

Journal of Mathematics Education at Teachers College

Spring – Summer 2010
Inaugural Issue

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the
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This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics and Education. Photographs are drawn from the Teachers College archives and personal collections.

This issue honors NCTM 2010 Lifetime Achievement Medalist, Dr. Henry O. Pollak, who has completed 22 years as a member of the Program in Mathematics and Education at Teachers College. Dr. Pollak has contributed so much to the mathematical preparation of the Program's graduates and to the communities of mathematics and mathematics education professionals in the United States and throughout the world.

David Eugene Smith, also pictured on the front cover, was the founding professor of the Teachers College Program in Mathematics and Education. Like Dr. Pollak, Professor Smith was widely respected by both mathematicians and educators.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics and Education. As a peer reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of Mathematics Education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2010-2011 issues are: *Teacher Education, International Education, Curriculum, Technology, and Equity*—all centered upon mathematics and its teaching. The *JMETC* will have a distinctive niche in the world of education publishing. Our readers are educators from pre K-12 and college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education.

Manuscript Submission

We seek conversational manuscripts (2500-3000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. All manuscripts may be submitted electronically at www.tc.edu/jmetc. This system will help keep the submission and review process as efficient as possible.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. All inquiries should be sent to Ms. Krystle Hecker, P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027.

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Journal of Mathematics Education at Teachers College

Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *International Mathematics Education*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to international or comparative mathematics education. Articles should be submitted to www.tc.edu/jmetc by September 1, 2010. The fall issue’s guest editor, Dr. Juliana Connelly, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2010, and final drafts of selected papers are to be submitted by November 1, 2010. Publication is expected in late November, 2010.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the Fall 2010 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citation checking, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form* from Ms. Hecker. Return the completed form to Ms. Krystle Hecker at JMETS@tc.columbia.edu or Teachers College, Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2011	Curriculum
Fall 2011	Technology
Spring 2012	Equity
Fall 2012	Leadership
Spring 2013	Psychology

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A Study of the Relationship Between Student Teachers' Expectations of Pupil Success and the Management of Classroom Discourse

Stuart Weinberg
Teachers College, Columbia University

The purpose of this article is to summarize the methodology and findings in the case studies of three student teachers in secondary school mathematics. The broad area of interest is the relationship between beliefs and practice. In the study, "beliefs" refers to expectations teachers have that students will be successful in learning with understanding. In regard to practices, a focal point was a student teacher's questions and instructions articulated while managing classroom discourse. An analysis of solicitations was intended to provide a measure of the kind of thinking in which student teachers expect their students to engage. During the study, a coding scheme was created, refined, and applied to lesson transcripts for the purpose of "making visible" the management of classroom discourse. The study may provide a basis for further research on student teachers' practice.

Introduction

In its Professional Standards for Teaching Mathematics, the NCTM (1991) called for a "shift toward classrooms as mathematical communities; logical and mathematical evidence as verification; mathematical reasoning; conjecturing, inventing, and problem solving; connecting mathematics, its ideas and its applications." The Professional Standards provided an agenda for creating a classroom in which mathematics is taught and learned with understanding. In this classroom, the teacher designs tasks that provide opportunities for students to "problematize" and that leave a "residue of mathematical value." The teacher understands "how much information to share and what kinds of information to provide" and leads students in the development of a "classroom culture in which reflection and communication are valued." The social culture is such that students' ideas and methods are valued, mistakes become opportunities for learning, and correctness is determined by mathematical arguments rather than the authority of the teacher (Hiebert, 1997, pp. 161-173). Toward this end, a teacher's role is critical. Supporting a social learning environment in which a teacher encourages and orchestrates mathematical discourse among students contributes to learning with understanding (Franke, Kazemi, & Battey, 2007).

Are teachers who have not experienced this kind of learning environment likely to create a classroom in which mathematics is taught and learned with understanding? What can be done to encourage student teachers to embrace current reforms in the teaching of mathematics?

Background—the Apprenticeship of Observation

Pre-service teachers' beliefs about mathematics teaching and learning are often shaped by their own

experiences in school. While undergoing years of what Lortie (1975) calls an "apprenticeship of observation," teacher candidates do learn much about teaching through "continuous contact with teachers and professors." It is not based on pedagogical principles. Instead, what they learn is "intuitive and imitative rather than explicit and analytical" (p. 62). Student teachers bring with them tacit assumptions, beliefs, and preconceptions about teaching and learning that affect their classroom practice.

The apprenticeship of observation may influence the expectations teachers have for their students' learning. One reason is that teachers' expectations are shaped, in part, by "memories of themselves as students." Teachers' own classroom experiences are assumed to be "representative," and, therefore, these experiences "inform their knowledge of student understanding" (Grossman, 1990, p. 11).

In his study of English teachers, Hillocks (1999) describes how teachers' beliefs about the likelihood of their students' success in learning influence instructional practices. He defined "optimistic" teachers as those teachers who have high expectations that their students will be successful. Optimistic teachers design instructional activities in which students are engaged in complex tasks, spend time constructing knowledge, and work independently or in groups more often than non-optimistic teachers. They spend less time on lecture and recitation, more time on content, less on mechanics, and design more complex curricula.

The Study

Participants in the study were three student teachers in a one year master's degree program leading to certification in the teaching of secondary school mathematics. A program requirement includes a minimum of 24 credits in mathematics at the undergraduate level, followed by 12

credits in graduate-level mathematics preparation. Student teachers complete a prerequisite methods course in secondary school mathematics. The methods course syllabus is guided by the NCTM Principles and Standards for School Mathematics (2000).

Data

Three student teachers were supervised by the author of this study for both their middle school and high school placements. Data included notes taken during the observations of lessons and during conferences, transcripts of at least three interviews, weekly journal entries by student teachers, and a reflection paper, in which student teachers discussed and analyzed lessons taught. The student teachers' reflections were divided into six sections: analysis of their students' prior learning, instructional planning, description of the teaching experience, assessment of student learning, analysis of the results, and reflecting on changes in teaching that could improve results. Post lesson conferences were semi-structured and began by asking the student teacher to reflect and comment on the lesson observed. The journals, seminars, and conferences are integral parts of a program to encourage student teachers to become "reflective practitioners."

Methodology and Procedures

Analysis of Student Teachers' Beliefs

To describe expectations student teachers have in regard to their students' learning, the study utilized Hillocks's (1999) notions and terminology. The following questions were written for the study and addressed in order to determine if a student teacher may be considered "optimistic." An affirmative response ("yes") to a question served to substantiate a designation of "optimist." All data were analyzed to answer the five questions.

1. Does the student teacher express confidence in his/her students' ability to grow?
2. Does the student teacher find ways for students to make important contributions in the construction of their own knowledge?
3. Does the student teacher convey an understanding of the special circumstances of students?
4. Does the student teacher convey the belief that students will learn under the right circumstances?
5. Does the student teacher reflect on his/her teaching and question its effectiveness?

The rationale for including question 5 is that optimistic teachers assume students will learn under the right circumstances. Because they may be surprised at student failure, optimistic teachers are more likely to reflect on their teaching and question its effectiveness. Non-optimistic teachers are more likely to place the onus

of failure on students by attributing failure to the deficiencies of the students rather than to themselves.

Analysis of Instructional Practice

All data were used to analyze practice. To provide additional analysis, transcripts of the last two lessons observed were coded. In the coding scheme, the basic unit of discourse in the classroom is called the "pedagogical move," defined as the utterance of an individual "for the pedagogical purpose of structuring the discourse, soliciting information or action, responding to a solicitation or reacting to a prior move" (Fey, 1966, p. 17).

The pedagogical moves were based on a method of discourse analysis created by Bellack (1966) and modified for the mathematics classroom by Fey (1966). Of the four primary moves, the solicitations were of particular interest because they were expected to reveal the types of thinking student teachers would like and/or expect students to engage in and provide a measure of cognitive demand inherent in classroom activities. Solicitation categories were created beginning with the seven types of questions defined by Cook and Rasmussen (1991) and printed in Brahier (2005). They appear in Table 1, listed in ascending level of cognitive demand inherent in the solicitation.

Table 1. Solicitation Categories

Solicitation	Types of thinking students may be expected to engage in
Knowledge	reciting, remembering, recognizing
Comprehension	understanding, translating, estimating
Application	using, demonstrating, solving
Creative thinking	elaborating, taking another point of view, brainstorming
Analysis	comparing and contrasting, inferring, attribute listing
Synthesis	hypothesizing, planning, creating
Evaluation	justifying, rating, judging using criteria

NOTE: Because of the prevalence of KNOWLEDGE solicitations in classroom discourse, the three subcategories, RECITE, REMEMBER, and RECOGNIZE, were recorded separately during the coding of lesson transcripts.

The Coders

Following instruction and practice, two retired educators coded the transcripts, working individually and then as a pair in order to reach consensus. The researcher met with the coders to provide context and answer questions. The coding scheme was revised on at least two occasions, followed by a re-coding of all transcripts. The

A STUDY OF THE RELATIONSHIP

final coding received the advantages of “experienced” coders. Support for this type of “collaborative” coding appears in recent research literature.

A Cognitive Demand Index was created to capture numerically the level of cognitive demand typically required by students to respond to a student teacher’s solicitation. The Index is a weighted average of values assigned to each solicitation, where the weights are determined by the number of times a solicitation occurs during a lesson.

Findings from the Case Studies

Based on analysis of all data in the case studies, Lena and Dan were considered to be optimists. Their teaching was considered closest to the constructivist model. In Lena’s and Dan’s classrooms, there is an emphasis on learning with understanding, looking for alternative solution methods, viewing mistakes as opportunities for further discussion, asking students to think and inquire, reason and communicate. Gail was considered a non-optimist who adhered to an objectivist model of teaching. In Gail’s classroom, there is simplification of content, telling students rather than eliciting, directing students toward a particular and often formulaic solution method, and a degree of classroom discourse that is not nearly as robust as what was observed in Lena’s classroom. Lena in particular reflects deeply on her lessons and questions the effectiveness of her teaching. Because she has high expectations that students will learn with understanding, that self-reflection deepens when students do not perform well. To see Lena teach is to see someone who reflects-in-practice as well as after the lesson has been taught. While teaching, she listens to students and responds. Because students’ ideas are valued, Lena will depart from her planned lesson in response to students’ queries and contributions.

The computed CD Index values and distribution of solicitations, seen in Table 2, suggest that solicitations in Lena’s and Dan’s lessons typically require a higher level of cognitive demand than those in Gail’s lessons.

The results of this exploratory study suggest that the level of cognitive demand typically required by students to respond to solicitations may be greatest in the classroom of a student teacher with high expectations for pupil success in learning with understanding.

What may be most useful about the study is that the methodology suggests procedures for helping student teachers, newly certified teachers, and even veteran teachers focus, reflect on, and develop their skills in orchestrating classroom discourse. For example, at Teachers College, whenever possible, at least two student teachers are assigned to each placement school. As a result, student teachers are able to critique each other’s lessons, guided by an evaluation protocol. The value of

Table 2: Percentages of Selected Solicitation Subcategories

	Lena	Dan	Gail
Knowledge	43.8%	53.7%	88%
Comprehension or application	45%	31.7%	12%
Creative thinking or higher cognitive level	11.3%	14.6%	0%
Recite	3.8%	7.8%	38.7%

this activity may be enhanced by application of procedures found in the study, including application of the coding scheme.

Although the coding of transcripts can be arduous, feedback from the coders at the conclusion of the study was positive. The coders reported sharpening their skills in the other role in which they serve—as supervisors of student teachers. One wrote,

I found myself evaluating more thoroughly the questioning techniques of student teachers. After an initial observation of one student teacher, I realized how much our post-lesson discussion dealt with questioning. I found myself pointing out low level questions, and engaging in discussions of how these questions could become more thought-provoking, thus contributing to more critical thinking by the student.

Student teachers became more aware of their questioning techniques and, in addition, “they started to evaluate the questions their students were asking.”

Suggestions for Further Study

A refinement in the methodology may prove to be useful in the analysis of classroom discourse that arises from pedagogical moves other than a teacher’s solicitations. For example, sequences of coded transcript lines may indicate when and how often students ask each other questions or when and how often a teacher encourages students to comment on each other’s work.

In this study, the same analytic tools were applied in all classrooms. However, given that models of discourse may be unique to the various mathematical domains, the mathematics of the lesson is something to be considered in a research study.

Franke et al. (2007) focus their attention on three features of classroom practice: discourse, norms, and developing relationships. They do so because “consensus is building that students need opportunities in classrooms to share their mathematical thinking, discuss alternative approaches to solve problems, use mathematical tools flexibly, and so on” (p. 248). It is important that students engage in discourse during a mathematics lesson. In order

for this to occur, appropriate classroom norms must be negotiated and relationships between students and teachers established and nurtured. Franke et al. call for further research on managing discourse, “particularly to relate how teachers manage discourse to the norms that govern discourse and the teacher-student relationships that are built and enacted” (p. 249). Research is needed to identify “routines of practice” (or “core activities” that should occur regularly in mathematics lessons) and examine how discourse, norms, and building relationships “work together in such routines.”

Among the questions Franke et al. (2007) pose is: “How do we make visible teachers’ work in managing discourse, establishing norms, and building relationships with students as we specify and study the enactment of routines?” (p. 251). In that regard, the methodology in this study may provide a starting point for additional research. It may be possible because the methodology provides a way to identify units of discourse (pedagogical moves) and, subsequently, enables one to analyze instructional practice generally and the orchestration of discourse in particular. Analysis includes making a statement about the level of cognitive demand inherent in questions and instructions articulated by the teacher. The teacher’s work in orchestrating discourse is made “visible” by the coding of transcript lines and providing an interpretation of codes and coding sequences.

Although not addressed in this article, changes in beliefs and practice that may have occurred during the one-year study were of interest. Generally, by end of year, student teachers were more knowledgeable about teaching strategies and methods, more confident with classroom management, and more proficient in planning lessons. Deeply held beliefs are more difficult to measure and are known to be resistant to change. In his introduction to Hillocks (1999), Shulman writes that the best opportunity for changing beliefs and, subsequently, practice occurs at the beginning of a teacher’s career. If “one could begin the socialization of teachers during their preparation period,” then there is “some hope” for change. (p. ix). Future research to examine aspects of teacher education programs that result in measurable changes in both practice and beliefs should be considered.

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