Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

TABLE OF CONTENTS

TABLE OF CONTENTS

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

Library of Congress Cataloging-in-Publication Data

Journal of mathematics education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and teaching—United States—Periodicals
QA11.A1 J963

More Information is available online: www.tc.edu/jmetc

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

TO OBTAIN COPIES OF *JMETC*

To obtain additional copies of *JMETC*, please visit the *Journal*'s website www.tc.edu/jmetc. The cost per copy delivered nationally by first class mail is $5.00. Payment should be sent by check to *JMETC*, Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Journal of Mathematics Education at Teachers College
Spring–Summer 2011, Volume 2

Modeling in the Common Core State Standards

Kai Chung Tam
Macau, PRC

The inclusion of modeling and applications into the mathematics curriculum has proven to be a challenging task over the last fifty years. The Common Core State Standards (CCSS) has made mathematical modeling both one of its Standards for Mathematical Practice and one of its Conceptual Categories. This article discusses the need for mathematical modeling in school mathematics and scrutinizes the strengths and weaknesses of CCSS with respect to current practices for teaching mathematical modeling.

Keywords: Common Core State Standards, teaching mathematical modeling, application of mathematics

Introduction

The Common Core State Standards (CCSS) of English Language Arts and Mathematics represent an effort to establish a shared set of educational standards for most states and territories (NGA, 2009). Their widespread adoption puts the topic of mathematical modeling on the surface of discourse. Over the last half-century, a significant effort has been put into developing appropriate applications of school mathematics in teaching and learning. Due to this emphasis, many thoughts and exploratory practices have been produced and, perhaps, incorporated into classrooms; however, a crucial step towards institutionalizing mathematics curricula with a major emphasis on modeling and application has not been made until this point.

The phrase *mathematical modeling* is, of course, not new to curriculum developers and theorists, as there is already extensive use of the words *model* or *modeling* in current standards documents (e.g., NCTM, 1989, 2000) and assessment work (NAGB, 2008). Consider some standards taken from the New York State Mathematics Core Curriculum, Standard 3 (NYSED, 2005):

> A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
>
> A.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics
>
> A.CN.5 Understand how quantitative models connect to various physical models and representations

The framework used in the NYS Mathematics Core Curriculum follows the NCTM's five-content-strand and five-process-strand model (cf. NCTM, 1989, 2000); every content strand crosses with each of the process strands. For example, A.CN.3 belongs to Algebra, one of the Content Strands, and also belongs to Connection, one of the Process Strands. The terms "*model*" and "*modeling*" mostly appear under two process-strands, Representation and Connection. Without further description of the phrases "*model situations*," and "*understand how...can be used*," especially how these requirements interact with the content, teachers cannot implement the standards listed above. All the standards in the Connection Strand within Algebra are identical to those within *Geometry* (coded as *G*) and those within *Algebra 2 & Trigonometry* (coded as *A2*), namely, A.CN.x = G.CN.x = A2.CN.x for any x from 1 to 8. This suggests that the standards associated with modeling are not necessarily clearly defined for teachers, and, as such, may not set very high expectations for students in the area of mathematical modeling. It is reasonable to require that students strive for connections within and between *Algebra 1*, *Geometry*, and *Algebra 2*, but there appears to be no significant expectation of improvement in those skills because no distinction exists to signal progress. While the effort put into writing the NCTM Standards (NCTM, 1989, 2000) cannot be ignored, especially the vivid illustrations of those standards regarding the Algebra, Connection, and Representation strands, the descriptions of the standards were, largely, too general for state curricula to be enriched and clarified.

In consideration of identifying "criteria" by which to evaluate, it is possible to propose some *necessary conditions* to help provide some analysis. On one hand, basic requirements that are intended for all students should be clearly defined so that they can be operationalized easily by educators. On the other hand, more general descriptions of an ideal should be provided so that educators can make sense of it and are able to connect the standards to other curriculum goals. This article will first introduce the need for teaching modeling in the mathematics curriculum, and then examine how the CCSS captures modeling as one of its important standards with respect to both clarity and generality and the requirements for these conditions.

Desire for Modeling in Mathematics Curricula

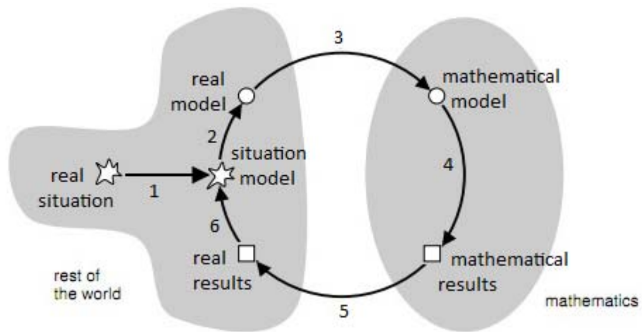
The need to apply school mathematics to the real world in an efficient way is important given the obvious gap between what needs to be taught and what is actually taught in schools. In fact, it is not easy to measure this gap, but many would agree that "pure" mathematics (as opposed to "applied" mathematics) continues to play a dominant role in mathematics classrooms; this may not be desirable, especially for those who perceive that school mathematics is not likely to be useful in their life, even for STEM professionals. For example, an associate professor at University of Washington observed that even engineers use "reasoning that look[s] very different from the activities of school math" (Stevens, 2007). Of course, this observation has to be carefully interpreted. Optimists might suggest that the engineers were invisibly influenced by their schooling in mathematics, even if they do not necessarily recognize it. Pessimists might insist that the most efficient education is to learn whatever directly precedes it in practice. Whether a student should learn a given mathematical topic in advance or on an *ad hoc* basis cannot be justified by reason alone, but needs to consider the results of rigorous experiments or carefully recorded experiences. The appropriate time for teaching mathematical content is at the heart of curriculum development in the inclusion or exclusion of any given topic. Certain content becomes debatable, when, for example, complex numbers are superficially introduced because engineers and mathematicians will need them for the future, whereas the average student likely will not. Delayed gratification is often a discouraging aspect of learning, and the hope that an average student could recognize its beauty is not likely the case, as Pollak (2007) pointed out.

Even so, it is still doubtful if such gratification is really delayed, or simply disappears. Freudenthal (1968) pointed out that learning whole-number arithmetic is not at all easy, but children usually grasp it quite well since they are able to *see* whole numbers in the world; in contrast, children rarely see fractions, and have difficulties mastering operations with them. Even physicists tend to stick with old approaches in the analysis of rigid bodies as if they have forgotten modern linear algebra, which is more convenient. Such discrepancies occur not because teaching pure mathematics is bad, but because it is not learned in the way that is connected to contexts.

In sum, the main reasons for teaching modeling are that every child can benefit from its power of application, and that mathematics can not only be learned in an isolated way but also be seen in the real-world.

Conventional Definition of Modeling and its Teaching

Before looking at how CCSS treats modeling, there is a concern with the vocabulary that is commonly used: *modeling* and *applications*. Many definitions (e.g., Pollak, 2003; Blum, 2005) describe modeling as a process depicted by one or more *modeling cycle(s)*, in which the following are performed: (1) understand and identify the issue in the real world, (2) formulate the structure of real-world situation, (3) translate to a mathematical model, (4) derive some mathematical facts from the model, (5) translate the resulting facts back to the real-world (this is called *interpretation*), and (6) validate the results (Figure 1). All but step (4) connect to the real-world.



(Adopted from Blum, 2005)

Figure 1

If the results of the modeling process are not satisfactory because of a discrepancy with what is observed, one can repeat the process by considering a simpler real-world situation, and/or modifying the model, and so on. Though every detail in this depiction of the modeling process might not be absolutely accurate in all situations, it is quite true that the essence of the modeling process "[goes] from the real to the conceptual or mathematical, and then back to the real" (Mario Juncosa, as quoted in Pollak, 2003). A person who is familiar with modeling, at least in a particular situation, should be able to move back and forth between the real-world and the mathematics in order to solve a problem. Another important feature of mathematical modeling is the fact that a mathematical model, as a product of the modeling process, is at least as valuable as the specific result: a model has the ability to regenerate solutions (hence it is *reproducible*), and it can also be reused for situations with similar structures, albeit in different contexts. For teachers, then, it is particularly useful to identify a few "classical" situations. In high school mathematics, for example, efficiency of airline routes, population growth trends, and the trajectory of a flying baseball are rich phenomena—as opposed to isolated applications—that can become motivation for learning many mathematical topics, such as Euler circuits, functions, vectors, probability and statistics. These situations, and the mathematics surrounding them, are even reusable in college mathematics courses, including differential equations and dynamical systems. Moreover, mathematical models developed from such classical situations are also likely adoptable to other contexts.

The framework for including modeling in the classroom can be classified into two contrasting types: *modeling as a vehicle* vs. *modeling as a goal* (Blum, 1993; Gershenfeld, 1999). The former relies on the conviction that mathematical modeling provides rich examples through which students can retain the mathematics that they have learned, and can extract important mathematical content. The latter treats the modeling process as a key part of mathematical content that needs to be taught and grasped, with recognition that the process is comprised of a different set of skills from what is needed for "pure" mathematics.

Place for Modeling in CCSS

The CCSS document for mathematics was organized into three parts: (1) a set of Standards for Mathematical Practice that contain general statements about attitudes and competencies that define proficiency in mathematics, regardless of content and grade level; (2) elementary-school standards, listed under each grade from K to 8; (3) high-school standards, listed under six Conceptual Categories (Number & Quantity, Algebra, Functions, Modeling, Geometry, and Probability & Statistics)

Modeling is an exception to this system of organization: it is both one of the Standards of Mathematical Practice and one of the Conceptual Categories. Also, the specific modeling standards are not listed under the same Category, but are integrated within all the other Categories. This indicates that modeling is perceived as needing unique consideration, but also may give rise to some confusion. Is modeling treated as a goal, since it is listed as a "standard"? Or is modeling a vehicle for learning other topics?

Given the importance and complexity of modeling, it is necessary to evaluate whether the CCSS offers a balanced stance for its use in classroom mathematics. Applying the two conditions stated in the introduction, the following two qualities in regard to modeling standards will be examined:

> *Definiteness*: Are the basic requirements of each modeling standard specific enough so that mastery can be operationalized? Also, are there coherent descriptions of what constitutes the expectations of an ideal student who is successful in modeling?
>
> *Relevance*: Are the ideal expectations of modeling reflected by the basic requirements? How do the expectations and requirements resemble a widely-accepted point of view?

All the Standards of Mathematical Practice, including modeling, apply to every grade. For grades K through 8, the whole modeling process is not required as its own standard, but many standards do reflect steps in the modeling cycle. Theoretically, these standards can appear in *every* domain. For each domain, various standards from grade 6 incorporate some of the modeling steps (Table 1).

Definiteness

The ubiquitous nature of mathematical modeling sets it as important enough to be included as a standard for each grade and each content domain, but not clarifying its particular role has similar drawbacks to those regarding NCTM's 'Connection Strand' of Standards. Fortunately, the CCSS do not repeat an identical description of modeling standards throughout the grades and domains; it attempts to describe what modeling is really about at each place. A

Table 1

Content Domain	Standard	An Excerpt of the Standard	Modeling Step(s) Involved (reference Figure 1)
Ratios and Proportional Relationships	6.RP.1	...use ratio language to describe ratio relationship between two quantities....	(2)(3)
The Number System	6.NS.7c	Write, interpret, and explain statements ... -3 ºC > -7ºC to express the fact that -3 ºC is warmer than -7 ºC	(3)(5)
Expressions and Equations	6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another...	(1)(2)(3)
Geometry	6.G.1	Find the area of ... polygons by composing into rectangles .... apply these techniques in the context of solving real-world and mathematical problems.	(1)(2)(3) (4)(5)
Statistics and Probability	6.SP.5d	...relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered	(5)

good example of definiteness is the Statistics and Probability section of grade 8. The four standards [8.SP.1 – 8.SP.4] provide a good idea of what eighth graders need to learn, such as how to interpret the slope and intercept of a linear model in an experiment. Another example is from the Kindergarten standards:

> K.CC.5 Count to answer 'how many?' questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 – 20, count out that many objects.
>
> K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)

These specific statements strongly relate to the modeling process, in which *counting* constitutes a representation of data, and *comparing* is useful for analyzing data and for interpreting and validating the results. Such definiteness and clarity is not surprising because the mathematical outcomes of grade K are generally more well-defined. It is clear, after all, that if a child cannot count to 10 when entering first grade, then s/he has some mathematical deficiencies. In contrast, consider the following standard:

> 8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables.

Certainly a variety of word problems exist for eighth graders to solve, but what are some restrictions? Do they *have to* or at least *know how to* use a system of equations to solve the problem, even if a one-variable approach or other informal approaches are possible? What context and level of depth is necessary for students of this age? Will 3/4 or $\sqrt{11}$ be included as a solution? It is still not a common practice[1] to determine specific details of mathematics content requirements as the grade levels increase. Even less determined are the standards of competencies for mathematical modeling, which have become a focus of recent research (e.g., Lesh et al., 2007). The vagueness pointed out above calls for a more thorough scrutiny of the specifications of standards. Higher grade levels are reasonably more demanding.

As for high school modeling standards, the degree of definiteness varies greatly between different standards. Some standards are written precisely, such as "Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments." (A-SSE.4). Others can be more obscure: "Use inverse functions to solve trigonometric equations that arise in modeling contexts [...] and interpret them in terms of the context." The standard lacks description of what contexts are preferred (artificial context? idealized situations? real industrial problems?), and how deep an interpretation is required.

Relevance

Throughout the CCSS document, *model* has two apparently different usages depending on the context: (1) in phrases such as *concrete models*, *to model* some mathematical concept by objects, by symbols, or by drawings, etc., the meaning of *model* (in its verb form) partially overlaps with that of *represent*, or *describe*; (2) *model* as in *mathematical modeling,* where the CCSS has defined mathematical modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." This description indicates that modeling is to be treated as a goal. The modeling cycle is defined in a way that is not very different from the conventional one; however, in contrast to the conventional definition, modeling in the CCSS does not necessarily include the whole process of solving a real-world problem. For example, *descriptive modeling* involves only the first three steps without deriving new facts from the model.

From grade 1 through grade 6, "models" are usually physical objects or graphical representations, the role of which are to visualize certain mathematical concepts, such as whole numbers, the four arithmetic operations, fractions, and decimals. In grade 7, however, the newly introduced *probability model* is itself a mathematical model. In grade 8, students are asked to "construct a function to model a linear relationship between quantities" [8.F.4]. These grade level standards imply not only that a model can be concrete or graphic and *represent* some mathematical ideas, but that it can also be abstract, and *be represented by* mathematical ideas and symbols. As a consequence, a more rigorous and explicit description of mathematical modeling is to be introduced at the high school level. Students who are able to grasp the evolution of modeling in CCSS, as the meaning moves from concrete manipulatives to abstract mathematical models, should benefit more from the mathematics they learn in the classroom and its application beyond.

As mentioned, high school modeling standards are embedded in other Conceptual Categories. This feature is reasonable because modeling emphasizes the connection between mathematical content and the applications, but it also raises the following problems about relevance:

(1) When a modeling standard is listed in another Conceptual Category, say, Geometry, should modeling be treated as a vehicle to teach geometrical concepts, or

[1] Here is an exception: automatically generated tests need very specified "test templates" in order to produce similar test items (Stocks and Carrington, 1993). Computer generated tests might not serve to test students' ability of modeling, but the way of specification might inform the writing of standards.

should a teacher treat modeling as a goal, seeing the geometrical concept as a product offered by the modeling process? The two different purposes are more or less competing unless a unified approach is developed. As one of the Conceptual Categories, modeling is to be treated as a goal. This is also evident in that many CCSS standards mention that the content should be understood in contexts; however, the contexts are not specified, making the goal obscure. After all, traditional word-problems could be thought of as mathematics in contexts, but when they are used as mere exercises of routines that students have learned, they are not adequate for the teaching of modeling (Pollak, 1969). Indicating some classical situations and models might help to define better what constitutes goal of modeling.

(2) When a standard is not indicated as a modeling standard, is it not required to be understood in context? This can be misleading. For example, the complex number system (N-CN) is not listed as a modeling standard, but it can model periodic signals[2] and also can model planar transformations very conveniently. Recognizing vectors as having both magnitude and direction (N-VM.1) is not a modeling standard, but there is an authentic question of determining whether a physical quantity is a vector or a list of scalars. For example, electric current apparently has direction and magnitude but is not a vector.

(3) There is potential misuse of the label "modeling" surrounding the standards about technology (e.g., A-REI.11, F-IF.7). Although technology is a key tool in teaching and learning mathematics, what relates it to mathematical modeling is the fact that computational tools aid in solving more sophisticated models, and that graphical tools help students to see mathematics in a more vivid way. Technology can help, but does not necessarily imply mathematical modeling.

(4) All of the standards within the Conceptual Category of *Statistics and Probability* are labeled as modeling standards. This is reasonable in theory, since statistics models *data* and their analysis, and probability models *chance* and *random behavior*. Actual practice, however, tends to treat statistics as a set of numerical tricks and using probability to model randomness in context is rarely addressed adequately in school mathematics (Moore, 1990). Even topics closely related to and aligned with mathematical models do not necessarily imply the actual use of modeling in the classroom!

In essence, mathematics that can be applied may not be learned in a way that it can be applied, and mathematics that is related to real applications[3] may not be taught as it relates to applications.

Final Remarks

Implementing, on a large scale, excellent mathematical teaching in the area of modeling and application is particularly challenging. Burkhardt (2006) provided Henry Pollak an overview of the development of modeling curricula to inform future directions (cf. Pollak, 2003, for a more detailed, historical review of modeling curricula in the United States; de Lange, 1996,for an international sketch). Establishments, including the production of materials, development of courses, and some reforms at the institutional level, were restricted to a small scale; most parts of the mathematics curricula continue to have only a weak connection to the real world. Burkhardt identified four barriers to curriculum change: systemic inertia; the real world (which often causes unwelcome complications in actual computations); limited professional development; and research of the development itself. These notions will continue to shape further developments. As Burkhardt mentioned, "key levers for tackling resistance to change are *curriculum descriptions*, supported by *well-engineered materials* to support *assessment*, *teaching*, *professional development* and *public relations* (in the literal sense) that are well-aligned with each other" (p. 190; italics and parentheses in original). A later summary of the Netherlands' "realistic mathematics education" by Vos (2010) also claimed that the Dutch have solved the first three of these barriers, though a lack of research exists. Indeed, what Vos described was that their success was based on a well-engineered practice rather than strong research. This is likely the case, since research can retain, replicate, summarize, or generalize successful practice at a higher probability, but it cannot replace the creativity and careful execution of a successful curriculum.

Logically, the "next step" following the CCSS is to bring modeling into classrooms. The "next step", indeed, has been started for decades. Many textbook series, such as *Connected Mathematics* (Lappan, et. al, 2003) and *Core-Plus Project*, (Schoen & Hirsch, 2003) were aligned with NCTM Standards and supported an interest of application. The Consortium for Mathematics and Its Applications (COMAP) has been producing materials in different formats for the purpose of modeling and applications since 1980. COMAP also developed a series, *Mathematics: Modeling Our World*, (Garfunkel, Godbold, & Pollak, 2000), which is of particular importance because it is both standard-based and focused entirely on mathematical modeling. COMAP is also supporting a current project that will result in *A Handbook of Mathematical Modeling* for the CCSS, which is based on the modeling standards and contains lessons designed to engage both teachers and students in modeling activities. As implied by Burkhardt's view, these efforts have not yet attained enough prevalence because the mere interaction between standards and textbooks might not be adequate. Systemic inertia has to be moved by a joint effort of curriculum developers,

---

[2] sine and cosine also work, yet they do not have mathematical properties as efficient as complex numbers.

[3] One belief is that all mathematics are more or less related to applications, especially when pure number theory has been applied to cryptography (Personal communication with Henry Pollak).

textbook writers, teachers and students. Although the CCSS modeling standards are not specific enough, the emphasis on modeling affords the opportunity for educators to think about how modeling and applications *should* look like in classrooms and, eventually, to reshape school mathematics to be more valuable for all students.

References

Blum, W. (1993). Mathematical modeling in mathematics education and instruction. *Teaching and learning mathematics in context*, 3–14.

Blum, W., & Leiss, D. (2005). “Filling up”—The problem of independence-preserving teacher interventions in lessons with demanding modeling tasks. In *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1623-1633). Presented at the European Research in Mathematics Education IV, Sant Feliu de Guíxols, Spain.

Burkhardt, H. (2006). Modeling in mathematics classrooms: reflections on past developments and the future. *ZDM*, *38*(2), 178–195.

Council of Chief State School Officers [CCSSO], & National Governors Association [NGA]. (2010). Common Core State Standards for Mathematics. Retrieved September 28, 2010, from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics*, *1*(1-2), 3–8. doi:10.1007/BF00426224

Garfunkel, S. A., Godbold, L., & Pollak, H. (2000). *Mathematics: Modeling our world (COMAP)*. WH Freeman and Co.

Gershenfeld, N. A. (1999). *The nature of mathematical modeling*. Cambridge Univ Pr.

Kliebard, H. M., & Franklin, B. M. (2003). The ascendance of practical and vocational mathematics, 1893-1945: Academic mathematics under siege. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 399–440). National Council of Teachers of Mathematics.

de Lange, J. (1996). Using and applying mathematics in education. In *International handbook of mathematics education* (pp. 49–97). Springer.

Lesh, R., Galbraith, P. L., Haines, C. R., & Hurford, A. (2009). *Modeling Students’ Mathematical Modeling Competencies*. Springer Verlag.

Lappan, G., Fey, J., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2003). *Connected mathematics*. Pearson Prentice Hall.

Moore, D. S. (1990). Uncertainty. In *On the Shoulders of Giants: New Approaches to Numeracy* (pp. 95–137). Washington: National Academy Press.

National Assessment Governing Board, U.S. Department of Education [NAGB]. (2008). Mathematics framework for the 2009 National Assessment of Educational Progress. Retrieved December 27, 2010, from http://www.nagb.org/publications/frameworks/math-framework09.pdf

National Council of Teachers of Mathematics [NCTM]. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Governors Association [NGA]. (2009, September 1). Fifty-one states and territories join Common Core State Standards Initiative, *News Release*, Retrieved March 15, 2011, from http://www.corestandards.org/news

New York State Education Department [NYSED]. (2005). Mathematics Core Curriculum, MST Standard 3, MST Standard 3. Retrieved December 25, 2010, from http://www.p12.nysed.gov/ciai/mst/math/documents/mathcore.pdf

Pollak, H. O. (1969). How Can We Teach Applications of Mathematics? *Educational Studies in Mathematics*, *2*(2/3), 393–404.

Pollak, H. O. (2003). A history of the teaching of modeling. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 647–672). National Council of Teachers of Mathematics.

Pollak, H. O. (2007). Mathematical Modeling—a Conversation with Henry Pollak. In *Modelling and Applications in Mathematics Education*, New ICMI Study Series (Vol. 10, pp. 109–120). Springer US.

Schoen, H. L., & Hirsch, C. R. (2003). The core-plus mathematics project: Perspectives and student achievement. *Standards-based school mathematics curricula: What are they*, 311–343.

Stevens, R. (2007). Where’s the math. Research that Matters, University of Winsconsin College of Education, (7), 14.

Stocks, P. A., & Carrington, D. A. (1993). Test templates: a specification-based testing framework. In *Proceedings of the 15th international conference on Software Engineering*, ICSE ‘93 (pp. 405–414). Los Alamitos, CA, USA: IEEE Computer Society Press.

Vos, P. (2010). The Dutch maths curriculum: 25 years of modelling. In *Modeling Students’ Mathematical Modeling Competencies* (pp. 611–620). Springer US.

SEARCH EXTENDED

Teachers College Columbia University
Department of Mathematics, Science, and Technology

MATHEMATICS EDUCATION FACULTY VACANCY

Teachers College invites applicants for a faculty position in mathematics education. A successful candidate will have an earned doctorate in mathematics or mathematics education at the time of appointment and a demonstrated ability to pursue an active research agenda in a significant area of mathematics education. Applicants should be qualified to teach graduate-level courses in at least two areas of mathematics and in two or more areas of mathematics education. Preference will be given to candidates with prior experience teaching school mathematics. All candidates are expected to demonstrate an ability to establish a research program and a potential to obtain external funding. Candidates also are expected to establish a record of continuous, scholarly productivity and a record of leadership. Minority candidates are strongly encouraged to apply.

Rank: Open Rank, Tenure Track

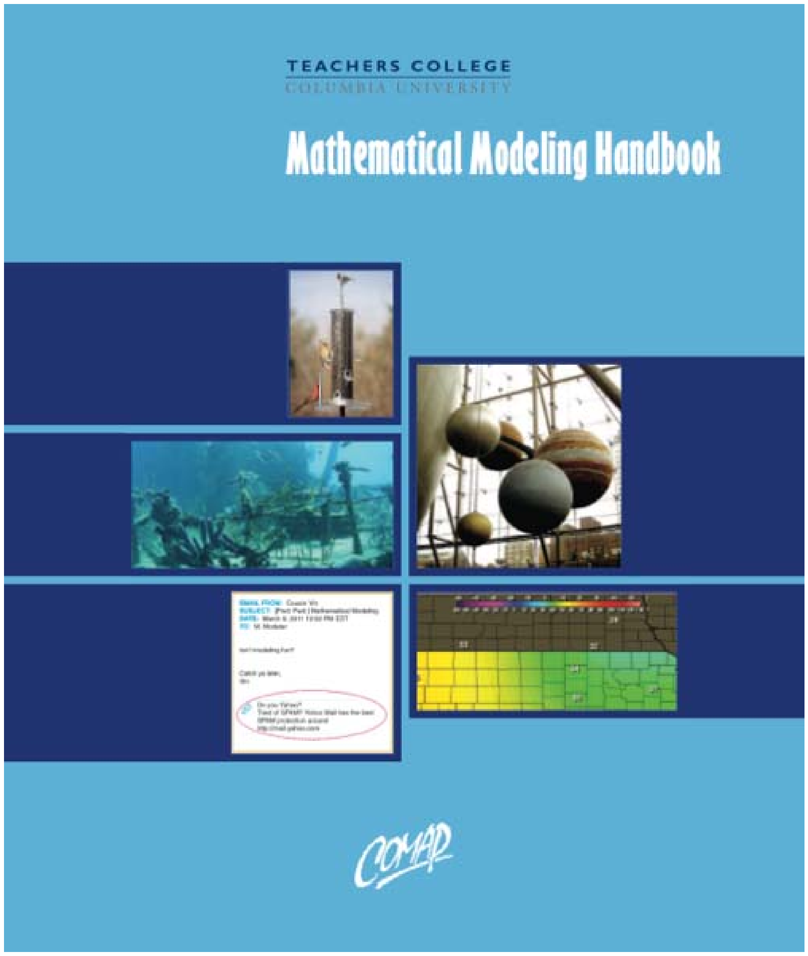
Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

Teachers College as an institution is committed to a policy of equal opportunity in employment. In offering education, psychology, and health studies, the College is committed to providing expanding employment opportunities to persons of color, women, and persons with disabilities in its own activities and in society.

Teachers College Columbia University
525 West 120th Street, New York, NY 10027
http://www.tc.columbia.edu

COMING - SUMMER 2011



Order from:

The Consortium on Mathematics and Its Application

BY MAIL:

COMAP, Inc.
175 Middlesex Turnpike, Suite 3B
Bedford, MA 01730
USA

BY EMAIL:

info@comap.com

Download a free Handbook Sampler of five lessons
at **www.comap.com/NCTM.html**