# Journal of Mathematics Education at Teachers College

Fall – Winter 2010

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This issue honors Dr. Alexander P. Karp, an Associate Professor in the Program in Mathematics at Teachers College. A native of St. Petersburg, Russia who is the author of more than one hundred publications including textbooks used throughout Russia, Professor Karp represents Teachers College at meetings and conferences throughout the world as well as through his role as managing editor of the *International Journal for the History of Mathematic Education*.

Former Teachers College Professor and Mathematics Education Chair, Howard Franklin Fehr, was among the most influential mathematics educators of his era. Through his many international contacts, he was the organizer of conferences, projects, and publications including the Congresses of Mathematics Education, a seminal conference on Needed Research in the field, and curriculum initiatives including the Secondary School Mathematics Curriculum Improvement Study.

#### **Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2011 issues are: *Mathematics Curriculum* and *Technology. JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

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Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at JMETC@tc.columbia.edu

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#### **Call for Papers**

The "theme" of the spring issue of the *Journal of Mathematics Education at Teachers College* will be *Mathematics Curriculum*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2000-2500 words describing research, experiments, projects, innovations, or practices related to mathematics curriculum. Articles should be submitted to Ms. Krystle Hecker at jmetc@tc.edu by January 1, 2011. The spring issue's guest editor, Nicholas Wasserman, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2011, and final drafts of selected papers are to be submitted by March 1, 2011. Publication is expected in mid-April, 2011.

#### Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form.* Return the completed form to Ms. Krystle Hecker at jmetc@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

#### **Looking Ahead**

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## Hua Loo-keng and the Movement of Popularizing Mathematics in the People's Republic of China

## Jean W. Richard

#### Borough of Manhattan Community College, The City University of New York

This paper examines the mathematical and teaching practices of the renowned self-taught Chinese mathematician Hua Loo-keng in the movement of popularizing mathematics in the People's Republic of China. In this movement, Hua Loo-keng taught industrial workers and peasants with a low level of education how to use fairly advanced mathematical methods in their workplace (agriculture, transportation, or industry). This paper explores the mathematical concepts that Hua Loo-keng taught the workers and farmers and how they applied them in their workplace. An important element in the movement of popularizing mathematics in the People's Republic of China is the non-traditional methods by which Hua Loo-keng taught the workers and farmers.

*Keywords:* Hua Loo-keng, popularization of mathematics, Optimum Seeking Methods, mathematics teaching and learning, Operations Research in China.

I am not sure, but I think that in the history of science there has never been such opportunity where such a large group of people would come to listen in a meeting about introductory methods of science.

— Hua Loo-keng (1972)

#### Introduction

This paper examines Hua Loo-keng's mathematical and teaching practices in the movement of popularizing mathematics in the People's Republic of China. "Popularization" is defined as the movement to bring formal and scientific knowledge to workers and peasants and adapting this knowledge to their daily experiences (Sheringham, 1984). In bringing scientific knowledge to workers and peasants with a low level of education, Hua Loo-keng taught them how to use fairly advanced mathematical methods in their workplace (agriculture, transportation, and industry).

The following principles were central to Hua Lookeng's philosophy and pedagogy: (a) mathematicians and scientists cannot be mere observers in the social and political development of China; (b) mathematics can still sustain its creative characteristics when practiced by industrial workers and farmers with a relatively low level of education, because by using mathematics, they will learn to value it and place it in context; (c) since pure mathematicians and industrial workers or peasants do not share common interests in mathematics, the role of the former is to identify what techniques should be taught and in what ways so as to bridge the gap between the abstract theories of mathematics and the concrete realities of industrial workers (Hua & Wang, 1989). The emphasis on problem solving, the teaching methods and the applications of mathematics in industries in the movement of popularizing mathematics fall within the purview of contemporary mathematics education. In 1989, the National Council of Teachers of Mathematics (NCTM), in summarizing expectations for an agenda of mathematical literacy, outlined several priorities in the effort to improve mathematical understanding and mathematical literacy. According to the NCTM, students should be able to (a) reason mathematically; (b) value mathematics; and (c) become problem solvers. By retracing the history of the movement of popularizing mathematics, we examine how many of these goals were achieved and their relevance to school mathematics.

This paper is divided into four sections. The first section is a biography of the self-taught Chinese mathematician Hua Loo-keng. The second section describes the principles of popularizing mathematics and its philosophy, which sought to discover what mathematical techniques needed to be popularized and how this could be accomplished. The third section retraces the history of the movement of popularizing mathematics. The fourth section describes the Golden Search Method as it was used in the movement and how the workers applied it in industrial settings. In this section, the relevance of the movement of popularizing mathematics to contemporary mathematics education will be discussed.

#### Biography of Hua Loo-keng

Hua Loo-keng was born on November 12, 1910, in Jintan, a city in Jiangsu Province. His parents were so poor that Hua Loo-keng dropped out of school so he could help his father manage the family grocery store. During this time, Hua Loo-keng continued to teach himself mathematics and, by the age of 20, published two mathematical papers (Wang, 1999). These publications attracted the attention of mathematicians at Tsing Hua University, who invited him to the university to work as a clerk in the mathematics department and in the library. At the university, Hua Loo-keng pursued his mathematical interests further, at the time focusing on number theory. Later, he was permitted to teach calculus with the position of instructor (Wang, 1999).

In 1935, Hua Loo-keng met Norbert Wiener, the American mathematician, who had accepted an invitation to lecture at Tsing Hua University. With Wiener's help, Hua Loo-keng would spend two years at Cambridge University under the guidance of the famous mathematician Godfrey Harold Hardy. Later, Hua Loo-keng returned to the People's Republic of China as a full professor at the Southwest Associated University at Kunming.<sup>1</sup> While in the People's Republic of China, Hua Loo-keng started a correspondence with the mathematician Hermann Weyl, who at the time was at the Institute of Advanced Studies at Princeton.

With Weyl's help, Hua Loo-keng became a fellow at the Institute of Advanced Studies, and later a professor at University of Illinois at Urbana Champaign. Hua Loo-keng returned to his country in 1950, whereupon he quickly embarked on reorganizing the Chinese Academy of Sciences, the highest institutional body responsible for supervising science education. Hua Loo-keng created areas of research in number theory, differential equations, functions of several complex variables, and algebra, and supported other research groups by being active in their seminars (Wang, 1999). To advance mathematics education at the university level, Hua Loo-keng wrote several textbooks.

Around 1958, Hua Loo-keng shifted his interests from pure mathematics in order to popularize linear programming and other Operations Research concepts among the farmers and industrial workers for their use in agriculture and transportation. At the time, the movement of popularizing mathematics was mostly based in Shandong Province. The simplex method and other Operations Research concepts were used extensively in farming and transportation (see Saaty, 1961; Waardenburg, 1965). Around 1965, Hua Loo-keng popularized a new set of methods known as the Operational Methods, and in 1970, another set of methods known as the Optimum Seeking Methods.

#### The Principles of Popularizing Mathematics

The methods of mathematics teaching and learning during the movement of popularizing mathematics were based on three principles: (a) For whom? (b) Which technique? and (c) How to popularize?

#### (a) For whom?

This principle is fundamental to the concept of popularizing mathematics. Since the popularization of mathematical concepts required the dynamics and the interaction of both presenters and learners, Hua Loo-keng estimated that without mutual interest between the two groups, the action of popularizing mathematics would be ineffective.

#### (b) Which technique?

To select mathematical concepts to be popularized, Hua Loo-keng relied on the following principles:

(i) The mathematical methods must be easy to explain to the workers and peasants and easy to understand as well as to apply.

(ii) No method can be popularized without determining a priori in which specific areas the method can be applied.

(iii) The methods must have theoretical foundations and guarantee the possibility of creating new methods based on previous ones.

#### (c) How to popularize?

According to Hua Loo-keng, the work is started in the workplace and carried out on a small scale in order to make it acceptable to the workers. If the presentations of the project obtain some results, then the project can be extended on a larger scale.

## The Movement of Popularizing Mathematics in the People's Republic of China.

In 1958, the movement of popularizing mathematics began in Shandong Province. Several factors contributed to the birth and expansion of the movement: (a) the birth of the communes and their educational models, which emphasized applied mathematics; (b) the ideology, which asserted that practice is an essential element of epistemology; (c) the political environment, which stressed the ideological reform of intellectuals. For Hua Loo-keng, the fundamental question was how mathematics could be linked to production. During this period, the mathematical concepts popularized were from Operations Research. According to Hua Loo-keng (1961): "This branch of science has become a shining pattern of mathematical

<sup>&</sup>lt;sup>1</sup>The Southwest Associated University was based at Kunming and was a merger during the Sino-Japanese war of three separate universities: Tsing Hua, Peking, and South-Open.

method linked to production practice<sup>2</sup> (p. 1). One of the main concepts popularized was linear programming used to obtain the optimal solution among all possible solutions.

In China, in 1958 linear programming was called *Yun ch'ou hsueh*, the "science of operations and programming" or "the science of planning and programming." Though these mathematical concepts can be very theoretical, they only require basic operations, such as addition, subtraction, multiplication, and division. In that same period, Hua Lookeng (1961) wrote an article, "*Yun ch'ou hsueh*, a new branch of science, applied in national economic construction." In the article, he described the methods used to popularize mathematics: the graphical representation (geometric) and the simplex method (called the *tableau method* in China).

*Yun ch'ou hsueh* was applied in the following areas: (a) communications and transportation to minimize costs; (b) commodity production distribution, where adjustment in minimizing travel time was needed; (c) industrial production for the rational distribution of materials and tasks; and (d) agricultural production for the arrangement and coordination of labor (Liu, 1960). This is how Hua Loo-keng (1961) described the movement at the time:

A great display was given to this branch of science and a mass movement was launched in a big way, resulting in all-round blooming of linear programming. The number of persons taking part in popularizing applied linear programming reached more than 400,000 in Shantung, including college, middle school and primary school teachers. workers, peasants. and managerial personnel. Through the diversified media of broadcasting, ... indigenous "movies." and songs, they made known linear programming to more than 8,000,000 people in the province with the result that linear programming was introduced to administrative areas, hsien, commune brigades and factory shops. (p. 12; see also Saaty, 1961; Swetz, 1977)

It was during that period that a famous mathematical result was discovered. The problem was the following: "Suppose we are given a specific starting position relative to a certain number of places which are linked together by roads. The problem is to devise a method or strategy to determine the shortest path which traverses all the roads, including finally returning to the starting point" (Wang, 1999, p. 251). Guan Meigu, a mathematician at Shandong Normal College, discovered a mathematical model that later came to be known as the *Chinese Postman Problem*. This problem has many applications, for example, in

<sup>2</sup>"Production practice" considers practice as an essential element of epistemology and emphasizes as well a relationship between theory and practice.

sanitation for the planning of street maintenance, in robotics, and website usability (Thimbleby, 2003).

Nevertheless, around 1962, the movement of popularizing mathematics faded away, and the reasons still remain unclear. Later in 1965, Hua Loo-keng restarted the movement with different mathematical methods, such as PERT-CPM (Program Evaluation and Review Technique/ Critical Path Method). In the People's Republic of China, these methods were called the Overall Planning Methods or Operational Methods. In 1970, Hua Loo-keng popularized the Optimum Seeking Methods, which were as easy to learn as in 1958.

In the period of popularizing PERT-CPM, the methods that found the most applications were search techniques that reduce the interval of uncertainty for the optimal solution. In this article, we select the Golden Search Method to demonstrate the use of optimization methods in the movement. The "Optimum Seeking Methods" are a collection of techniques with the objective of searching for the optimal solution (Wilde, 1964). In the movement of popularizing mathematics, Hua Loo-keng (1989) defined the Optimum Seeking Methods (*You Xuan Fa* in Chinese) as "a method to find technological production processes that are best in some sense, while using as few experiments as possible" (p. 57).

#### A Case of Teaching Methods of Hua Loo-keng: Golden Search Method

The Golden Search Method is a modified version of the Fibonacci Method (Wilde, 1964). In the Fibonacci Search Method, the interval of uncertainty, as well as the number of experiments to be performed, needs to be known a priori. In the Golden Search Method, the number of experiments to be conducted need not be known in advance. The main advantage of the Golden Search Method is that it requires a minimal amount of information for the implementation of the search (Wilde, 1964).

This technique is called the Golden Search Method because of its use of the Golden Number 0.6180339887.

How did an industrial worker with a low level of education understand this concept? Hua Loo-keng's concern was the following: "How can we make ordinary workers understand the [Golden Search Method] well and then use it in their work?" (Hua & Wang, 1989, p. 8). To teach the method to industrial workers, he used the *Folding Paper Technique*:

Step 1: Take a strip of paper of a certain length.

Step 2: Ask the audience to memorize the number 0.618.

As an example, consider any technological process for which the optimal temperature lies somewhere between 1000 and 2000. We may perform one thousand experiments at 1001, 1002..., and then we are bound to find the optimal temperature.

(Hua Loo-keng would show the workers a strip of paper). Suppose this is a strip of paper with a scale from 1000 to 2000 on it. Notice that there are two endpoints, with one point larger than the other one. Remember the following formula in order to conclude where exactly to place the point 0.618 on the strip of paper:

#### $(Large number - Small number) \times 0.618 + Small number$

In the case of this example where Large number = 2000 and Small number = 1000, the result is:



The first trial point is taken at 0.618 of the whole range, at 1618. (At this point, Hua Loo-keng used a cigarette to scorch a hole on the strip.)

What about the second point?

Step 3: (Then Hua Loo-keng simply folded the strip in half.) The second trial point is taken at the point opposite the first point, i.e., at 1382 (see Figure 2).



Then the results at these two points are compared. But the result of the second point could also be found with the following formula:

(Large number + Small number) – Middle number

Let us illustrate: Large number = 2000, Small number = 1000, Middle number = 1618. Therefore,

2000 + 1000 - 1618 = 1382

Step 4: Evaluate the two results, and tear the strip at the worst point and keep the part containing the point that produces a better result.

Suppose the section 1618/2000 was discarded; the large number becomes 1618, the small number is 1000; therefore, 1382 becomes the middle number. Applying the formula, we obtain:

1618 + 1000 - 1382 = 1236

Step 5: Do the test again and remove the undesired part. Repeat Step 3 for the remaining strip until the optimal solution is reached. (Hua & Wang, 1989).

#### The Relevance of the Movement of Popularizing Mathematics to Contemporary Mathematics Education

In 1989, the National Council of Teachers of Mathematics (NCTM) suggested that a program of mathematical literacy should help students (a) learn to value of mathematics, (b) become confident in their ability to do mathematics, (c) become mathematical problem-solvers, (d) learn to communicate mathematically, and (e) learn to reason mathematically (p. 7). These goals were strikingly similar to those of Hua Loo-keng, who believed that the essence of mathematical applications lay in problem solving and that an effective long-term approach to problem solving demands that new methods be created, since situations may change and traditional methods may be inadequate to solve important types of problems.

Besides teaching the different optimization techniques to the industrial workers, Hua Loo-keng (1972) emphasized various components in problem solving, which teachers can implement in a classroom settings: (a) identifying the problem and understanding it, (b) drawing a plan and evaluating the different methods to decide the best one to use to solve the problem; (c) evaluating the results to determine if mistakes were made before starting production. For the last part, teachers can specify to students that evaluation of results is necessary before proceeding to the next problem.

More importantly, Hua Loo-keng emphasized work experiences or past knowledge as fundamental in solving problem. All these components of problem solving can become a set of skills in a mathematics classroom. The methods and philosophy that Hua Loo-keng implemented in the movement of popularizing mathematics have resonance in contemporary mathematics education. In fact, NCTM (2009) defines two concepts that are important for a high school curriculum: reasoning and sense making. According to NCTM, "reasoning often begins with explorations, conjectures at various levels, false starts, and partial explanations before a result is reached" (p. 4). Sense making is defined "as developing understanding of a situation context or concept by connecting it with existing knowledge" (p. 4).

According to Hua Loo-keng, when one studies, there is a path between learning and knowing. The concept of knowledge involves relating present learning with past learning experiences. In the movement of popularizing mathematics, Hua Loo-keng believed that the workers and peasants in their daily practices had acquired some mathematical knowledge and that his role was to build on these experiences and teach new methods to which the workers and peasants could easily relate. In one of his speeches, Hua Loo-keng (1973) emphasized the creativity of the workers in applying mathematics: "In Fujian, in 33 days we had 2400 results. In Hubei, in almost two months we had 8776 results....I did not obtain these results. The workers did the applications themselves. How can someone like me who only has some knowledge about mathematics, who does not have any industrial knowledge obtain so many results?" (p. 3).

#### Conclusion

Hua Loo-keng's approach to mathematics education, as manifested in his efforts to popularize mathematics in the People's Republic of China, provides important contributions to the study of mathematical practices in the workplace. The mathematical concepts selected were theoretically difficult for workers with a low level of education, but Hua Loo-keng provided methods of teaching to put these challenging problems within reach.

Hua Loo-keng's philosophy of mathematics education was based in the belief that particular work environments and everyday contexts can and do play a crucial role in the evolution of mathematical thinking. While this article focused on the teaching of the Golden Search method through an application, the movement of popularizing mathematics manifested in a vast range of problem-solving contexts, in the fields of chemical engineering, drug manufacturing, coal mining, textile industry, etc.<sup>3</sup>

In articulating the essential goals of contemporary mathematics education, NCTM (1989) emphasized that mathematics education professionals should examine the following: (a) how applying mathematics will steer students to appreciate its role; (b) how applying mathematics will guide students toward understanding its impact on their lives; and (c) the importance of problemsolving. Hua Loo-keng (1972) in his campaign to popularize mathematics emphasized that problem solving required (a) understanding the problem, (b) evaluating the different approaches to select the appropriate method; (c) evaluating the results to determine their feasibility. Hua Loo-keng's philosophy of problem solving embodies the principles espoused by NCTM.

Several avenues exist for further research on the movement of popularizing mathematics and its relevance for contemporary mathematics education. NCTM (1989, 2009) already signaled a transition from industrialized societies to information societies and the necessity to redefine the paradigms of research in mathematics teaching and learning. One area of concern is mathematical literacy, its definition and its significance for today's citizens and democracy. The movement of popularizing mathematics featured many characteristics of mathematical literacy and may offer valuable insights into this aspect of contemporary mathematics education.

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<sup>&</sup>lt;sup>3</sup>For further readings, see: Fitzgerald, A., & MacLane, S. (Eds.). (1977). Pure and applied mathematics in the people's Republic of China: A trip report of the American Pure and Applied Mathematics Delegation. Washington, DC: National Academy of Sciences. See also Halberstam, H. (Ed.). (1983). Loo-keng Hua: Selected papers. New York: Springer-Verlag.

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