We seek conversational manuscripts (2500-3000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. All manuscripts may be submitted electronically at [www.tc.edu/jmetc](http://www.tc.edu/jmetc). This system will help keep the submission and review process as efficient as possible.

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. All inquiries should be sent to Ms. Krystle Hecker, P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027.

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## *Journal of Mathematics Education at Teachers College*

### **Call for Papers**

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *International Mathematics Education*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to international or comparative mathematics education. Articles should be submitted to [www.tc.edu/jmetc](http://www.tc.edu/jmetc) by September 1, 2010. The fall issue’s guest editor, Dr. Juliana Connelly, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2010, and final drafts of selected papers are to be submitted by November 1, 2010. Publication is expected in late November, 2010.

### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the Fall 2010 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citation checking, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form* from Ms. Hecker. Return the completed form to Ms. Krystle Hecker at [JMETS@tc.columbia.edu](mailto:JMETS@tc.columbia.edu) or Teachers College, Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### **Looking Ahead**

Anticipated themes for future issues are:

Spring 2011	Curriculum
Fall 2011	Technology
Spring 2012	Equity
Fall 2012	Leadership
Spring 2013	Psychology

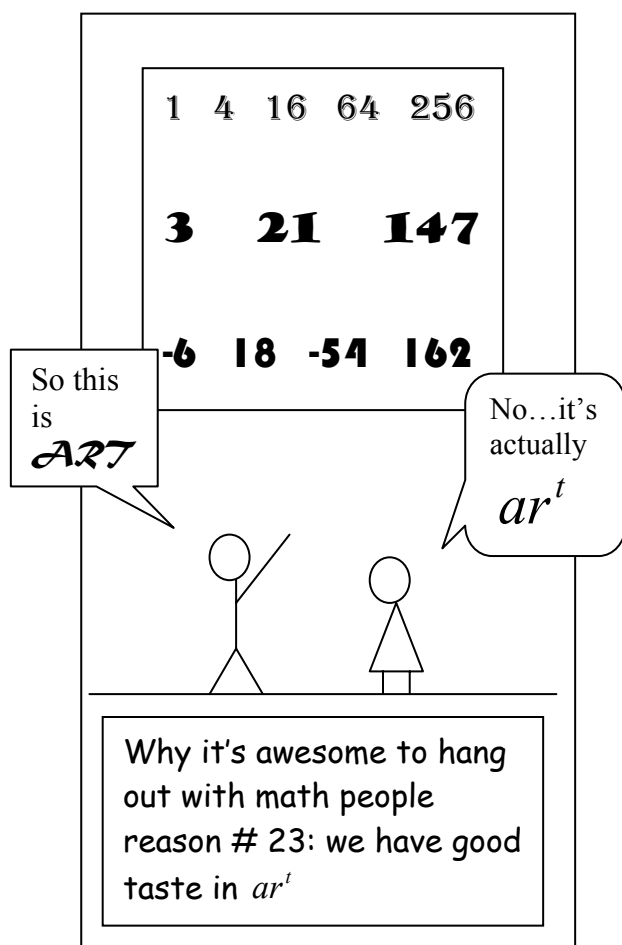
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David Liang

**Mathematics as Art**

My cooperating teacher was a huge fan of hand-drawn stick figures, for it allows him to insert some humor and creativity into content. For me, it serves the same purpose – in addition to keeping the students entertained during the lesson, it’s a way for me to entertain myself (as well as other teachers). I’m not quite fully sure what gave me the idea to draw a comic illustration depicting two stick figures discussing art in relation to geometric figures. If had to guess, it would most likely trace back to the prototypical image of a teacher taking a class on a field trip to the museum. Also, I find humor in irony; so what better way is there to show that than to have a student correct the teacher? The comic was produced using basic tools in Microsoft Word, such as the line, circle, triangle, and textbox tools.



**Student Research in  
Community College Calculus**

Community College mathematics often concludes with courses in differential and integral calculus. Although innovative approaches to the calculus include calculator and computer facilitated courses, the “same old—same old” approach to the calculus survives in many college and high school classrooms. Tried and true instructional methods are still common. The heterogeneous classes—heterogeneous both in terms of mathematical preparation and career objectives—found in most community college calculus classrooms present challenges to the instructors. Upon which demographic should emphasis in the calculus be placed—the marginally prepared student or the well-prepared, future economics majors or engineers? Often it is the most able student and the most likely to continue the study of mathematics in four-year colleges who is “short changed” by the instructor’s efforts to reduce failure rates and avoid administrator criticism.

The Department of Mathematics at the Borough of Manhattan Community College in New York City has devised a calculus sequence that is mindful of the needs of the best prepared as well as the least prepared calculus student. Activities intended for the “best and brightest” students of the calculus emphasize opportunities for student research. Two talented BMCC Mathematics faculty members, Dr. Toni Kasper and Dr. Sofya Nayer, provided JMETC with numerous examples of original research products produced by their Community College students. JMETC’s space restriction precludes many impressive examples submitted by Drs. Kasper and Nayer. However, the ones JMETC has included set a very high standard for calculus instruction in the nation’s community college classrooms. Many thanks with much admiration to Toni and Sofya for sharing their research notes from the calculus classroom.

**PROJECT SUMMARIES**

**Project #1**

(Title: *The Wedding Band: An Application of Integration in Daily Life*) completed by Youssef J.

As Yousseff states in his Introduction, he found himself drawn to mathematics, even in elementary school, despite the fact that no one in his family had an interest in mathematics or science. He came to the United States as a basketball player representing his country (Morocco) in the World University Games at Buffalo, but was inspired to return to the U.S. later to pursue his interest in mathematics.

This project demonstrates the applicability of mathematics in everyday life. In particular, we find the most efficient way to construct a wedding ring of fixed inner radius where the thickness (outer radius) and height of the band are constrained by a given relationship. This

## NOTES FROM THE CLASSROOM

approach to learning mathematics was completely new to the student.

### **Project #2**

(Title:  $\pi$ ) completed by Weng C.

This student spent most of his life in Hong Kong where, he tells us in his Introduction, mathematics was stressed, but it frightened him since he found it difficult to understand. He got poor grades in the subject and disliked it very much. But, as he explains, it was only by working on his project that he actually came to enjoy mathematics and appreciate it as a source of beauty. The project was a turning point for this student, both in terms of building confidence in his ability to do higher level math and in appreciating the beauty of the subject.

This project explores different ways to calculate the irrational number  $\pi$ , which arises in geometry, probability, and calculus. For thousands of years, people have been computing  $\pi$  to more and more decimal places. Weng discusses how  $\pi$  was approximated by the ancient mathematician/scientist Archimedes, in the Bible (*The Book of Kings* and in *Chronicles*), and by the ancient Egyptians. In his paper, the student shows how to use the modern methods of infinite series and the integration procedure to estimate  $\pi$ .

### **Project #3**

(Title: *Investigations in Polar Coordinates*) is the work of Fabio F.

Fabio did not graduate from high school; he came to BMCC after getting his GED. At first he rejected the suggestion to work on a project; he agreed to do so after additional encouragement by a fellow student. The student concluded that doing this research made him feel like “a little [Isaac] Newton,” and said that this experience will stay with him forever.

In this project, the student examined a large number of functions in the polar coordinate system. His goal was to identify characteristics of their graphs, including symmetries, intercepts, and range. In addition, he sought to discover a formula that computes the number of so-called *petals* for a variety of polar functions.

### **Project #4**

(Title: *Was Galileo Right...Again?*) completed by Jingzhou Z.

Jingzhou entered less than one week after arriving in the United States. Because of his poor command of English, he was placed in remedial courses except for mathematics, this due to the universality of mathematical symbols and syntax. As he explains in his Introduction to the project, he had ambivalent feelings about mathematics, but with help and encouragement, he undertook this project.

The purpose of this work was to examine the equation of the catenary, the curve assumed, for example, by a bridge cable suspended between two towers. In the seventh century, Galileo had suggested that the catenary had the shape of a parabola. In this project, the student shows that the equation of this curve is not that of a parabola but that

the difference in arc length (between a catenary and corresponding parabola) is less than two percent, which suggests that Galileo’s conjecture, while not correct, nevertheless shows remarkable intuition.

### **Project #5**

(Title: *An Application of Newton’s Second Law to Rocket Propulsion*) completed by Millibani B.

Millibani came to the U.S. from Africa. As he describes in his Introduction, he was surprised—and delighted—to find an opportunity to do research below the graduate school level. The project showed him “how mathematics can be applied to any science and especially to physics.”

The question explored by this project was: Which is the more efficient way to propel a rocket, to inject small amounts of fuel frequently, or larger amounts less often? The student was able to prove, mathematically, that the rocket velocity increases with smaller, more rapid bursts of fuel and that the velocity approaches its theoretical maximum speed (not calculated in the report) when the number of rapidly-timed bursts approaches infinity. He concluded that “the engineering challenge is to make  $n$  as large as possible so that the mass of fuel burned ... will be as small as [practicably] possible.”

### **Project #6**

(Title: *The Golden Spiral*) completed by Tsealla Y.

Tsealla came to the United States from Israel. She professes an interest in spirals that began at age three by observing the movements of a snail.

In this project, the student draws a line that connects Fibonacci numbers to the so-called Golden Ratio, then to the “golden rectangle” within which lies the Golden Spiral. The project concludes with a summary of where, in nature, the golden ratio, rectangle, and spiral can be seen.

### Conclusion

These examples are among the many projects that lend themselves to independent (albeit guided) student research. While these projects certainly do not lead to new discoveries in mathematics, they are an easily administered educational tool that (1) helps build student confidence and interest in pursuing mathematical work; (2) exposes the student to the beauty and power of mathematics; (3) adds to the student’s knowledge base; (4) introduces the student to seminar/presentation type activities; and (5) enhances the student’s written and spoken verbal skills. We might add also that the work our students have completed adds to our enjoyment of teaching in a community college and to our confidence that our BMCC courses are equivalent to those offered in many universities.

*Toni Kasper  
Sofya Nayer*