Journal of
Mathematics Education
at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the
Program in Mathematics and Education at Teachers College
Columbia University in the City of New York.

This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology.*

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Journal of Mathematics Education at Teachers College

Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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Journal of Mathematics Education at Teachers College
Spring–Summer 2011, Volume 2

The Common Core State Standards: Comparisons of Access and Quality

Nicholas H. Wasserman
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Last year the United States unveiled the Common Core State Standards (CCSS) in English and Mathematics for grades K–12. In particular, the authors included two possible sequences of 8–12 mathematics courses that would fulfill the standards. Most notably, the courses titled "3a" and "3b" in these two sequences have become gatekeepers to Pre-Calculus (and consequentially Calculus). Taking "3b"would not prepare students to take Pre-Calculus, but at that juncture students would be prepared for a variety of other possibilities among mathematics courses—Statistics, Finance, Modeling, Linear Algebra, Discrete Mathematics, and Computer Science (those in "3a" would have access to all of these and Pre-Calculus). Employing Harvey & Knight's analytic framework on educational quality, this article compares who has access to taking various high school mathematics courses in three countries: the U.S., Finland, and Singapore. Using the framework as a lens to discuss various statistics and different measures of quality, the new CCSS offer a relatively wide variety of courses for high school students, aiming to make the mathematics classes required useful to the students who take them, while simultaneously keeping options open for higher level mathematics.

America has always strived toward a democracy that believes education is the way to level the playing field. In order to do so, our educational system should be consistent, offering every student—rich or poor, majority or minority—an equal opportunity to succeed. It should not afford more privilege to one over another. It should be consistent for everyone. Jimmy having a chance to take Calculus and Juan not having that opportunity is unacceptable in this system.

Our country has also always strived toward a democracy that believes education should be the way to prepare individuals to achieve their own dreams. You can be anything you want to be, do anything you want to do—the "American Dream." As such, our educational system should be exceptional, serving each student's needs and helping each person fulfill his/her potential. It should cater to everyone's needs. Jimmy having a chance to take Calculus and Juan not having to take it is imperative in this system.

These two opposing ideals cannot be reconciled completely; they represent one tension in defining educational quality (Harvey & Knight, 1996). For a country like Sweden, whose national curriculum offers very little differentiation, the desire for consistency in education is apparent. The assumption that no one has particular gifts or talents that need to be developed over others, however, could be written off as unresponsive in a diverse democracy. On the other end, in a country like China, high-stakes testing and tracking at a very early age give an exceptional education and afford opportunities to some students. Pre-determining students' academic futures and career options by testing or other means, and denying similar opportunities to others, could be considered unjust. The United States has been content "to pass" on facing this divide, leaving it in the hands of decentralized, individual states. In an age of national testing and accountability, in competition with other high performing countries that do not have such a decentralized system, Americans have come to a crossroads—the proposition of the Common Core State Standards (CCSS) for K-12 education in English and Mathematics. While the current system fails to produce a relevant and quality education for all students, many Americans fear that essentially national standards would reinforce consistency at the cost of exceptionality. The question becomes, does the CCSS initiative present students with a quality education in mathematics, particularly compared to other countries?

Access to Mathematics

For many centuries mathematics has been regarded as inherently valuable—that everyone should study mathematics because the process of logical reasoning and thinking was naturally beneficial and transferable. While this view has been challenged and disputed vigorously in the last century, mathematics still plays a very important role in much of modern society—business, investment banking, computers, data encryption, the sciences, engineering, and much more. Many good jobs require a solid background in mathematics, yet, not all do. Herein lies the debate: who should have what type of preparation in mathematics, how do you distinguish among them, and at what age?

When considering access to mathematics, attempting to answer such questions drives decisions, standards, and policy. What kind of preparation in mathematics should be required for all students? What should be available for those interested or gifted in a field? What level of mathematics are students cognitively ready to handle and at what age? When should the curriculum be differentiated to prepare students for individual futures? What types of mathematics courses are necessary and useful to citizens and workers? What types of mathematics courses prepare citizens to compete globally? Looking at the proposed CCSS in mathematics through the lens of access to mathematics courses, the aim of this article is to understand how these potential standards compare to what Finland and Singapore are offering their students.

CCSS in Mathematics

The proposed CCSS are greatly "whittled down" compared to current state versions that are burdened with detailed descriptions of seemingly endless concepts. These new standards have been organized into six broad categories for all of K-12 mathematics—Number & Quantity, Algebra, Functions, Geometry, Statistics & Probability, and Modeling (CCSSI, 2010). Within each of these categories for the secondary level is a list of standards that all students should learn in order to be ready for college and career. Additional standards for students pursuing careers in the fields of Science, Technology, Engineering and Mathematics (STEM) also are included.

Based on the CCSS, the authors also included two sample mathematics pathways, or curricula, that might fulfill them (CCSSI Appendix A, 2010). In general, the pathways prescribe four mathematics courses; however, depending on the school, students may take a fifth class (e.g., Calculus) or may not need to take a fourth course. (See Figure 1.) Broadly speaking, the first two courses cover typical Algebra I and Geometry material. The third course, similar to Algebra II, is partitioned into a high and low track. The high track covers some of the additional STEM standards, whereas the low track does not. From this point, those in the low track would not be prepared to take Pre-Calculus but would be led to a variety of other options for a fourth year mathematics course—Statistics, Finance, Modeling, Linear Algebra, Discrete Mathematics, and Computer Science; those in the high track would have access to all of these courses and to Pre-Calculus. The third course, titled "3a" and "3b" in these two sequences, essentially acts as a gatekeeper to Pre-Calculus and, consequentially, to Calculus.

The line between what everyone should know and what only those pursuing careers in specific disciplines should know is very thin; however, it must be drawn somewhere. Inevitably, certain students will need to acquire knowledge that others will not. The desire to balance both consistent education, giving all students equal access to curriculum, and exceptional education, giving selected students the opportunity to pursue advanced curriculum, is evident in these standards. The authors mention balancing four things: preparation for the workforce, preparation for college (both STEM and non-STEM), international benchmarking, and keeping students options open as long as possible (CCSSI, 2010). The CCSS standards and content should be analyzed in relation to the curricula of other countries. Do the CCSS require students to learn enough mathematics knowledge? Do they give gifted students access to rigorous courses?

As answers to these questions about access to mathematics curriculum are explored, the CCSS and, in particular, the suggested possible pathways and courses will be compared to mathematics curricula in Finland and Singapore—two countries that have proven to be leaders in mathematics on international tests like the TIMSS and PISA (PISA 2003; PISA 2006; TIMSS 2007; TIMSS 2003; Simola, 2005). The educational systems and mathematical requirements for each country will be discussed briefly, as well as who and what percentage of the population they represent has access to various courses. Due to the difficulties in obtaining complete statistical information on an educational landscape as diverse and disjointed as the United States, some generalizations and assumptions were made in determining estimates for the American population of students. While the statistics might be general, the study aims for broad comparisons between the three countries, where estimations of this kind are sufficient for application. Using Harvey and Knight's analytic framework (Kubow & Fossum, 2007), which was developed to assess educational improvement that is often characterized by conflicting forces, aspects of the effectiveness and quality of access to mathematics curricula in the United States (current, and under the proposed CCSS) will be compared to that of both Finland and Singapore.

Finland and Singapore:
Secondary Education Systems

Finland

In Finland, the National Board of Education is responsible for the national curriculum. Compulsory education goes through age 16 (Figure 2). It then branches into two tracks of further secondary education: general upper secondary education and vocational education. Despite the fact that neither of these is required, about 92% of the 16-19 age population pursues one of these two educational opportunities—approximately 50% are admitted to general upper secondary education, and 42% attend vocational schools (Statistics Finland, 2009).

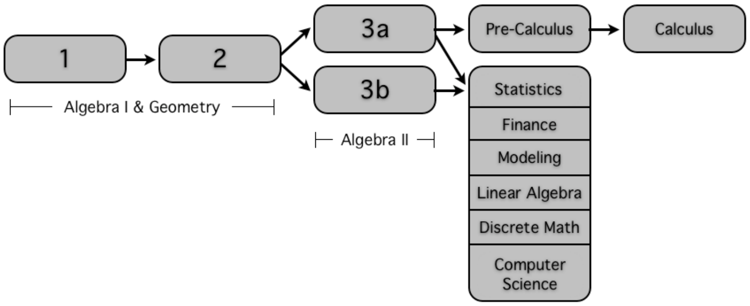
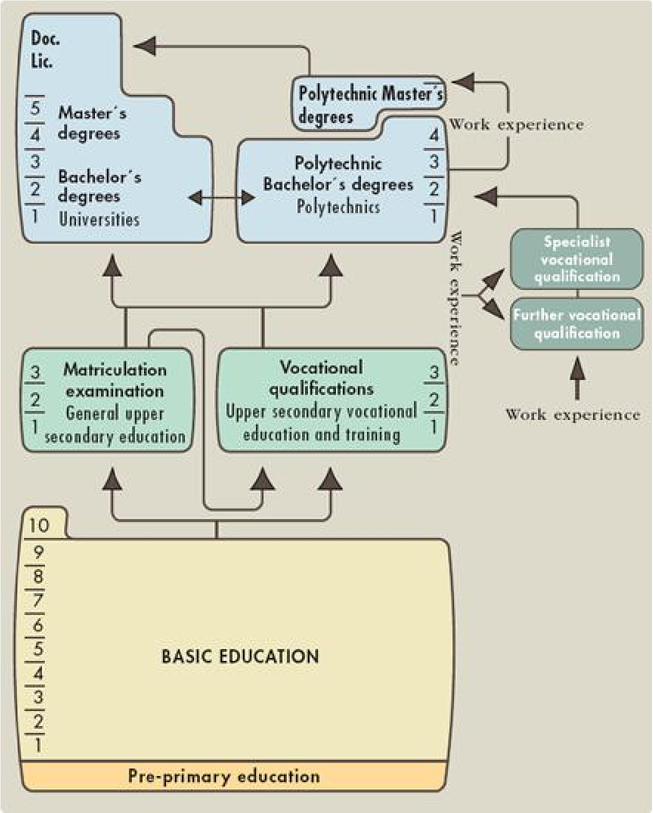


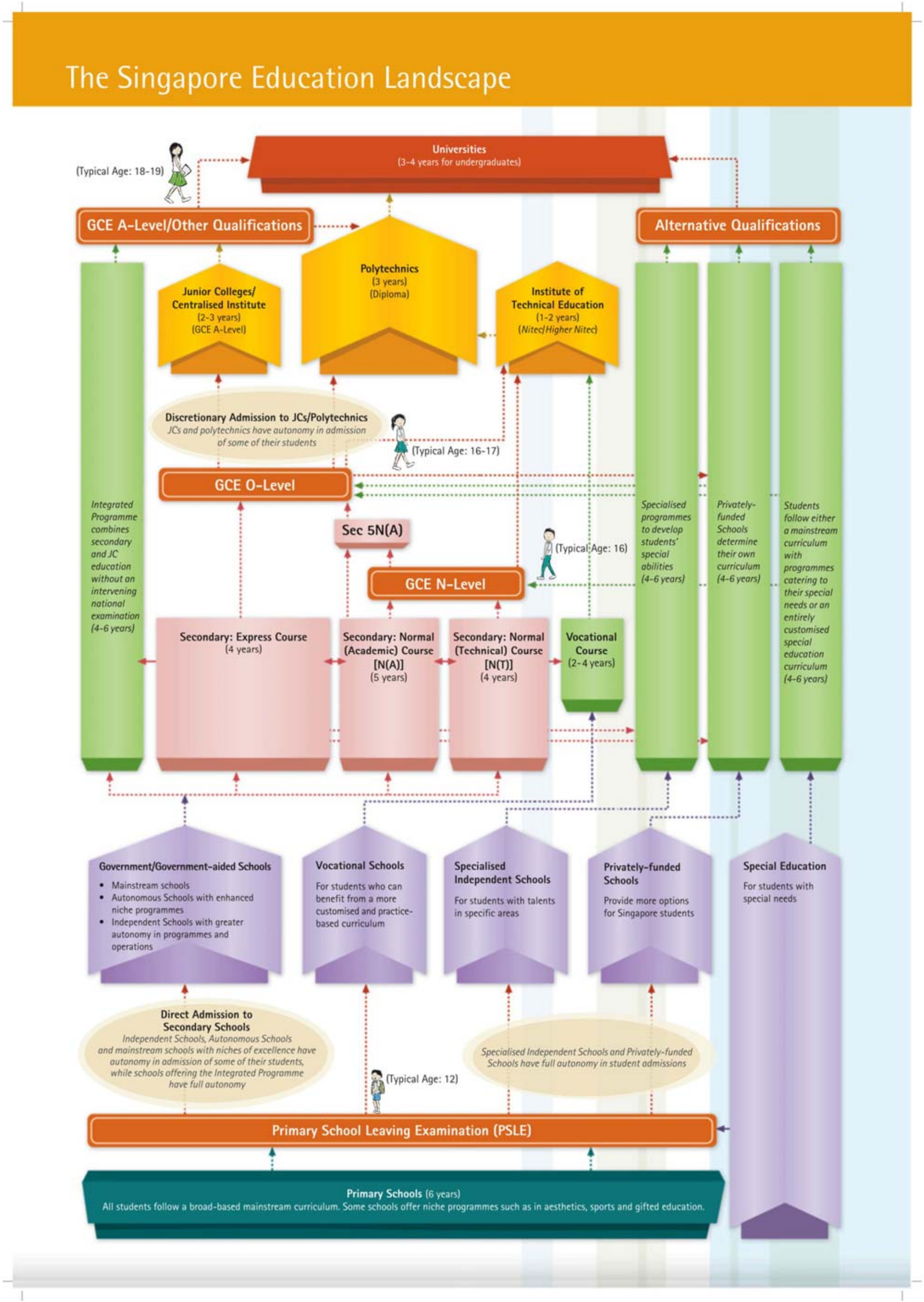
Figure 1. Suggested CCSS secondary mathematics pathways, adopted from CCSS Appendix A



Source: Finnish National Board of Education (www.oph.fi)

Figure 2. Finnish Educational System

With regard to mathematics, those in vocational education learn the equivalent of Algebra I and Geometry, with specializations depending on the particular vocation (Vocational education and training in Finland, 2004). Some, like the Technology, Communication & Transport and Natural Sciences sectors, may cover related mathematics topics, but only in so far as required to know and perform in a typical job in that field. Those in the general upper secondary education track have two options regarding mathematics: the short and long syllabus. Approximately 42% of students complete the long syllabus in mathematics (Statistics Finland, 2009). For those completing the short syllabus, all graduates will have the equivalent of up to Algebra II with a further specialization in the mathematics of Finance/Business or Vectors/Trig/Computers. Those completing the long syllabus are required to complete Calculus with a further specialization in Advanced Calculus or Number Theory/Logic (Finnish National Board of Education, 2003).



Source: Ministry of Education (MOE), Singapore (www.moe.gov.sg)

Figure 3. Singaporean Educational System

Singapore

In Singapore, the Ministry of Education is responsible for the national curriculum, and, in 2009, required the many Private Education Institutions (PEI's) to register with the Council for Private Education (CPE). This subsidiary of the Ministry of Education has regulatory power over the private education sector, even further centralizing the educational system. Compulsory education goes through age 16 as well (Figure 3), but at age 12 it is partitioned into different academic tracks: Express or Special, Normal Academic (N(A)), Normal Technical (N(T)), and Vocational. Depending on the academic track, students have various opportunities to pass the three different levels of examinations: the N-, O- and A-levels. It is through these three examination levels that students are granted access to a variety of other forms of higher education. The Express or Special track prepares students to take the O-level examination, and then, most likely, two years of study at a junior college to prepare for the A-level examination. There are also Integrated programs that combine the secondary and junior college education into a 6-year program in preparation for the A-level examination. The Normal track is divided into two tracks, Academic and Technical, where students first must pass the N-level examination, and then possibly take the O- and A-level examinations to pursue further higher education. Approximately 56% of the population enters the Express or Special tracks, 26% enter the Normal Academic, and 13% enter the Normal Technical, leaving about 5% who enter the Vocational track (Statistics Singapore, 2009).

Regarding mathematics, the level varies greatly across the four tracks. Those few students in the Vocational track are not required to take any advanced study in mathematics; only those skills that would be relevant to a specific job are required. Students in the lowest academic track, N(T), gain roughly the equivalent of Algebra I and Geometry with no option for additional mathematics courses. The Express and N(A) tracks include many Algebra II topics and an option to study additional mathematics, including Pre-Calculus and Calculus. This knowledge is not required for the O-level examination however. Students passing this examination can pursue preparation for the A-level examination in a junior college, which could include the study of Calculus, Advanced Calculus and Statistics (Ministry of Education, Singapore, 2006). For A-level examinations, students must take four content subjects from two main categories, Humanities & Arts (Art, Economics, Geography, History, English Literature, Music, Theatre) and Mathematics & Sciences (Biology Chemistry, Physics, Mathematics, Computers).

Application of the Harvey and Knight Framework

Using Harvey and Knight's analytic framework as a guide for this discussion (see Kubow & Fossum, 2007), various aspects of the quality of access to mathematics courses in the United States (current, and under the proposed CCSS), Finland, and Singapore will be compared. Harvey and Knight (1996) define quality using five notions, each connoting different visions of educational improvement: Exceptionality, Consistency, Fitness for purpose, Value for money, and Transformation. Using this framework, the discussion will focus on various portions of the CCSS and the suggested course offerings, as well as how they deal with some of the tensions involved in quality.

Exceptionality

Specifically for the mathematics curriculum, access to high-level courses for students is one means of assessing exceptionality. In particular, who has access to a widely influential course such as Calculus is noteworthy in comparing these countries' curricula. Finland, whose educational system through age 16 is, for the most part, consistent and universal for all students, has two tracks that branch off for secondary education. It is within these branches that differences in curricula begin to appear, especially regarding mathematics. Only those who enter the general upper secondary education track, and who choose the long syllabus in mathematics, will have access to Calculus materials. This means that of any one age group, approximately 21% of students potentially will see Calculus concepts in secondary school (Statistics Finland, 2009). The educational paths and tracks in Singapore create a vastly different academic roadmap, but only those taking O- and A-level examinations could see Calculus concepts in their schooling. No significant portion of the population from the Vocational track would prepare for these examinations. From those in the Normal track(s), N(A) and N(T), approximately 9% of an age group would pass the O-level examination to take further courses in junior college, possibly including Calculus (Parliamentary Replies, 2010). From the Express track, 56% of an age group would have access to taking rigorous mathematics, including Calculus, but most of these students would probably not do so until the junior college level. Therefore, roughly 65% of an age group would have access to taking Calculus, but given the requirements for A-level examinations, perhaps only 40%[1] would take any of the

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[1] Students choose to take four A-level examinations: three from Humanities & Arts and one from Mathematics & Sciences, or vice versa. Given the lack of data found on which subject tests students prepare for in taking the A-level examinations, the author uses an estimate that roughly half opt for three from Humanities & Arts, and

Mathematics courses or examinations, resulting in approximately 26% of students potentially learning Calculus concepts prior to a University setting. In the United States, about 6% take the AP Calculus examination(s), and a total of around 11% of an age group cohort takes some form of Calculus prior to college (College Board, 2009; National Science Board, 2002). While there is no way to assess how the CCSS curriculum might influence this statistic, it is hard to imagine the numbers drastically changing. However, it could be argued that if roughly half of the population split at the "3a" and "3b" courses, and that roughly half of those in "3b" continued on into the Pre-Calculus and Calculus track, that up to 25% of an age group might be on pace to take Calculus; although based on the current figures of 6% and 11%, only about half of those would take it as a part of their secondary school curriculum (Table 1).

Table 1. Comparison of Access to Calculus

Percentage of an Age-Group having Access to Calculus			
Finland	Singapore	U.S.	U.S. Core Standards
21%	*26%	11%	*13%

* based on statistical estimates

Exceptional education is marked by how well a system serves those students who are particularly gifted, which in the case of mathematics, is often having access to rigorous courses such as Calculus. Both Finland and Singapore have done well on international comparisons, in part likely due to the rigor of their curricula and the high percentage of students who gain access to challenging mathematics courses. However, a question comes to mind: is the high number of students who take Calculus a function of how good the mathematics is, or how limited the options in mathematics are? If there were more alternatives, would as many students pursue Calculus as opposed to taking other courses in mathematics? Of course, there are also other explanations for why more students in Finland and Singapore might take Calculus. For example, some cultural distinctions might explain these differences, such as how much importance is placed on mathematics and mathematics education within a country. Other differences among the populations also persist. Many working class families in Singapore reside just across the border due to the cost of living, and thus are not necessarily included in the educational system; the United States, on the other hand, is required to enroll every student—even those who do not speak English well. So while from the outset it seems that the United States lags behind in access to exceptional mathematics courses, our definition of quality regarding mathematics education needs to be broadened to consider other factors.

Consistency

For the purpose of utilizing Harvey and Knight's framework, the level of consistency seen in mathematics education can be discussed as the level of mathematics that is required of all students. From this perspective, the educational system in Finland is very consistent and requires nearly identical training through age 16. Regarding mathematics specifically, this would include the study of concepts typical to Algebra I and Geometry courses. Roughly half of the population at this point either does not pursue secondary schooling or enters vocational training that has little additional mathematics content. About 29% of the population will cover Algebra II concepts and possibly specialize in Finance/Business or Vectors/Trig/Computers mathematics through the short syllabus, and another 21% of the population will complete the long syllabus that includes Calculus (Statistics Finland, 2009). In Singapore, about 95% of the population pursues some form of secondary, academic schooling, including 13% in the Normal Technical track. Roughly, this involves about 18% of the population learning just Algebra I and Geometry topics. For the rest of the population, about 26% might end up pursuing Calculus in junior college based on an earlier estimate, and the other 56% would end up completing the equivalent of either Algebra II or Pre-Calculus. For the United States, about 10% of an age group will never finish high school, severely limiting their mathematical studies (U.S. Census Bureau, 2010). Most students, however, would be required to complete at least three years of high school mathematics, which in many states would be the equivalent of Algebra II. For estimation purposes, it will be assumed that a significant portion of the remaining 90% of the population completes at least Algebra II, and only about 11% of those make it to Calculus. If using the CCSS as a guide, and the same dropout rate, it might be that half split into "3a" and "3b," where half of those in "3a" go onto Pre-Calculus and the other half take a different fourth year course, and where half of those in "3b" take a fourth year course and the other half don't take any (Table 2 and Figure 4).

Regarding consistency, both Finland and Singapore have very centralized systems that dictate national curricula. Currently in the United States there is a range of standards that fluctuate from state to state and district to district, as well as differing requirements for public and private education; the CCSS could become a way to unify *some* of these differences. With regard to the consistency of mathematics education achieved, Finland requires study

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the other half opt for three from Mathematics & Sciences. With five choices in Mathematics & Sciences, one being Mathematics, for the half that opt for only one in Mathematics & Science, only about 10% of students would take Mathematics; and for the half that opt for three from mathematics & sciences, approximately another 30% are likely to take mathematics.

Table 2. Comparison of Highest Secondary Mathematics Training

Highest Secondary Mathematics Training for an Age-Group				
	Finland	Singapore	U.S.	U.S. Core Standards
Algebra I & Geometry	50%	18%	*20%	*20%
Algebra II	29%	56%	69%	*20%
Other				*35%
Pre-Calculus				*12%
Calculus	21%	*26%	11%	*13%

* based on statistical estimates

up to Algebra I and Geometry topics for everyone, and 95% of the population in Singapore chooses to pursue secondary education in some form that would include Algebra I and Geometry topics. In the U.S., most states require three years of mathematics study that would include Algebra II topics for many American students. Presumably this obligation would remain with the CCSS as well. The requirements in the U.S. seemingly demand more mathematics; however, with a 10% dropout rate and some students not required to graduate with Algebra II content in the U.S., about 20% might have only Algebra I and Geometry content. In Singapore, however, while only Algebra I and Geometry are required, most of the population pursues mathematics further, leaving about 18% of the population with only Algebra I and Geometry content. So while the U.S. requires more in theory, comparatively, more Singaporeans finish with higher levels of mathematics in practice. Finland, in contrast, has a significant amount, about half, of the population who finish with roughly the equivalent of Algebra I and Geometry content. Despite this, those in Finland who continue are relatively strong mathematics students, since approximately 21% make it to Calculus. Based on these numbers, it might be easy to conclude that, overall, American and Singaporean students achieve higher levels of mathematics than do Finnish students; however, about the same percentage of the population in Finland and Singapore end up taking Calculus. Such results impose the question if pursuing Algebra II and other topics are really worthwhile or unnecessary for all students, in regard to giving the best students access to higher levels of mathematics.

Fitness for Purpose

Fitness for purpose is perhaps the most difficult to compare, since all three countries have various paths for students who desire to pursue more or less mathematics. The main indicator of a mathematics education being fitting would be how well students gain access to the mathematics they need for their future. This would most likely be evidenced by a variety of course offerings and options regarding mathematics. Singapore has the most possible educational tracks; however, which path one follows is dependent mostly on test scores, and not necessarily interest. If test scores are a good measure of both aptitude and interest, then perhaps these examinations are useful for tracking students into the various paths. The Vocational and N(T) tracks in Singapore are very practical for catering mathematical knowledge and needs around specific career interests, but often severely limit peoples' options. Many tracks do not require significant mathematics beyond Algebra I and Geometry. As long as those students who are in the Vocational and N(T) tracks are not there solely because of test scores, but because of their choices and interests, this system seems to be suitable for students' purposes. In Finland, the students who pursue vocational careers and schooling over general upper secondary schooling do not seem to be hindered by lack of access to higher level mathematics. Evidently, vocational career people manage to do their jobs sufficiently with only knowledge of Algebra I and Geometry curriculum. Although a high percentage of the population does not pursue more advanced mathematics topics, it also appears to be suitable. Those who do pursue mathematics do very well, with about half continuing on to Calculus. Yet if wanting to keep all students options open as long as possible, neither of these systems, which begin tracking at age 12 or 16, may be best. The current and proposed system in the U.S. addresses this idea fairly well, since it requires most students to complete topics through Algebra II. Another notable inclusion in the CCSS that is indicative of trying to meet students' mathematical needs is the number of options available for further mathematics. The courses proposed—Statistics, Finance, Modeling, Linear Algebra, Discrete Mathematics and Computer Science—represent a much larger variety than in either Finland or Singapore (or the current U.S. system). Since the U.S. requires more mathematics of more students—keeping future options open for students—the CCSS, which offer

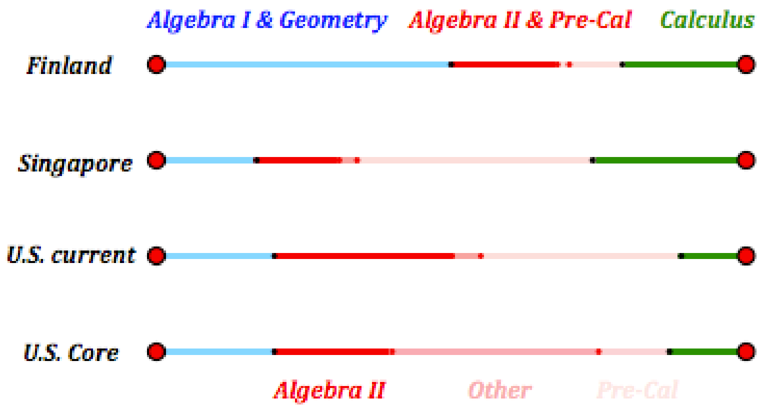

Figure 4. Graphic Representation of Highest Secondary Mathematics Training

more options in mathematical coursework, might be a viable way to help guarantee that the education students receive in mathematics is beneficial for their futures. In terms of fitness for purpose regarding mathematics education and access to mathematics curriculum, it is possible that different ideas in different countries might drive the factors that determine suitability.

Value for Money

Precisely determining the value of funds spent on mathematics education is wrought with personal and philosophical differences that, at a minimum, incorporate different perspectives on education as a private or public good. In order to demonstrate value in an educational system in terms of the mathematics curriculum accessed, one might compare where students typically are in their mathematics development at some common age. This would demonstrate some degree of value, whereby a country offering students higher mathematics by a certain age, say 16, would be considered better, because the same student growing up in both countries would be given a different education based solely on his/her residence. At age 16, for example, students in all countries would have probably learned Algebra I and Geometry topics, with very few in Finland having more knowledge, some in America having taken Algebra II material, and some in Singapore having completed Calculus topics. In this case, Singapore might be given the best value for educational system, because a particular student could potentially gain more from his/her educational experiences in Singapore than in Finland or the United States.

Transformation

It is difficult to compare the extent to which transformation of the mathematics education in various countries is viewed as important. One could plausibly link the notion of transformation with the various tracks offered to students. The most likely form of transformation in education would be including further options (e.g., tracks) for students as opposed to redefining an entire educational curriculum. Based solely on this distinction, Singapore, which has roughly four tracks beginning at age 12, might be considered to view transformation of the mathematics curriculum as more important than Finland, which has a single track up to age 16. The likelihood of transformation would be expected to be smaller in Finland and more likely in Singapore given its ability to create and redefine the various existing tracks. In the current U.S. system, which has not only 50 different states, but hundreds of districts responsible for making up the public educational landscape (let alone the private school system that also plays a role in defining curricula), transformation might also be considered important. The ability for California to alter curriculum is not tied to the entire country adopting the same ideas (e.g., 8th grade Algebra mandate). Yet obligating students to attain a set level of mathematics, basically Algebra II in the proposed CCSS, simultaneously confines major changes to a certain degree. It is reassuring to know, however, that this mathematical requirement is similar in nature to those found in Finland and Singapore. While nationalization of the CCSS might detract from flexibility, one striking aspect is the variety of options discussed for fourth year courses in high school. This, in and of itself, represents a broad transformation of and shift in the types of mathematical knowledge appropriate and useful for secondary students. If these courses embody relevant mathematics for different students, this could cause a far-reaching change in how high school mathematics is envisioned.

Conclusion

Looking at the proposed CCSS through Harvey and Knight's five elements of quality comparison with other high-achieving mathematics countries has yielded some insights. In particular, the standards still offer opportunities for gifted students to pursue high-level mathematics courses, like Calculus, but simultaneously open doors to a potentially broader range of mathematics courses. While the U.S. might never have an equal percentage of students who take Calculus as their counterparts in Finland and Singapore, the wider range of offerings might be particularly appropriate for giving more students opportunities to pursue mathematics that is applicable to their interests and potential futures. The compulsory level of mathematics proposed in the CCSS, being basically Algebra II, seems on par with, if not ahead of, both Finland and Singapore. It is reassuring to know that the U.S. is not proposing something drastically different from what other high performing countries are doing. The question as to how useful knowledge of Algebra II content (or other common mathematics courses) is for students is still yet to be decided, particularly since Finland manages without this requirement. If anything, the transformative ideas behind the proposed CCSS seem to be most evident in the variety of potential course offerings, building on the need for students to be offered diverse and relevant mathematical opportunities, not just Calculus.

In dissecting the CCSS, it is understood that any set of standards can never be all things to all people. Standards are meant to establish a common foundation (or core) that leaves room for some flexibility but not so much to be considered unjust or unequal. They are meant to provide a rigorous mathematics framework, but that also is flexible and relative to future interests. Simultaneously the CCSS have both condensed and extended the proposed secondary curriculum by clearly articulating one route to advanced mathematics courses in high school, and by expanding potential course offerings. The organizational layout of the proposed courses (refer back to Figure 1) also suggests that the first two courses are potentially meant to be universal, i.e. no distinction between honors or regular levels (tracking begins at the division between "3a" and "3b"). Such an idea could possibly level the playing field for many students who are at a disadvantage in the current U.S. system. Having a universal curriculum through the equivalent of Algebra I and Geometry, which divides into an appropriate Algebra II course based on aptitude and interest, that further branches into more appropriate courses of mathematical study depending on individual preferences, strikes a balance between exceptional and consistent education. This is certainly better than tracking after elementary mathematics. And it is certainly better than not offering any differentiated or advanced courses, even for the brightest minds. So while the idea of a national curriculum might be worrisome, at least the secondary mathematics standards that it could be based on hold up in an international comparison of access and quality in mathematics education.

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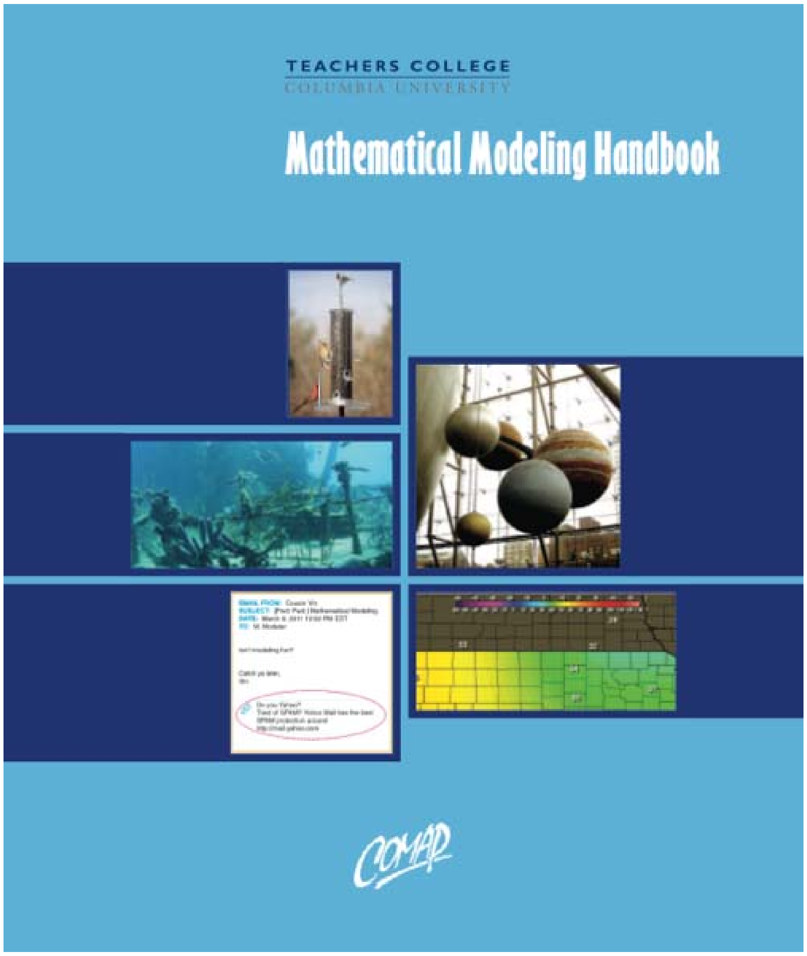

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