

Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the
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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Roszkopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Roszkopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Journal of Mathematics Education at Teachers College

Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the *Consortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* “cognitive category” of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a “disposition to mathematize,” that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory’s Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher’s notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a “disposition” toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

Notes from the Curriculum Laboratory begins with a brief view of the Laboratory’s interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory’s modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

Modeling Lessons and the Common Core State Standards

Benjamin Dickman
Brookline, Massachusetts



The Common Core State Standards for Mathematics (CCSS) places a heavy emphasis on modeling with mathematics (CCSSI, 2010). Included as a *Standard for Mathematical Practice* and associated with over one-third of all high school standards, modeling is defined as “the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions” (CCSSI, 2010, p. 72). Singling out modeling as a mathematical skill to be developed in students may be a new concept for many mathematics teachers and educators. In the interim between the Standards’ release in June 2010 and forthcoming publications of CCSS-aligned textbooks and curricular materials, there is a clear and present need for access to “modeling lessons.”

A good starting point for the development of a modeling lesson is the identification of a real world problem or question. As stated in the CCSS, “proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace” (CCSSI, 2010, p. 7). In our own example to follow, we will begin with a real world problem, as it can often provide both the inspiration for a lesson plan as well as the motivation for engaging in modeling.

Example: What’s the best place to park?

Suppose you are going out shopping and are wondering what the best place to park would be. What sort of considerations go into such a decision? From this question, many others arise:

- Where are we allowed to park?
- What do we mean by “best”?
- How many things are we buying?
- What sorts of things are we buying?
- Are there time constraints? And so forth.

Expecting students to make the transition from identifying a real world problem to developing a fully-functional model in a single day is often unreasonable. Thus, modeling lessons will likely require a minimum of two days. One way to allot time for such an endeavor is to allow for a general exploration on day one (e.g., eliciting and exploring open questions) and then provide more specifics during the following class meeting. For obvious reasons, a teacher will be able to rely less on a detailed

plan during the general exploration phase of a modeling lesson. Note further that the instructor and students are likely to differ on their answers for some of the exploration questions. For example, the teacher may consider “best” to mean “the least work done”, where “work” is defined as the product of weight and distance. Meanwhile, a student might think in terms of how fragile, expensive, or unwieldy (shape-wise) an object is. A good teacher will not shy away from alternative answers, and will instead pursue them as well as the resulting mathematics.

Day two provides an opportunity to include more specifics from a lesson plan. For the problem above, the teacher can suggest using her own definition of best (i.e. least work done) and seeing where it leads. With only one object to be purchased, students are likely to realize that the closer the parking space, the better. But what about for two objects? Some students might think that parking halfway between them is the best solution; others might suggest parking closer to the heavier object. If the goal is that each trip from and to your car requires the same amount of work, how would this affect the ideal parking space? What if you are only concerned with total work done? Instructors might also look for ways to incorporate student responses from the previous class: what if we replace “weight” with “price”? How would this affect our model?

A good modeling lesson lends itself to a variety of related problems. In our example, one might consider the case where you have three objects to buy. If they are of similar weight, how might you represent this with a diagram? (This may be a good opportunity to incorporate technology into the lesson plan.) Even proficient students are unlikely to guess the ideal parking space for such a scenario; a common guess may be the centroid, which is not generally correct. In fact, the full solution to this problem is known (e.g., de Villiers, 2009). For this particular lesson, one might also include a physical model to help illustrate underlying mathematical concepts. This can be done as follows: drill three holes in a flat, wooden board; run three strings of equal length through these holes; tie the string-ends above the board together; and tie each of the string-ends below the board to three equal weights. Using a minimal energy argument, one concludes that the resting spot for the knot above the board will be located at the “best” place! In addition to allowing students to see a real-world manifestation of the theory they have developed, this activity can lead to a number of interesting questions, e.g., Does the physical model serve to confirm the mathematical result? Or is it the other way around?

Finally, it is worth considering other scenarios in which the models students have developed can be called upon. As stated in CCSS, one important insight “provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations” (CCSSI, 2010, p. 72). For example, suppose you are faced with the task of deciding

where to lower a diver so that he can salvage sunken treasure from various sites. Although the context is markedly different from that of going shopping, students should be aware of the flexibility of their mathematical models.

Modeling provides a unique opportunity to identify real-life problems and questions and investigate them mathematically. At present, the dearth of materials for mathematical modeling amplifies the need for capable instructors and curriculum designers. Through a combination of creativity and flexibility, teachers can work alongside students to explore and expand upon issues encountered in daily life, illuminating underlying mathematical structures in the process. In this way, we hope to instill in students the belief that they can use formal mathematics to great effect well beyond the classroom walls.

References

- Common Core State Standards Initiative (2010). *Common Core State Standards for Mathematics*. Retrieved from <http://www.corestandards.org/the-standards>. (30 March 2011).
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Meteorology: Describing and Predicting the Weather—An Activity in Mathematical Modeling

Heather Gould

Stone Ridge, New York



Websites such as weather.com usually don't give the actual current temperature at your location—it's an educated estimate! This modeling activity begins with students exploring the distribution of temperatures across a "map" and ends with the students using mathematical models to estimate the temperature at any given point on that map.

On the first day of the two-day modeling activity intended for algebra classes, students are asked to consider a map with points at which the temperature is known (known-temperature points) and use the map and their experience with temperature to determine how temperature changes over distance from known-temperature points. They should conclude that the temperature changes linearly. The challenge arises when they must find a way to estimate temperature at a point *not* collinear with any

two other points at which the temperature is known (Figure 1). Creative students will solve this problem employing a variety of methods, but each must be based on the assumption of linearity. To determine students' understanding, they are asked to use the model to check if there will be freezing rain along a given route.

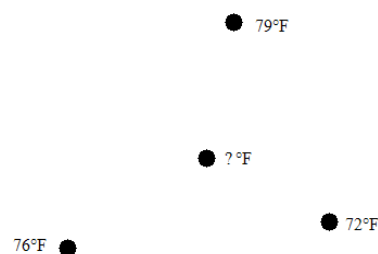


Figure 1

On the second day, students are encouraged to use the graph of a linear function to model the same situations as in the previous day. By doing this, students learn about the properties of linear functions and their graphs. Students who chose to use this method will deepen their understanding and students who chose to use different models are able to make connections between their model and linear functions.

Teachers who employ modeling activities will be surprised at the variety of methods and models that students create, as well as the wealth of discussions that they cause. For example, in determining how the temperature changes with respect to distance, students use their experience to determine that temperature change must be continuous: it is unreasonable to assume that there might be a sudden "jump" in temperature or there might be a "hole" in which no temperature exists. This helps students understand the definition of mathematical continuity. Debate may result during the process of identifying the rate at which temperature changes over a distance: such discussion gives students the opportunity to explore characteristics of various functions. Students should conclude that a function with a constant rate of change—a linear function, as they will learn—is the *most reasonable* function to begin modeling temperature change. This may change once the model has been constructed and tested, as students may find it necessary to refine the linear model, particularly when applying it to the real world; in doing so, students learn to consider which variables to ignore for the sake of ease and the particular constraints of their model. They learn that models can be refined to include these variables and constraints once a preliminary model is produced and tested.

Further discussion should address the different methods used when the point at which the temperature needing to be estimated is not collinear with two known-temperature points. Two examples of models students may create are shown in Figure 2. On the left, a student uses the concept of linearity to approximate the temperature twice:

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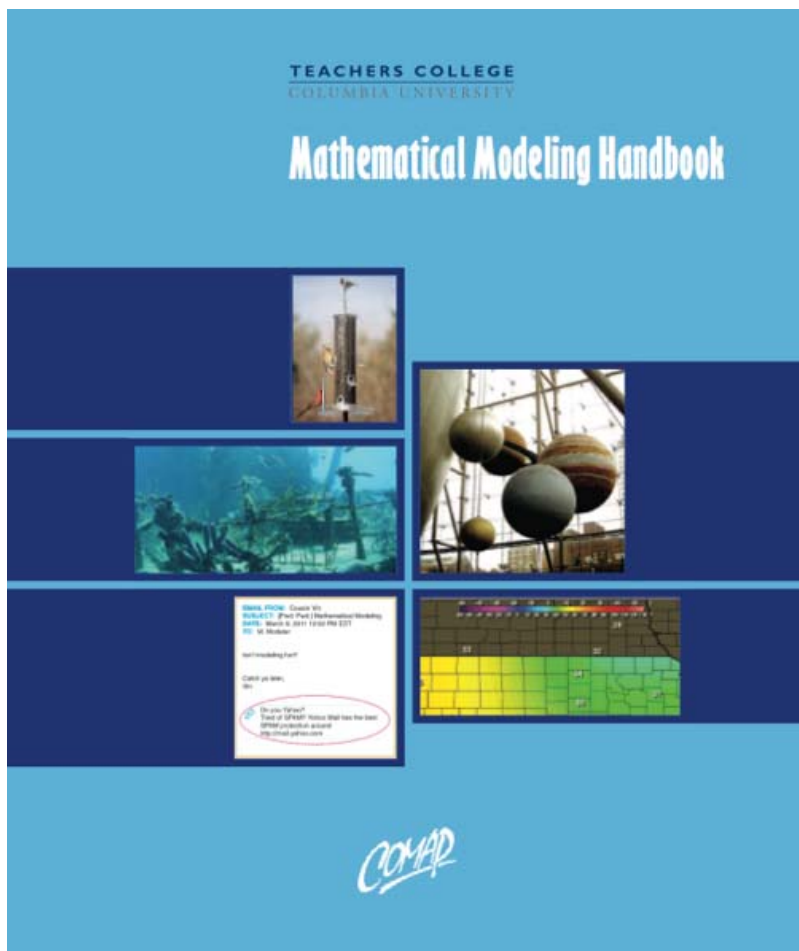
Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

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