# Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

# TABLE OF CONTENTS

## Foreword



## Preface



## Editorial Point-Counterpoint



## Articles

# TABLE OF CONTENTS

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.



This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

## Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology.*

*JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

## Manuscript Submission

*JMETC* seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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### Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

### Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### Looking Ahead

Anticipated themes for future issues are:

| | |
|---|---|
| Fall 2011 | Technology |
| Spring 2012 | Evaluation |
| Fall 2012 | Equity |
| Spring 2013 | Leadership |
| Fall 2013 | Modeling |
| Spring 2014 | Teaching Aids |

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# NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the C*onsortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* “cognitive category” of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a “disposition to mathematize,” that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory’s Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher’s notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a “disposition” toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

*Notes from the Curriculum Laboratory* begins with a brief view of the Laboratory’s interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory’s modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

to its gauge than any other gauge. (Having students "construct" this definition of region by calling on relevant geometry is a key feature of the lesson.) We weight the rainfall measured at each region's gauge according to the portion of the territory's area in that region. The estimate of rainfall for the territory is the weighted average of the rainfall readings.

In finding the rainfall estimate, we have created a Voronoi diagram in which the boundaries of regions are determined from their centers of influence; in this problem, centers are the rainfall gauges. Every Voronoi boundary is a portion of the perpendicular bisector of a segment joining two centers.

This lesson can lead to a consideration of various methods for computing area of polygonal and other figures. (One possible method is to superimpose on the "map" of the territory a transparency with a coordinate grid.)

Question to pose to the class after finding the weighted average: How does the Voronoi diagram change if we add a fourth gauge D? Let us examine a couple of different locations, including 1) D in the interior of triangle ABC and 2) D in the exterior of triangle ABC.

Another kind of problem arises when we are given a Voronoi diagram and wish to determine the centers of the Voronoi regions, something to be considered in subsequent lessons.

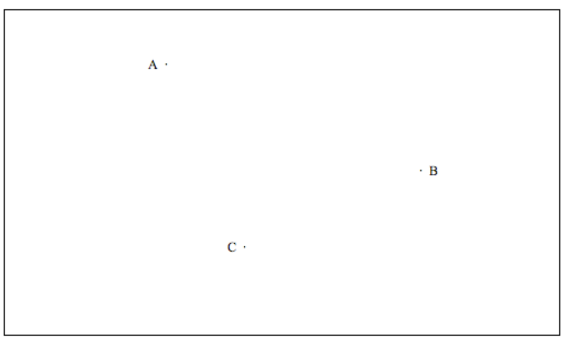


Figure 1. Territory map with rain gauges

*References*

The Consortium for Mathematics and Its Application (COMAP) (1998). *Mathematics: Modeling Our World*. Cincinnati, OH: South-Western Educational Publishing.

# The Buckyball Has Relatives: A Classroom Approach to Polyhedra

Anahu Guzman
LIM College

The construction of fullerene models using the following group activity is an opportunity to foster creativity and collaboration and to introduce some basic graph theory definitions in an enjoyable way. Fullerenes are a family of carbon-cage molecules in the form of a hollow sphere, ellipsoid, or tube. Spherical fullerenes are also called buckyballs. From a mathematical point of view, a fullerene can be thought of as a convex 3-dimensional polyhedron with 3 edges at each vertex, and faces consisting of 12 pentagons and k hexagons ($k \neq 1$).

<u>Number of Group Members</u>: 3

<u>Required materials</u>:
1 pack of 2.5 inch assorted color rubber bands
1 12 inch or larger balloon
1 small pack of dental floss

Begin the discussion by examining a soccer ball; notice the pentagonal and hexagonal faces. Two faces share each edge and every 3 faces share a common point (vertex). A soccer ball resembles the "Buckyball," one of the most famous fullerenes (C60). Since fullerene graphs are 3-valent (3 edges at every vertex), any circuit (begins and ends at the same vertex, alternating between different vertices and edges) will use two of the edges at every vertex of the circuit, which allows use of the missing edge to code the circuit using binary numbers. Code circuits using 0 for edges that point in and 1 for edges that point out of the circuit. A shell diagram associated with the code $(0011)^3(01) = 00110011001101$ is shown in Figures 1 and 2. The outer part of the circuit can be thought of as the "icing" with faces and the inner as the "filling" with faces (Figure 3).

***Step 1.*** *Laying out the (0011)³(01) circuit (Figure 1)*

Select 3 different rubber band colors to represent 0's, 1's, and the path edges. Each rubber band represents one edge. Place 14 rubber bands on a table in a circular shape, these are the path edges. Place the first "0 rubber band" in the inner part of the "circle" next to an intersection of two path edge rubber bands; this is the starting point rubber band. Place another "0 rubber band" in the next intersection, followed by a "1 rubber band" outside the circular structure on the next two intersections. Continue the rest of the process clockwise following the (0011)³(01) circuit. Take time to reinforce the idea of binary coding.

***Step 2.*** *Connecting the (0011)³(01) circuit*

Connect the adjacent path edge rubber bands with a piece of dental floss; be sure to tie three knots to secure the structure. Use floss to connect to the circuit of path edges all of the 0 rubber bands on the inside and all the 1's on the outside, keeping the (0011)³(01) circuit. Stress to students the fact that there are three rubber bands connected at each knot (3-valent).

***Step 3.*** *Filling (Figure 2) and Icing (Figure 3)*

Identify the starting point rubber band; place a piece of tape on it. Tie the next two inner rubber bands together, clockwise from the starting point. Connect the next two pairs of inner rubber bands. Notice that the starting point rubber band is not connected. You need four additional rubber bands of a different color to complete the filling. Take three of them and tie them together at a single point. Open the structure to form a "y" shape. Connect the starting point rubber band with the left part of the "y" structure. Clockwise from the starting point, take the first connected pair of rubber bands and tie them together with the right part of the "y" structure. Take the next pair and tie them together with the bottom part of the "y" structure. Take the remaining rubber band and connect it to the starting point and the last pair of inner rubber bands, see Figure 2. Notice that we end up with one hexagonal face and six pentagonal faces and all inner knots are connected to 3 rubber bands (3-valent). Use the same process to close the fullerene, see Figure 3, and inflate the balloon within the structure.

*Possible questions:*

- How many hexagonal faces and pentagonal faces are there?
- How many faces, edges, and vertices are there?
- Can you find other circuits in your construction and write down their "codes?"

For higher grades, introducing Euler's Polyhedron formula (V-E+F=2) shows students a fundamental result in non-metrical geometry. The construction of fullerenes using the proposed process is an opportunity to show students important geometry that goes beyond issues of "distance" and angles in a way that encourages collaboration and connection to other topics (e.g. polygons and polyhedra).

*References*

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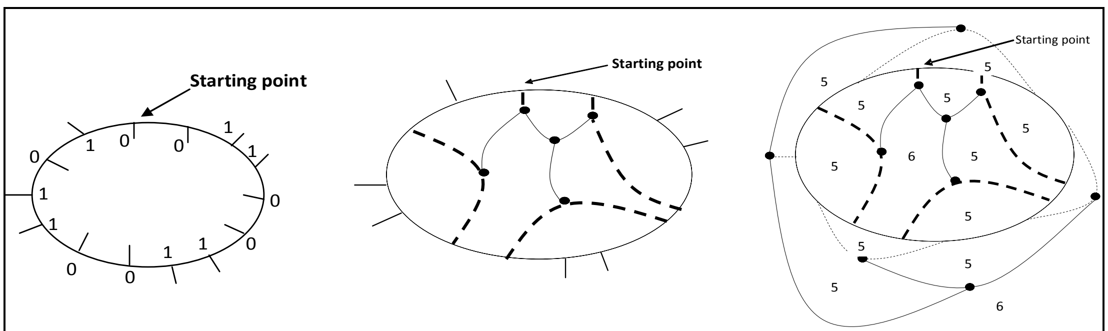


Figure 1 Figure 2 Figure 3

SEARCH EXTENDED

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## Department of Mathematics, Science, and Technology

# MATHEMATICS EDUCATION
# FACULTY VACANCY

Teachers College invites applicants for a faculty position in mathematics education. A successful candidate will have an earned doctorate in mathematics or mathematics education at the time of appointment and a demonstrated ability to pursue an active research agenda in a significant area of mathematics education. Applicants should be qualified to teach graduate-level courses in at least two areas of mathematics and in two or more areas of mathematics education. Preference will be given to candidates with prior experience teaching school mathematics. All candidates are expected to demonstrate an ability to establish a research program and a potential to obtain external funding. Candidates also are expected to establish a record of continuous, scholarly productivity and a record of leadership. Minority candidates are strongly encouraged to apply.

**Rank:** Open Rank, Tenure Track

**Send** CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

*Teachers College as an institution is committed to a policy of equal opportunity in employment. In offering education, psychology, and health studies, the College is committed to providing expanding employment opportunities to persons of color, women, and persons with disabilities in its own activities and in society.*

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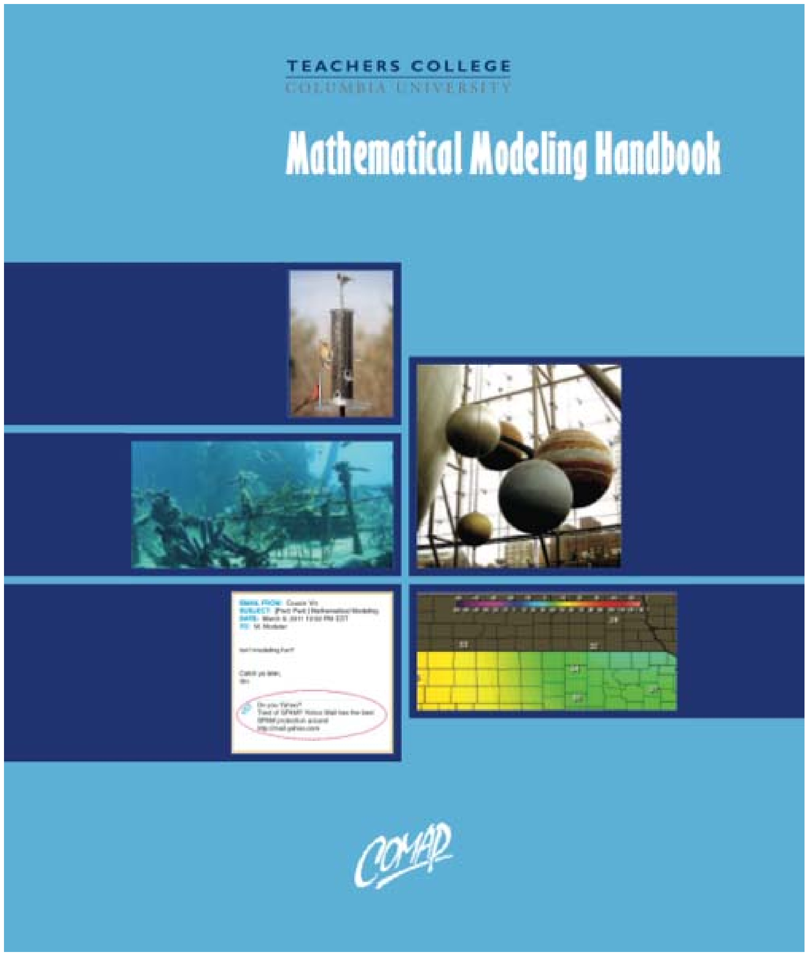


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