

# Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN  
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the  
Program in Mathematics and Education at Teachers College  
Columbia University in the City of New York.

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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Roszkopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Roszkopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

*JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

**Manuscript Submission**

*JMETC* seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at [www.tc.edu/jmetc](http://www.tc.edu/jmetc).

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at [JMETC@tc.columbia.edu](mailto:JMETC@tc.columbia.edu)

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**Library of Congress Cataloging-in-Publication Data**

Journal of mathematics education at Teachers College  
p. cm.

Includes bibliographical references.

ISSN 2156-1397

EISSN 2156-1400

1. Mathematics—Study and teaching—United States—Periodicals  
QA11.A1 J963

**More Information is available online:** [www.tc.edu/jmetc](http://www.tc.edu/jmetc)

## *Journal of Mathematics Education at Teachers College*

### **Call for Papers**

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at [JMETC@tc.columbia.edu](mailto:JMETC@tc.columbia.edu) by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at [hecker@tc.edu](mailto:hecker@tc.edu) or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### **Looking Ahead**

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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## Slouching Toward a National Curriculum

Jeremy Kilpatrick  
University of Georgia

The U.S. school mathematics curriculum changed slowly during the last century, and only in the past few decades have there been serious efforts to establish it nationally rather than having it be, at least in principle, locally determined. Waves of change have periodically swept the curricular ocean, but on the seabed, in the classrooms where students encounter their curriculum, much less has changed. A variety of recent efforts have been undertaken to change the curriculum nationally, and the country may be close to adopting a common core of standards for the school mathematics curriculum. In view of the complexity of how the curriculum operates and how it might change—and in particular, in view of the critical role teachers of mathematics play in making any change—the effects a national curriculum might have on classroom practice are impossible to predict.

*Note:* Based on a colloquium presentation at Teachers College in December 2009.

### Introduction

In 1968, Joan Didion published a collection of essays entitled *Slouching Toward Bethlehem*, a title she took from the last line of William Butler Yeats' (1921) famous poem "The Second Coming," which ends as follows:

And what rough beast, its hour come round at last  
Slouches towards Bethlehem to be born?

I have borrowed the image of slouching from Didion and thereby from Yeats as a way of discussing the prospect of our moving toward a national curriculum in school mathematics. In this paper, I discuss: (a) issues of curriculum structure, (b) views of curriculum change, (c) some recent efforts to change the school mathematics curriculum in the United States, and (d) the question of whether we are indeed slouching toward a national curriculum.

### Curriculum Structure

We begin with the question of what a curriculum is. A couple of popular characterizations are, first, a set of experiences designed to promote learning and, second, the course that pupils follow. There are other ways to think about it, but I especially like the image of a linear path through a multidimensional domain. If you think about our field of mathematics, it really is multidimensional. And there is no way we can put into schools everything in that multidimensional domain, so we have to make a selection. From the learner's point of view, however, the curriculum appears linear. Students come to school every day and work on one piece of school mathematics at a time. So from their perspective, it is linear, whereas from our perspective, standing aside and looking at the field, it is multidimensional. It has various facets, various parts. The key role for the curriculum maker, therefore, is to find a path to take through the domain so that students learn

important things that need to be learned. And as everybody knows, that is a nontrivial task.

In the Second International Mathematics Study for the International Association for the Evaluation of Educational Achievement (IEA; Robitaille, 1980; Travers & Westbury, 1989), the researchers looked at three levels or aspects of the school mathematics curriculum: (a) the *intended* curriculum, which is the administrator's point of view; (b) the *implemented* curriculum, which is the teacher's point of view; and (c) the *attained* or *realized* curriculum, which is the pupil's point of view. That distinction in views was helpful to the IEA researchers. They were doing an international study, so they could address the question of what the official documents in various countries said the curriculum was. They could use questionnaires to try to understand what teachers thought they were teaching, what they said they were teaching. And they could use tests to examine what the students had actually learned. For a large-scale study that relies on questionnaires and tests, such a parsing of the curriculum may be useful, but it rests on certain assumptions that can be questioned in other contexts.

The *intended-implemented-attained* decomposition implies that the power of the curriculum starts with administrators, flows to teachers, and then flows down to pupils. It is a unidirectional top-down orientation that takes a narrow view of education and of the curriculum. Whose intentions are we talking about? What about the teacher's intentions? What about the students' intentions? What do we mean by an intended curriculum? Is there only one? I would say no, there is not. And this parsing casts the teacher in a role of an obedient employer. Here is your curriculum; now teach it. It is a top-down view not only of the curriculum but also of how the curriculum might change.

In my view, the so-called intended curriculum is not a curriculum at all; instead, it is a blueprint for a curriculum to be realized. The word *curriculum* comes from the Latin word *currere* (to run) meaning course or career. It refers to

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actual experience—not intentions, but reality. I would argue that the intended curriculum is to the real curriculum as the architect’s plan is to the building.

Curriculum systems, as Ian Westbury (1980) has argued, have a *deep structure* requiring:

the recognition that many of the elements of the surface structure of those systems (for example, syllabi, guidelines, given kinds of examination, [and, we might add, standards]) may have, in particular cases, a problematic influencing relationship on both the curriculum as it is found and attempts to modify or change that curriculum. (pp. 15–16)

### Views of Curriculum Change

One should recognize that systematic curriculum change in any country is a 20th-century phenomenon. It was not practiced before about the middle of the last century when educators began devising projects to engineer desired change—much as projects had been used to develop medicines, weapon systems, or spaceships. Systematic curriculum change began, I would argue, with the new math efforts, which showed how much any change depended on the teacher. At the time of the new math, projects were organized to change the curriculum in a systematic way by revising courses and textbooks systematically, and all those projects discovered that the teacher played a critical role:

Every teacher is involved in curriculum development, whatever curriculum he [or she] follows, and there are obvious reasons why he [or she] should know as much as possible about its construction and be able to examine it critically. (Howson, Keitel, & Kilpatrick, 1981, p. 259)

One of the major lessons that curriculum developers learned, if they learned anything at all from the new math experience, was that if the teacher did not understand or know what the proposed curriculum was, it did not get implemented the way the people who created it thought it should.

One can think about any educational system as a complex of nested structures, with the classroom at the center where the mathematics is taught and learned, with classrooms nested within schools and schools nested within various units such as school districts. Depending how a country’s educational system is organized, schools and districts will be nested within various state and regional structures, all of which are nested within a country.

We tend to view the structures as forming a hierarchy, with the decisions at the top filtering down to classrooms. But I would argue that it is more accurate to see the systems as interlocking and interpenetrating. The vector of

change can go in any direction: not just top-down, but also bottom-up, as well as from side-to-side. The ocean is a good metaphor for thinking about the school mathematics curriculum because one often gets a lot of surface change going on at the top, whereas down at the bottom, where the curriculum really lives in classrooms, teaching and learning are unchanged or relatively unchanged. Consequently, the surface of the curricular ocean may sometimes appear to have been swept by a tsunami, but at the depths, life goes on much as before.

There have been three main waves of U.S. school mathematics curriculum “reform” in the 20th century. First, there was a wave at the beginning of the century around the University of Chicago to unify the secondary mathematics curriculum. Second, there was the new math wave in the middle of the century. And then there was the standards movement near the end. I would argue that none of those waves actually reformed the curriculum. Even though people talked about reform, the forms stayed pretty much the same. There were changes, but most were not the changes that the reformers had intended. The rhetoric of reform often masked disunity, contradiction, misinterpretation, and indifference. I have heard various estimates of how many U.S. teachers of mathematics have been involved in these curriculum change efforts—whether to unify the curriculum, bring in the new math, or apply standards. And 10 percent seems to be a rough upper limit for the number of teachers who might have been seriously involved in any of these efforts, which indicates that school mathematics forms a very stable system. The metaphor suggesting that the bottom of the ocean is where the students are suggests that whatever change there is, is not dramatic or revolutionary.

### Efforts to Change the School Mathematics Curriculum

Recent efforts to change the U.S. school mathematics curriculum have been different from those during the new math era. Although the National Council of Teachers of Mathematics (NCTM) played a part during the new math era in moving curriculum efforts forward, only in the last several decades has NCTM been seriously involved at a national level in talking about and working toward curriculum change. In 1980, NCTM published a document, *An Agenda for Action*, whose theme was to make problem solving the focus of school mathematics. Of course, readers of that document did not necessarily understand what that meant, but it sounded good. And basic skills were to be elevated to more than computation. That was NCTM’s first real effort to go beyond its membership and actively influence public policy. At the time of the new math and during the immediately following decades, people were very suspicious of any national movement to determine the curriculum. Instead of having the federal government get involved or having states get together,

NCTM, our national teachers organization, began to push for curriculum change—a phenomenon unfamiliar to people in other countries, where the curriculum is typically set by the national government. In the early 1980s, NCTM officers began to discuss how the council might make curricular recommendations. They appointed several committees on selecting textbooks and evaluating programs. In 1984, a task force was appointed by NCTM to set guidelines for the K–12 school mathematics program.

### *NCTM Standards Documents*

The 1984 NCTM task force began the so-called standards-based reform—or standards-based change—in school mathematics. NCTM published a series of standards documents: the curriculum and evaluation standards (NCTM, 1989), the standards for teaching (1995), the assessment standards (1995), and the principles and standards for school mathematics (2000). The 1989 document was the first effort to propose national standards, and it led other subject matter domains like history and science to set up standards for their fields. These documents, and especially the one from 2000, have been very influential publications. Most states in the country have tried to take at least some, if not all, of the NCTM standards into account in formulating their own.

### *The Mathematics Learning Study*

Another national effort that resulted in some influence on curriculum change in school mathematics was the Mathematics Learning Study of the National Academy of Sciences (NAS). During the last half of the 1990s, the National Research Council (NRC) had conducted a study commissioned by the U.S. Department of Education and Department of Human Services for the NAS on the difficulties that young children were having in reading (Snow, Burns, & Griffin, 1998), and the NAS decided that if they could settle the reading wars, they could also settle the math wars (neither of which they actually did). In 1998, the Department of Education and National Science Foundation asked the NAS to establish a committee to conduct a study of mathematics learning. The study was completed in a year and a half, and in 2001, the committee released its 480-page report entitled *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001). It is available in hard copy or at the National Academy Press Web site <http://books.nap.edu>. To disseminate the results to a broader audience, the following year the committee released a 52-page so-called popular report (Kilpatrick & Swafford, 2002), also in hard copy or on the Web. The longer report is sometimes called the unpopular report, but it has turned out to be influential in many places.

The goal of the Mathematics Learning Study was to make recommendations for improving student learning of

mathematics in the grades from pre-kindergarten to 8, and specifically to (a) synthesize the rich and diverse research on pre-K to 8 mathematics education; (b) provide research-based recommendations for teaching, teacher education, and curriculum for improving student learning and identify areas where research is needed; and, as if that were not enough, (c) give advice and guidance to educators, researchers, publishers, policymakers, and parents.

A major theme of the reports of the Mathematics Learning Study is the idea of *mathematical proficiency*. If anything from the study has been influential, it has been that idea. The committee members discussed many different words and phrases that could be used to characterize what we meant by “successful mathematics learning.” We thought of *mathematical literacy*, but that did not seem to capture the whole spectrum of what school mathematics should be. We then thought of *numeracy*, which is a term often used in the United Kingdom to capture successful mathematics learning, but it is not a term used in this country. Someone proposed *mastery*, but we quickly realized that its history in U.S. education is somewhat checkered. The term *competence* was another possibility, but it, too, has an unfortunate history.

Our aim in formulating the idea of mathematical proficiency was to get beyond the skill-understanding dichotomy in which people were saying, “Oh, we should seek skill in mathematics before understanding.” “We should seek understanding in mathematics before skill.” “Or we should seek only one or the other.” The committee did not want it to be a binary proposition, so what we did was to settle on the term *mathematical proficiency*, which we defined as composed of five interwoven strands that were to be developed simultaneously. We did not want the question, Is it skill, or is it understanding? We said, “It is skill, it is understanding, and it is more than either of those.” We used the term to define learning goals, talk about proficiency in teaching, and organize both our synthesis of research and our discussion of where the research falls short.

The five strands are as follows: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The strand we eventually termed *adaptive reasoning* was something that the mathematicians on the committee especially wanted because of a widespread concern that reasoning had been undervalued in the NCTM standards documents. The mathematicians initially proposed that we call it *logical reasoning*, but committee members argued that mathematics involves more than just logical reasoning, so we ended up with this not-so-clear term, *adaptive reasoning*, to try to allow for other kinds of reasoning, such as inductive reasoning and plausible reasoning, as well as deductive reasoning. Furthermore, some of the mathematicians resisted including the strand *productive disposition*. They said, “What’s that doing there? That’s affect; it’s attitude. It doesn’t belong as part of



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proficiency.” But the teachers on the committee said, “We can’t have something called mathematical proficiency if students are turned off by mathematics. Proficiency has to have some kind of affective component.” So the five strands are what we settled on; they are what we could get agreement on.

In the book (Kilpatrick et al., 2001), we used a drawing of a braid or rope to portray the interlocking, interpenetrating, integrated nature of the five strands. Earlier we had thought about using the metaphor of a prism breaking a ray of light to show how the strands might fit together. But we ended up with the metaphor of braiding because we wanted to say that the five strands interlock and develop together. Right after the book came out, I happened to go to Singapore, where I learned about the framework that people at their National Institute of Education had adopted for portraying the school mathematics curriculum (Dindyal, 2005). They used a pentagon with mathematical problem solving in the center, and the five sides were concepts, skills, processes, metacognition, and attitudes. We on the Mathematics Learning Study committee did not know about the work done in Singapore, and they did not know about our work. Completely independently, each group came up with a five-component portrayal of school mathematics. The two do not correspond perfectly. For example, the Singapore people used the term *metacognition*, which we had thought about using but then avoided because we thought too few in our audience would understand it. Strategic competence is not the same as metacognition, and adaptive reasoning is not the same as processes, but there are some close resemblances across the components, which suggests that maybe we and the people in Singapore were at least in the same ballpark in thinking about mathematics learning. They put problem solving at the center of their framework, and we said that problem solving was involved in all of the five strands. Both their framework and our strand model get at the same notion: that proficiency in mathematics is more than simply skill or understanding and that learners need to develop all five components simultaneously.

### *Focal Points and Focus in High School Mathematics*

After NCTM published the 2000 *Principles and Standards* document, they began to get criticism that they needed to spell out what mathematics should be taught at each grade and complaints that international studies were showing the U.S. mathematics curriculum to be, as the phrase goes, a mile wide and an inch deep. In response, NCTM (2007) published a document dealing with Pre-K to Grade 8 mathematics called *Curriculum Focal Points* that tries to give for each of the elementary and middle school grades some foci for the mathematics curriculum, so that the curriculum is not trying to include everything at every grade. That document appears to have been reasonably well accepted by the profession. I think most teachers and

other mathematics educators have generally been approving, but one of the unfortunate things that happened was in the media. The media like to tell their own story about these things, and they portrayed the document as marking an end to the math wars, with NCTM conceding defeat. *The Wall Street Journal*, for example, portrayed it as “a remarkable reversal” by NCTM, with teachers getting “new marching orders” to go back to basics. The NCTM had spent weeks with a reporter, trying to explain what the focal points document was all about, but the reporter ended up writing an article that was not what NCTM had in mind when they were explaining to him what they were doing.

In 2009, NCTM published a secondary school volume, *Focus in High School Mathematics: Reasoning and Sense Making*. It is not as comprehensive as the earlier focal points document. It does contain grade-by-grade examples of curriculum focal points in reasoning and sense making, but there are two additional volumes yet to come—one on probability and statistics, and one on algebra and geometry. These documents reflect or will reflect NCTM’s efforts to focus the school mathematics curriculum.

### *National Mathematics Advisory Panel Report*

The so-called National Mathematics Advisory Panel (NMAP) issued a report in March 2008 called *Foundations for Success*. The panel had been established in April 2006 by President George W. Bush with a very extensive charge that included the following items to be reported on:

- (a) the critical skills and skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics;
- (b) the role and appropriate design of standards and assessment in promoting mathematical competence;
- (c) the processes by which students of various abilities and backgrounds learn mathematics;
- (d) instructional practices, programs, and materials that are effective for improving mathematics learning;
- (e) the training, selection, placement, and professional development of teachers of mathematics in order to enhance students’ learning of mathematics;
- (f) the role and appropriate design of systems for delivering instruction in mathematics that combine the different elements of learning processes, curricula, instruction, teacher training and support, and standards, assessments, and accountability;
- (g) needs for research in support of mathematics education;

- (h) ideas for strengthening capabilities to teach children and youth basic mathematics, geometry, algebra, and calculus and other mathematical disciplines;
  - (i) such other matters relating to mathematics education as the Panel deems appropriate; and
  - (j) such other matters relating to mathematics education as the Secretary may require.
- (NMAP, 2008, p. 7)

The first of those points—the critical skills and skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics—put the emphasis very squarely on algebra, and we are still living with that emphasis. How did the panel define *algebra*? First, they reviewed state standards, current textbooks, the 2005 12th-grade National Assessment of Educational Progress objectives, the American Diploma Project benchmarks, and the Singapore standards to see how algebra was portrayed in each of those documents. Then they produced a list of 27 major topics of algebra organized into six categories. The panel was careful to say in a footnote that the list “is meant as a catalog for coverage, not as a template for how courses should be sequenced or texts should be written” (NMAP, 2008, p. 15). But regardless of how the list was meant, it reveals that the panel was assuming that school algebra is school algebra now and forever, here and everywhere.

In other words, the panel assumed they could define what school algebra is by listing a set of topics. They did not say what was behind those topics or how those topics were to be handled in school. They just listed the 27 topics that would tell teachers what they should be teaching, what their algebra course is or ought to be. Consider, for example, the topic of logarithmic functions. How are such functions to be introduced, understood, or used? Apparently, the NMAP thought it was not important to address such a question.

The 12th Study of the International Commission on Mathematical Instruction concerned the future of the teaching and learning of algebra. Writing in the report of that study, Margaret Kendal and Kaye Stacey (2004), who had surveyed the teaching of algebra in a number of countries, made the following observation: “Don’t take your country’s curriculum and approach to teaching algebra for granted and don’t assume all other educational jurisdictions operate in a similar way—they conspicuously do not” (p. 345). Kendal and Stacey found substantial differences across countries in: who takes algebra, whether it is integrated or layered across the years, how much emphasis is put on ideas like generality or pattern, how much attention is paid to multiple representations, and what role is played by technology when algebra is taught. These are not topics; they are ways in which algebra can be and is being handled. Consequently, one cannot simply list 27 algebra topics and assume that one has said

everything there is to know about them. The bottom line is that algebra cannot be well defined by listing topics. The algebra curriculum, like the school mathematics curriculum in general, has to be characterized differently.

### *The White Papers Project*

A lesser known project that tried to influence the school mathematics curriculum is the White Papers Project of the National Academy of Education (NAEd). Begun in March 2008, it was supported by several foundations and included support from the National Academies of Science. The project involved six working groups, each of which was asked to produce a so-called white paper that would review the available research evidence on a specific topic in education and present policy options. The six topics were as follows:

- Reading and literacy
- Science and mathematics education
- Time for learning
- Teacher quality
- Standards, assessments, and accountability
- Equity and excellence in American education

The idea was to get the papers drafted in time to present them to the incoming administration and Congress after the November 2008 election.

I was co-chair, with Helen Quinn of Stanford University, of the group that wrote the science and mathematics education white paper. After a March 2008 meeting of all the chairs and the advisory board, our group held several conference calls to discuss issues. We met at Stanford in May 2008, with some people calling in, and several of us met later in October 2008 at the University of Washington. Most of our work, however, was done by email. We finished our paper on Election Day 2008, and had a definitely final draft on the first of December. We had been asked to write 20 pages, but our paper turned out to be 33 pages with 26 additional pages of notes and references. So we sort of overdid it, but we did get it done in time.

Several weeks after the election, we held a public forum “Education Policy in Transition” at the National Academy of Science building in Washington, DC, for people from the executive transition team and the new Congress to inform them as to what was in our white papers. Former Colorado Governor Roy Romer, who had played a major part in getting the project going, talked about how the white papers should form a bridge between policymakers and the academic community, and called on President-elect Obama to act on meaningful education reform in his first hundred days in office. Mark Lampkin, who was Romer’s counterpart on the Republican side, said that the papers would give the President-elect and the 111th Congress a guide to much needed school reform. Many important ideas were shared by participants in the

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forum. The NAEd Web site <<http://www.naeducation.org>> has a videocast of the forum along with the briefing sheets for the white papers that were given out there.

Although the final draft of our white paper had been finished in 2008, the process of review, final editing, and approval set up by the NAEd took until mid October 2009, when the paper was posted on its Web site (Kilpatrick & Quinn, 2009). As published, it is 12 pages with six recommendations. Only four of the white papers were eventually posted, and ours was the third. The idea of capturing the attention of the incoming administration and Congress as soon as they got to Washington did not work out as planned, but there are some indications that even though the papers were late, they influenced some policy people and agencies.

In our white paper, we used ideas about proficiency in science taken from a study by the National Academy of Science that had resulted in a book *Taking Science to School* (Duschl, Schweingruber, & Shouse, 2007). The book listed four characteristics of proficiency in science, and we added a fifth so as to include proficiency in engineering:

A student with proficiency in science

- Knows and can apply major scientific ideas
- Can collect and analyze data (experiments and observations)
- Understands science as a way of knowing (and can apply this to their own thinking)
- Can participate effectively in scientific practices (argue from evidence, design tests of an idea, formulate testable questions, write and draw diagrams to illustrate and explain one's thinking and record one's investigations, read about science and interpret the written text effectively, gather information from a variety of resources, use technology as appropriate, etc.)
- Understands the designed world and can engage in the process of design

Perhaps not surprisingly, we had a similar set of characteristics of proficiency in mathematics that were taken from *Adding It Up* (Kilpatrick et al., 2001):

A student with proficiency in mathematics has

- *Conceptual understanding* - comprehension of mathematical concepts, operations, and relations
- *Procedural fluency* - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence* - ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning* - capacity for logical thought, reflection, explanation, and justification
- *Productive disposition* - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Our first recommendation is the following:

The federal government should strengthen the pre-kindergarten through 8th grade science and mathematics curriculum by supporting the National Science Foundation to fund the development of several curricula that focus on core concepts and skills, thereby preparing all students to succeed in high school. The materials should include related curriculum support materials, professional development tools, and assessments.

This was not originally our first recommendation, but during the review and revision process, it was moved to the front. The important thing about it is that people from the science education community and the mathematics education community were able to agree on how the federal government should strengthen the pre-K to Grade 8 curriculum in science and mathematics. It seems that at those grades, there is agreement within each community on how to proceed. In contrast, although the mathematics education community seems to be close to some consensus on what the curriculum should be for Grades 9 to 12, the science education community cannot agree on what science courses ought to be offered in those grades or even what topics should be included. Mathematics educators may think they have problems with the nature and form of high school mathematics, but science educators seem to be in even deeper trouble.

Consequently, our second recommendation is the following:

High school course sequences and curricula in science and mathematics should be rethought and redesigned.

I skip the other recommendations except for the last. The sixth recommendation, which was originally our first recommendation, may be the most important point in the paper:

Federal and state policy makers should establish a research and development cycle to sustain and improve science and mathematics education nationally.

We are certainly not the first people ask for such an iterative cycle, but it needs to be said repeatedly that rather than scrapping everything and starting over, science and mathematics educators should try continuous improvement the way engineers do, the way things work in other areas. Why does the field not have something closer to that instead of saying, "Well, we have to revolutionize everything in a radical and dramatic reform effort"? That is not the way other countries do it, and our working group thought we should not do it either.

*Common Core State Standards Initiative*

The Common Core State Standards Initiative is a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The Initiative officially began in July 2009 with an effort to develop a common core of state standards in English-language arts and mathematics, first for college and career readiness and then for Grades K–12. The college and career readiness standards were to be “research and evidence-based, internationally benchmarked, aligned with college and work expectations, and include rigorous content and skills” (NGA, 2009). The initiative began with 48 states (all but Alaska and Texas), together with the District of Columbia, Puerto Rico, and the Virgin Islands.

The process of developing standards proceeded at a very fast pace. On July 1, 2009, the development work groups and feedback groups were announced. For the college and career readiness standards, the mathematics work group had 15 members led by Bill McCallum, of the University of Arizona; Phil Daro, of America’s Choice; and Jason Zimba, of Bennington College. The mathematics feedback group had 19 members and included many prominent mathematicians and mathematics educators who had been involved in standards setting at the state level. Draft college and career readiness standards were unexpectedly released on July 22, 2009, and officially released for a month of public comment on September 21, 2009. Three days later, a so-called validation group, with 29 members, was announced whose role was to oversee the process of standards formation and revision.

On November 10, 2009, the work groups and feedback groups for the K–12 standards were announced. The mathematics work group had 51 members, again led by McCallum, Daro, and Zimba, and the mathematics feedback group had 19 members. Draft K–12 standards went to the validation group on November 20, 2009, and on March 10, 2010, a public draft was sent to the states and posted on the Web at <http://www.corestandards.org/> to allow for commentary and review. The final set of standards was launched on June 2, 2010, less than a year after the process had officially begun.

One feature of the Common Core Initiative is that the states are working with Achieve (an independent education reform organization that has sponsored the American Diploma Project), ACT (originally the American College Testing program), and the College Board to develop means of assessing the standards once they are in place. Unlike other countries, the United States has never had an end-of-high-school test. Instead, we have had college-entrance tests, the most widely used coming from ACT and the College Board. The Common Core Initiative is asking these organizations, “What do students need to be able to do at the end of high school so that they are adequately prepared to go into a career or college?”

Achieve has been promoting second-year algebra as the culminating mathematics course needed for high school graduation, and some states are beginning to implement that. There are some reasons to believe that if Achieve could start over, they would not say Algebra Two. As we point out in our white paper (Kilpatrick & Quinn, 2009), the original second-year algebra course was never designed for every student, let alone developed to be required for high school graduation. Most mathematics educators, I think, would want a final required high school course to be broader and more potentially useful than the traditional Algebra Two course. It is an interesting difficulty: people would like such topics as finite mathematics and statistics to be incorporated into every student’s curriculum, and yet at the same time they would like the curriculum to have fewer topics and be more focused. Currently, much of Algebra Two is just Algebra One over again, which also needs to be fixed. The mistake is to say that Algebra Two is the capstone course for every high school student, and the only solution may be to redefine what Algebra Two is.

Once the Common Core standards were launched, they could be officially adopted by any state or territory as long as any additional standards it added did not amount to more than 15% of the total. I think the percentage originally proposed was much higher, but it was lowered to 15% in part, I think, as a reflection of everyone’s desire to have a more focused and uniform curriculum.

Most states and territories have officially adopted the Common Core standards, spurred on in part because adoption of the standards earns a state extra points in the competition for federal Race to the Top funds. One can anticipate that some states will embrace the standards without any question and attempt to implement them exactly as they are. One can also anticipate that some states will object to adopting or implementing them for one reason or another. It will depend upon the state’s education situation and its politics, how much they think they can get away with, whether they think they need the Race to the Top money, and whether they can get that money by other means. A state may find strategies to say, “Well, in principle we endorse these standards, but we happen to like our present standards, so we are going to follow them.” Teachers in some states are likely to be sent mixed messages about state standards, and there may well be lawsuits. For example, in July 2008, the California State Board of Education voted to require all eighth graders in the state to take Algebra One. That requirement was not implemented everywhere in California, but that fall some individual districts began requiring every eighth grader to take Algebra One, whatever Algebra One is. Fifty years ago, when I taught Algebra One in California, we all knew what the course was. Today it is not at all clear what Algebra One consists of, because there are so many versions of it around. In any event, the Education Legal Alliance of the California School Boards Association

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(CSBA) challenged the requirement, got a trial court injunction in December 2008 to prevent its implementation, and in 2010 won against the state board in appellate court (CSBA, 2010). As the Common Core standards come into effect, therefore, there will quite likely be lawsuits, there will be districts that will not follow the state standards, and there will be states that do what they can to get the money from the federal government because they need the money desperately. It is going to be a very complicated issue.

Everyone who has worked on the Common Core standards knows that they are far from perfect, but no mechanism or process has been set up for improving them rather than simply junking them and starting over. Adoption is one thing, but how the standards will play out in the state, the local districts, and the classroom is another. If they are immediately revised, that will distress everyone who is trying to reach some sort of steady state in implementing them, but they will need to be revised at some point, especially the parts that are not working well.

One possibility for revision of the Common Core standards is to examine how countries with a national curriculum manage the process of curriculum change. Many countries appear to have in place a sensible process for getting the school mathematics curriculum revised on a regular basis in a way that does not traumatize teachers, does not require everything to be overturned, does not require completely new textbooks, and so on. If we can borrow curriculum ideas from other countries, we can also borrow their ideas for changing the curriculum.

One natural consequence of the adoption of common national standards is likely to be the adoption of one or more nationwide high-school-leaving examinations. We have a strong psychometric tradition in this country that may make it difficult for such exams to be comprehensive and thought-provoking. There is likely to be strong pressure to make the exams cheap, easily scorable, and highly reliable, at the possible expense of validity. Other countries have a tradition of incorporating teacher judgment into the scoring of high school exit exams, but we do not have such a tradition.

Another problem posed by the adoption of common standards is that publishers are likely to find it increasingly difficult to produce innovative mathematics textbooks and instructional materials. In the past, there have been some opportunities, mostly spurred on by the National Science Foundation, for publishers to produce ground-breaking materials. National standards may, however, lead every publisher to produce more or less the same thing. Publishing houses have been consolidating at an increasingly rapid pace. With a smaller number of publishers going after the school mathematics market, the competition for sales will likely mean that, like the automobile industry, their products are going to look very much alike—more alike than they have in the past.

### A National Curriculum?

When U.S. Speaker of the House Tip O'Neill lost his first election—to the Cambridge, MA, city council—he said that his father told him, “All politics is local. Don't forget it.” In my view, all curriculum change is local—and personal. If the U.S. school mathematics curriculum is to change, it will have to be local change. It will also need to be collaborative. Yet we seem clearly to be headed for a national curriculum. How are these ideas to be reconciled?

Some years ago, three of my doctoral students and I studied the development of an innovative precalculus course at the North Carolina School of Science and Mathematics (NCSSM; Kilpatrick, Hancock, Mewborn, & Stallings, 1996), whose mathematics faculty, by the way, Henry Pollak advised and whose mathematics program he greatly influenced. In our study, we found that the mathematics faculty, which had started out to change the curriculum by developing a course, ended up changing as well their practice as teachers, and they made those changes by working together. We studied four high schools, including the NCSSM, that were offering the course, and in every one of those schools the teachers had teamed up to change their practice. Nobody had done it on his or her own. Teachers had worked together, especially at the NCSSM, to develop and implement the curriculum.

I would argue that no teacher can get very far trying to change the curriculum alone. Curriculum change needs to involve a group of teachers working together collaboratively. The same thing is true, I think, if teachers are going to change their teaching. The work in lesson study in various countries is a good illustration of how lessons can be changed if teachers work together (Fernandez & Yoshida, 2004; Lewis, 2002; Stigler & Hiebert, 1999). And the same seems to be true if teachers want to change the activities they are using in class. An example I recently stumbled across is the BGfL (the Birmingham, England, Grid for Learning <<http://www.bgfl.org/>>). The BGfL is a Web site where teachers post ideas for activities in their classes and collaborate with one another in developing, using, and refining those activities. There are many such sites on the Web, which underlines the point that curriculum change is collaborative.

Consequently, one should not assume that the apparent movement in the United States today toward a more centralized, uniform curriculum will mean that curriculum change will begin coming from the top down. Despite claims to the contrary, school systems are very much alike. There is always a gap between official pronouncements and actual practice. For years, France has been seen as having a very centralized system. The claim was that theoretically the French minister of education could say something like the following: “At this moment in every classroom in France, every student in a certain grade is studying the Pythagorean theorem.” In contrast,

several decades ago, before England had a national curriculum as they do now, they were a good example of a decentralized system. But centralized and decentralized systems are not necessarily as different as they might seem, which may give us some hope that even though our system is today fairly decentralized, it may not become all that centralized. As a French school inspector once said, “In France, every teacher is supposed to be doing the same thing, but nobody is, and in England, where everyone is supposed to be going his own way, nobody is” (quoted by Howson et al., 1981, p. 58). That comment suggests that centralized or not, the education system is never quite what people may think it is.

I was at a conference recently with a colleague from England who said that now, just as the United States appears about to embrace a national curriculum in mathematics, England and Wales seem to be moving away from theirs. That is another ray of hope that perhaps the movement toward a national curriculum will not last forever. The part to be played by individual teachers always seems to be critical:

Curriculum...must mean more than syllabus—it must encompass aims, content, methods and assessment procedures. One cannot truly talk, then, of a ‘national curriculum’, for it depends upon individual teachers, their methods and understanding, and their interpretation of aims, guidelines, texts, etc. The part played by the individual teacher must, therefore, be recognized. (Howson et al., 1981, p. 2)

No one can say now what school mathematics will be in a decade or so, but it can be safely predicted that it will not be what people are predicting. Responding to the chaos of his time, Yeats says in “The Second Coming”:

Things fall apart; the centre cannot hold;  
Mere anarchy is loosed upon the world,  
The blood-dimmed tide is loosed, and everywhere  
The ceremony of innocence is drowned;  
The best lack all conviction, while the worst  
Are full of passionate intensity.

Surely some revelation is at hand.

Today, some revelation about school mathematics seems to be at hand. We will soon see how the competition for Race to the Top funds changes what states think they are doing in school mathematics. The adoption of common standards will change the rhetoric nationally, but it remains to be seen how it changes what happens in individual classrooms. We can certainly hope it helps teachers move toward more effective instruction in mathematics; we can hope for that.

## References

- California School Boards Association. (2010, May 4). *ELA victory officially ends 8th-grade Algebra 1 mandate* [Press release]. Retrieved from [http://www.csba.org/NewsAndMedia/Publications/CASchoolNews/2010/May/ElectronicOnly/ELA\\_AlgebraWin.aspx](http://www.csba.org/NewsAndMedia/Publications/CASchoolNews/2010/May/ElectronicOnly/ELA_AlgebraWin.aspx)
- Didion, J. (1968). *Slouching toward Bethlehem*. New York, NY: Farrar, Straus & Giroux.
- Dindyal, J. (2005, August). *An overview of the Singapore mathematics curriculum and the NCTM Standards*. Paper presented in Topic Study Group 3 (Secondary Curriculum) at the Third East Asia Regional Conference in Mathematics Education, Shanghai, Nanjing, and Hangzhou, China.
- Duschl, R. A., Schweingruber, H. A., & Shouse, A. W. (Eds.). (2007). *Taking science to school: Learning and teaching science in grades K–8*. Washington, DC: National Academies Press.
- Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Erlbaum.
- Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development in mathematics*. Cambridge, UK: Cambridge University Press.
- Kendal, M., & Stacey, K. (2004). Algebra: A world of difference. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The future of the teaching and learning of algebra: The 12th ICMI Study* (pp. 329–346). Boston, MA: Kluwer, 2004.
- Kilpatrick, J., Hancock, L., Mewborn, D. L., & Stallings, L. (1996). Teaching and learning cross-country mathematics: A story of innovation in precalculus. In S. A. Raizen & E. D. Britton (Eds.), *Bold ventures, Vol. 3: Case studies of U.S. innovations in mathematics education* (pp. 133–243). Dordrecht, Netherlands: Kluwer
- Kilpatrick, J., & Quinn, H. (Eds.). (2009). *Science and mathematics education* (Education Policy White Paper). Washington, DC: National Academy of Education. Available: [http://www.naeducation.org/Science\\_and\\_Mathematics\\_Education\\_White\\_Paper.pdf](http://www.naeducation.org/Science_and_Mathematics_Education_White_Paper.pdf)
- Kilpatrick, J., & Swafford, J. (Eds.). (2002). *Helping children learn mathematics*. Washington, DC: National Academy Press. Available: <http://books.nap.edu>
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press. Available: <http://books.nap.edu>
- Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia, PA: Research for Better Schools.
- National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author.

## SLOUCHING TOWARD A NATIONAL CURRICULUM

- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2007). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making*. Reston, VA: Author.
- National Governors Association. (2009, July 1). *Common Core State Standards Development Work Group and Feedback Group announced* [Press release]. Retrieved from <http://www.nga.org/portal/site/nga/menuitem.ef1a671e4253877ae8ebb856a11010a0/>
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- Robitaille, D. F. (1980). Intention, implementation, realization: Case studies of the impact of curriculum reform. In H.-G. Steiner (Ed.), *Comparative studies of mathematics curricula: Change and stability 1960–1980* (pp. 90–107). Bielefeld, Germany: Universität Bielefeld, Institut für Didaktik der Mathematik.
- Snow, C. E., Burns, M. S., & Griffin, P. (Eds.). (1998). *Preventing reading difficulties in young children*. Washington, DC: National Academy Press. Available: <http://books.nap.edu>
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the worlds' teachers for improving education in the classroom*. New York, NY: Free Press.
- Travers, K. J., & Westbury, I. (Eds.). (1989). *The IEA Study of Mathematics I: Analysis of mathematics curricula*. Oxford, UK: Pergamon.
- Westbury, I. (1980). Change and stability in the curriculum: An overview of the questions. In H. G. Steiner (Ed.), *Comparative studies of mathematics curricula: Change and stability 1960–1980* (pp. 12–36). Bielefeld, Germany: Institut für Didaktik der Mathematik-Universität Bielefeld.
- Yeats, W. B. (1921). *Michael Robartes and the dancer*. Churchtown, Ireland: Cuala Press.

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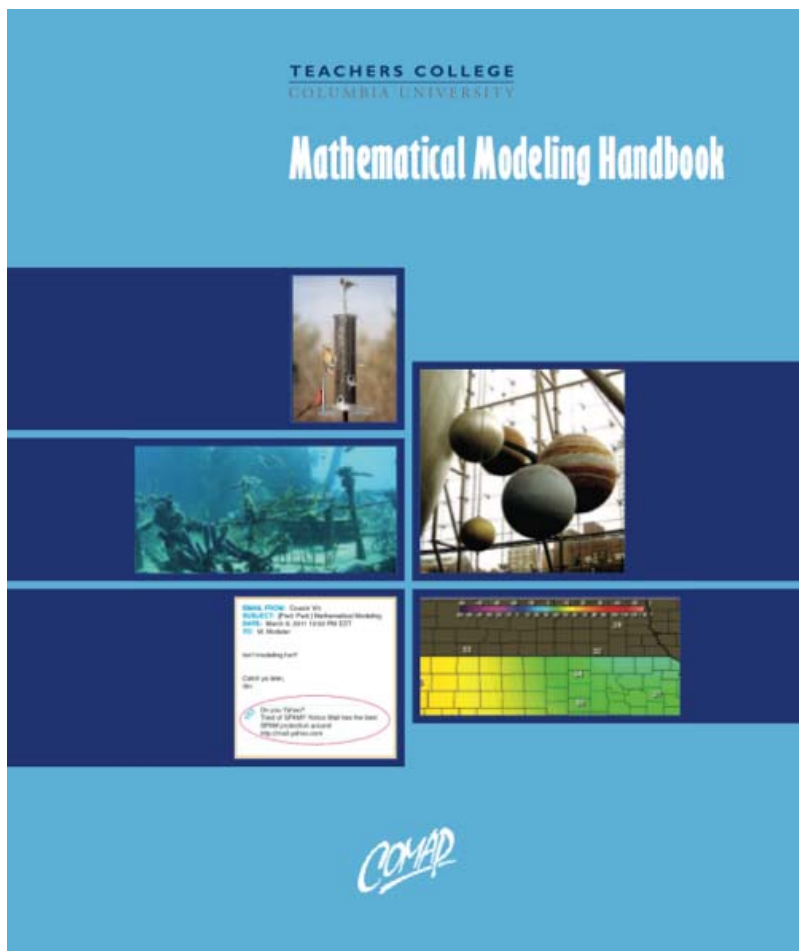
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