## Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

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## The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

#### **Guest Editor**

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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

#### **Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the JMETC include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

#### **Manuscript Submission**

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at JMETC@tc.columbia.edu

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#### Journal of Mathematics Education at Teachers College

#### **Call for Papers**

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

#### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

#### **Looking Ahead**

Anticipated themes for future issues are:

Fall 2011 Technology
Spring 2012 Evaluation
Fall 2012 Equity
Spring 2013 Leadership
Fall 2013 Modeling
Spring 2014 Teaching Aids

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#### **ABOUT THE AUTHORS**

**Jeremy Kilpatrick** is Regents Professor of Mathematics Education at the University of Georgia, and an international leader in the field. Professor Kilpatrick began his career in mathematics education at Teachers College Columbia University.



**Nicholas Wasserman** is a graduating doctoral student at Teachers College Columbia University and a high school Mathematics Teacher at Marymount School of New York. As a graduate from the UTeach program at the University of Texas at Austin, he enjoys teaching and developing mathematical understanding through algebraic reasoning and problem solving. His primary research interests include the fields of mathematics teacher education, secondary mathematics education, and international education.



**Kai Chung Tam** was born in Hong Kong and brought up in Macau. Having studied physics and mathematics in Tsinghua University, Beijing, he is now a doctoral student in mathematics education at Teachers College Columbia University. His interests include fractals, application of mathematics, mathematical intuition in problem solving, and the design of mathematics curriculum. His dissertation is about the effect of context on students' mathematical performance.



**Shikha Takker** is a research scholar pursuing her Ph.D. in Science Education at the Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, India. She has a Masters' in Education (Elementary) from the Tata Institute of Social Sciences, India. Her experience as an elementary school teacher motivated her to pursue research in mathematics education. Her research interests include issues concerning mathematics education, teacher education, childrens' thinking, and classroom practices.



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#### ABOUT THE AUTHORS



**David Webb** is assistant professor of mathematics education at the University of Colorado at Boulder and is also the Executive Director of Freudenthal Institute USA, an international research collaborative for mathematics education. Dr. Webb's research interests are in the areas of teachers' formative assessment practices and the design of professional development activities. Recent research projects have focused on studies of teacher change in classroom assessment, the design and use of assessment in math and science, and the impact of reform curricula on student learning and achievement. Dr. Webb was a middle and high school math and computer applications teacher in Southern California. He currently teaches methods courses for prospective middle and high school mathematics teachers, and graduate level courses that focus on assessment design and practice and the nature of mathematics and mathematics education.



Henk van der Kooij is research faculty at the Freudenthal Institute, University of Utrecht, and has a long history of designing instructional materials and assessment tasks in mathematics curriculum reform projects. He has served as the coordinator of mathematics and science in the Dutch National Examination Board (CEVO). He also was member of the design team of the HAWEX project (1987-1991) and coordinator of the TWIN project (1996-2000), which were both secondary mathematics curriculum projects that were implemented nationwide in the Netherlands. Van der Kooij has also been involved in NSF-supported projects in the United States including coordinator of the Dutch team that designed all assessment materials for the ARISE project (COMAP, 1993-1998).



**Dr. Monica Geist** teaches mathematics at Front Range Community College in Westminster, Colorado. She earned her M.S. in Applied Mathematics at the University of Colorado-Denver, and her Ph.D. in Applied Statistics and Research Methods at the University of Northern Colorado. Her interests include evaluation, assessment, and adapting her teaching using Realistic Mathematics Education and progressive formalization.



**Khairul Amilin Tengah** was born in Brunei in 1978. He is a lecturer of Secondary Mathematics Education at Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam. He received his B.Sc.Ed. from Universiti Brunei Darussalam and M.Ed. in Mathematics Education from Kings College London. His research interests include Lesson Study approach in mathematics education, Recreational Mathematics, Creative approaches in teaching mathematics, and Technology use in the mathematics classroom. Currently, Khairul is a Ph.D student in Mathematics Education at Teachers College Columbia University.

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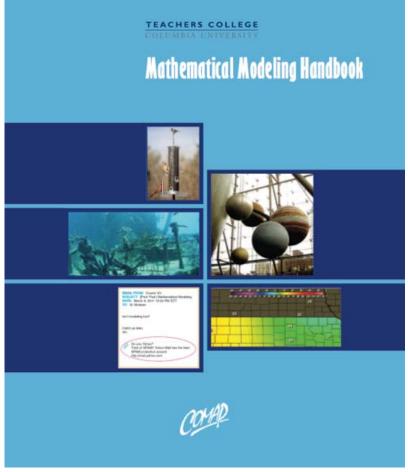
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Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

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