

Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

TABLE OF CONTENTS

Foreword

- iv** **Honoring the Past—Anticipating the Future**
J. Philip Smith, Bruce R. Vogeli, Erica Walker

Preface

- v.** **Mathematics Curricula: Standards and Implementation**
Nicholas H. Wasserman

Editorial Point-Counterpoint

- 6** **Will Common Core State Standards facilitate consistency
and choice or lead to unexpected outcomes?**
Nicholas H. Wasserman and Jacob Koelher

Articles

- 8** **Slouching Toward a National Curriculum**
Jeremy Kilpatrick, University of Georgia
- 18** **The Common Core State Standards: Comparisons
of Access and Quality**
Nicholas H. Wasserman, Marymount School of New York
- 28** **Modeling in the Common Core State Standards**
Kai Chung Tam, Macau, PRC
- 34** **Reformed Curriculum Framework: Insights from
Teachers' Perspectives**
*Shikha Takker, Homi Bhabha Centre for Science Education,
TIFR*
- 40** **From Curriculum Guides to Classroom Enactment:
Examining Early Career Elementary Teachers' Orientations
Toward Standards-Based Mathematics Curriculum
Implementation**
Joan Gujarati, Manhattanville College
- 47** **Design Research in the Netherlands: Introducing
Logarithms Using Realistic Mathematics Education**
David C. Webb, University of Colorado at Boulder
*Henk van der Kooij, Freudenthal Institute for Science and
Mathematics Education University of Utrecht, The Netherlands*
*Monica R. Geist, Front Range Community College Westminster,
Colorado*
- 53** **Using Simplified Sudoku to Promote and Improve
Pattern Discovery Skills Among School Children**
Khairul A. Tengah, Universiti Brunei Darussalam

TABLE OF CONTENTS

63 NOTES FROM THE CURRICULUM LABORATORY

Bruce R. Vogeli

What is Mathematical Modeling?

Henry O. Pollak

Modeling Lessons and the Common Core State Standards

Benjamin Dickman, Brookline, Massachusetts

Meteorology: Describing and Predicting the Weather— An Activity in Mathematical Modeling

Heather Gould, Stone Ridge, New York

Packing Oranges

Kai Chung Tam, Macau, PRC

Arithmetic and Algebra to Solve Fairness Problems

Joseph Malkevitch, York College

Finding Average Rainfall

Stuart Weinberg, Teachers College Columbia University

The Buckyball Has Relatives: A Classroom Approach to Polyhedra

Anahu Guzman, LIM College

Other

72 ABOUT THE AUTHORS

The *Journal of Mathematics Education at Teachers College* is a publication of the
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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Roszkopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Roszkopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

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|-------------|---------------|
| Fall 2011 | Technology |
| Spring 2012 | Evaluation |
| Fall 2012 | Equity |
| Spring 2013 | Leadership |
| Fall 2013 | Modeling |
| Spring 2014 | Teaching Aids |

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NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the *Consortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* “cognitive category” of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a “disposition to mathematize,” that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory’s Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher’s notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a “disposition” toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

Notes from the Curriculum Laboratory begins with a brief view of the Laboratory’s interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory’s modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

What is Mathematical Modeling?

Henry O. Pollak

People often say that the mathematics they learned in school and the mathematics that they use in their lives are very different and have little if anything to do with each other. The textbook or the teacher may have asked, for example, how long it takes to drive 20 miles at 40 miles per hour and accepted the answer of 30 minutes. But when you live 20 miles from the airport, the speed limit is 40 mph, and your cousin is due at 6 PM, does that mean you leave at 5:30? Your actual thinking may be quite different. This is rush hour. There are those intersections at which you don't have the right of way. How long will it take to find a place to park? If you take the back way, the average drive may take longer, but there is much less variability in the total drive time. And you'd better find out when the plane is expected. But don't forget that the arrival time they give you is the time the plane is expected to touch down on the runway, not when it will start discharging passengers at the gate. And so forth.

Our expectation is that the CCSS Modeling Standards will help to bridge the gulf between reasoning in the mathematics class and reasoning about a situation in the real world. So what do we mean by "mathematical modeling"? What's different about reasoning in the outside world from reasoning in mathematics? The real situation usually has so many "angles" to it that you can't take everything into account, so you decide which aspects are most important and you keep those. At this point, you have an idealized version of the real-world situation which you translate into mathematical terms. Now you have a *mathematical model* of the idealized question. Then you apply your mathematical instincts and knowledge to the model and get interesting insights, examples, approximations, theorems, and algorithms. You translate all this back into the real-world situation and you hope to have a theory for the idealized question. But you have to check back: are the results practical, the answers reasonable, the consequences acceptable? If so, great! If not, take another look at the choices you made at the beginning and try again. This entire process is what is called *mathematical modeling*.

Is all this new? Yes and no. Many scientific disciplines use mathematics in their development and practice and when scientists are careful they do indeed check which aspects of the situation they have kept and which they have chosen to ignore. Engineers and scientists, be they physical, social, or biological, have not expected mathematics to teach this for them within a scientific framework, although preparing for this kind of reasoning is part of mathematics. Can we just leave mathematical

modeling to the scientists? We absolutely cannot. Mathematics education is clearly responsible for teaching how to use mathematics in everyday life and in intelligent citizenship, and let's not forget it. But actually any separation from science is a delusion. Let's admit it, both everyday life and intelligent citizenship often also involve scientific issues. So what really matters is learning and practicing the mathematical modeling process. The particular field of application—whether it is everyday life, being a good citizen, or understanding some piece of science—is less important than experiencing this thinking process.

Don't we do this already? Don't we already have "word problems? We have seen that "mathematical modeling" is not just a new and pretentious name for "word problem" or "problem solving" in the traditional sense. Word problems embody the hope that external images will lighten the atmosphere in which mathematics is done, but let's not kid ourselves: the purpose of a word problem is only to practice the mathematics of the current chapter. Therefore an answer to a word problem is considered correct if the student finds the applicable algorithm and carries it out successfully. But mathematical modeling demands more. Yes, the mathematics you decided to use has to be done correctly, but, as we saw, the results must be examined in terms of the original external situation to check that they make sense. The fact that the mathematics that was carried out within the mathematical model is correct is not enough to make the total modeling process a success: the real-world results must also be reasonable in that they check with the real world.

Don't get the impression that all of this is an unnatural demand on mathematics education. Far from it, it strengthens the affinity between pure mathematics and its applications. The heart of mathematical modeling, as we have seen, is problem *finding* before problem *solving*. So often in mathematics, we say "prove the following theorem" or "solve the following problem." When we start at this point, we are ignoring the fact that finding the theorem or the right problem was a large part of the battle. By emphasizing the problem finding aspect, mathematical modeling brings back to mathematics education that aspect of our subject and greatly reinforces the unity of the total mathematical experience.

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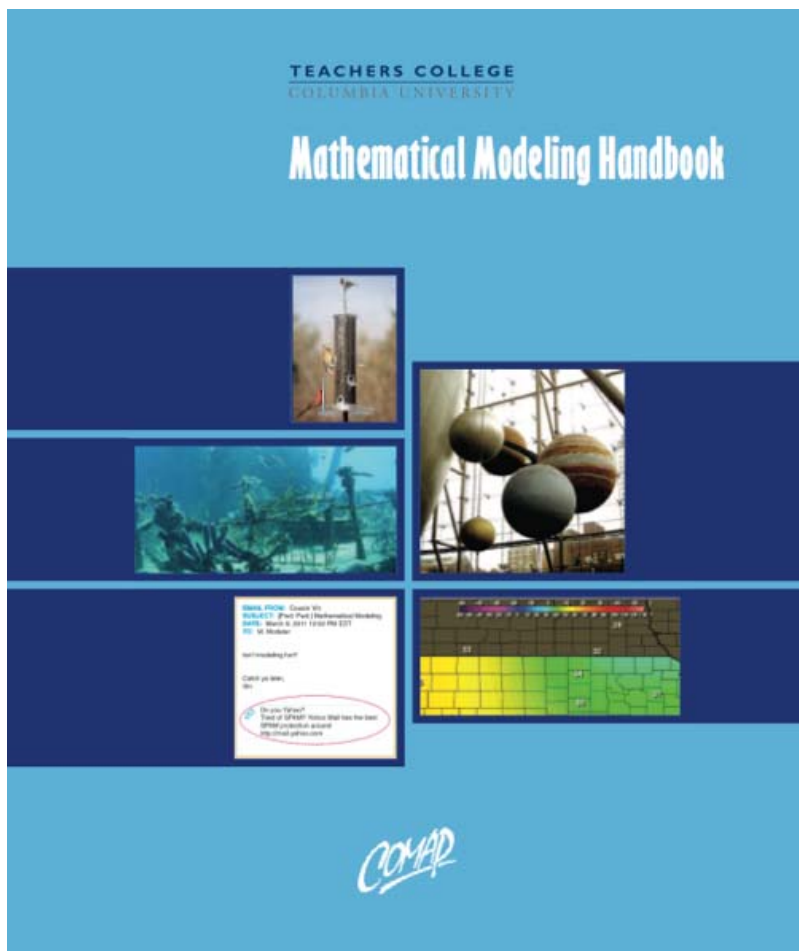
Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

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