Journal of Mathematics Education at Teachers College

Fall – Winter 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

Dr. Robert Taylor was selected by the Teachers College sponsored Teachers for East Africa program to teach mathematics of Uganda's Makerere University. He returned to TC as an instructor in the Department of Mathematics, Statistics, and Computing in Education where he developed an innovative programming language (FPL) intended to introduce educators to the then - new field of computer programming. His seminal work entitled *Computers: Tutor, Tool, Tutee* led to leadership in the new field of computers in education. Dr. Taylor completed 33 years as a member of the Teachers College faculty in 2009.

Dr. Carl N. Shuster completed the doctorate at Teachers College in 1940 under the guidance of William David Reeve. Shuster joined the TC faculty at Reeve's invitation and soon was recognized as the nation's leading advocate of the use of traditional technology, especially measurement technology, in the mathematics classroom. Dr. Shuster served as President of the National Council of Mathematics from 1946 to 1948 and concluded his career as Distinguished Professor of Mathematics at Trenton State University.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select key words from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

Library of Congress Cataloging-in-Publication Data

Journal of mathematics education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and teaching—United States—Periodicals
QA11.A1 J963

More Information is available online: www.tc.edu/jmetc

TABLE OF CONTENTS

TABLE OF CONTENTS, continued

Journal of Mathematics Education at Teachers College
Fall–Winter 2011, Volume 2

Mathematical Visualization

Jonathan Rogness
University of Minnesota

Advances in computer graphics have provided mathematicians with the ability to create stunning visualizations, both to gain insight and to help demonstrate the beauty of mathematics to others. As educators these tools can be particularly important as we search for ways to work with students raised with constant visual stimulation, from video games to MTV. Computer generated images, animations and interactive demonstrations permeate all areas of mathematics education, making it important for educators and researchers to determine how best to harness these tools to increase student learning. This survey article discusses what we know, and do not know, about creating effective visualizations and describes possible avenues for future work in the area.

Note: Based on a colloquium presentation at Teachers College in April 2011.

Keywords: Visualization, Möbius Transformations, Animations, YouTube.

Introduction

Although well-drawn diagrams have long been a part of mathematical education, the advent of computer graphics in the last thirty years has enabled an explosion of mathematical visualizations for use in research and teaching. Computer based images and interactive visualizations are now widely used in both K-12 and postsecondary courses; an online search for the phrase "interactive calculus applet" currently returns nearly nine million web pages. This trend seems likely to continue as modern systems make it even easier for educators to create compelling visualizations for students.

However, quantity does not ensure quality, and there are many open questions about which visualizations are most effective in increasing student learning, and about how they should be used in the classroom. This article serves as a short survey of these issues and as a primer for those who wish to use or even create visualizations. The first section describes three particularly effective visualizations to provide context for the rest of the article. The second section discusses what makes for an effective visualization. The third section describes why visualization is important for mathematics education, followed by a fourth section discussing open questions and possible research in the field. Finally, a short appendix lists some of the current tools available to create your own visualizations.

Examples

There are far more students and instructors in calculus courses (and below) than in junior- or senior-level undergraduate mathematics courses, so it would stand to reason that the vast majority of visualizations are created for those lower level courses. The examples in this section are somewhat unusual in that they are based on higher level material. This choice was made for two reasons: (a) to remind readers that even advanced "theorem and proof" based mathematics courses can benefit from visualizations, and (b) to point out that a truly exceptional visualization can take a high level concept and make it understandable to a less advanced student, or even the general public.

The examples in this section are freely available online, and readers are highly encouraged to download them. The static images in the figures below do not do them justice.

Example 1: *Geodesics*. A *geodesic* on a surface is a generalization of a straight line on a plane. On a small enough scale, a geodesic will always indicated the shortest path between two points on the surface. A careful mathematical definition of *geodesic* involves fairly sophisticated notions from metric space theory or multivariable calculus, but informally one can think of a geodesic as the path a small ant would trace out on the surface while always crawling straight ahead, never turning left or right.

This characterization of geodesic paths is illustrated wonderfully in the opening scene of (Polthier, Schmies, Steffens & Teitzel, 1997), which may be freely downloaded at http://page.mi.fu-berlin.de/polthier/video/Geodesics/Scenes.html. The one-minute video shows a motorcycle following a wild path around a three-dimensional figure-eight surface, looping around the surface and through its holes. A few well-chosen camera angles make it clear that the handlebars on the motorcycle never turn to the left or right. See Figure 1. In other words, the motorcycle is driving straight ahead, and the seemingly random path on the surface is in fact a geodesic. This powerful visualization makes a complicated idea from differential geometry accessible to students in middle or even elementary schools. They may not be able to write

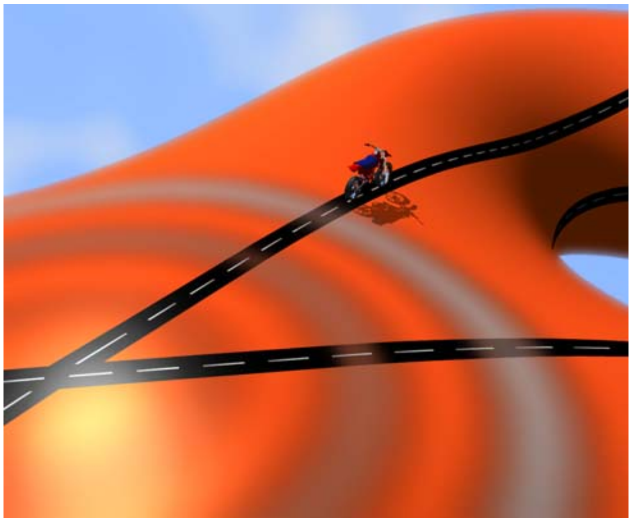

Figure 1.

down a rigorous definition involving derivatives of parametric paths in space, but students who watch the video can gain an accurate understanding of what a geodesic is.

Example 2: Curved Spaces. We live in a three-dimensional universe—or, at least, are limited by human vision to seeing a three-dimensional universe. Many people assume that the universe is shaped like $\mathbb{R}^3$, i.e. a three-dimensional space which stretches forever in every direction. Surprisingly, this might not be the case. To understand why, it can help to drop down a dimension and think about video games.

The two-dimensional space $\mathbb{R}^2$ is a flat plane which stretches out forever in every direction. There are other spaces which have a finite area and yet allow a two-dimensional object to move forever in any given direction. For example, in the video game *Asteroids*, if the player's spaceship reaches the top of the screen it reappears on the bottom; similarly, if it flies off the side of the screen, it reappears on the opposite side. The ship can fly forever without ever changing direction, and yet its "universe" is limited to the area of the computer screen.

Mathematically we say the opposite edges of the screen are *glued* to each other. If the screen were made of a bendable material, we could physically glue these edges together, revealing that *Asteroids* is in fact played on a torus which has been cut open and laid out flat so that we can display it on a computer screen. See Figure 2, which shows a series of frames from a video available at http://www.math.umn.edu/~rogness/visualization.shtml. After the torus is cut open and laid flat, portions of the spaceship appear in all four corners of the resulting rectangle.

Moving back up a dimension, the possible shapes of our universe are mathematical objects known as *3-manifolds*, which are typically not defined in the American curriculum until advanced undergraduate or beginning graduate level courses in topology. The Euclidean space $\mathbb{R}^3$ is just one of the possibilities; in others, it could be possible to fly in a spaceship and, as in *Asteroids*, eventually return to one's starting point without having changed direction.

We can construct a space by generalizing the two-dimensional example above. Instead of a rectangular computer screen, imagine a cubical room. The room has

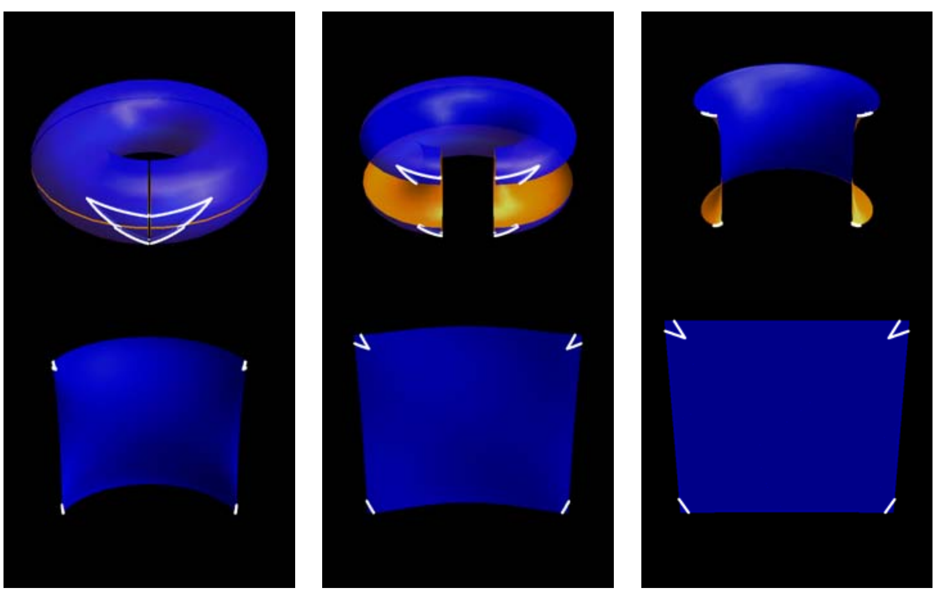

Figure 2.

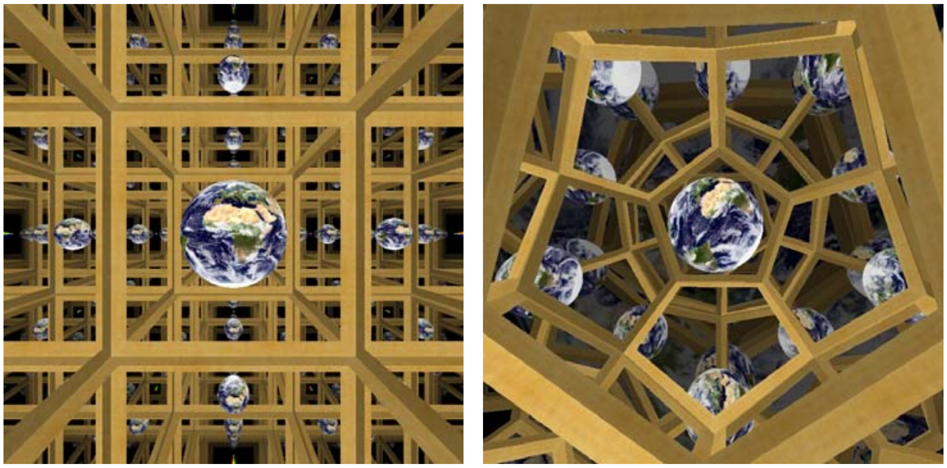

Figure 3(a) and 3(b).

the peculiar property that, if you walk through any wall, you reappear on the opposite side of the room. Similarly, if you move through the floor, you find yourself coming through the ceiling at the top of the room. This special room is known as the *3-torus*. Mathematically we again say the opposite sides of the 3-torus are glued to each other, but the limitations of our human senses make it impossible to create an illustration analogous to Figure 2. One cannot draw an accurate three-dimensional picture in which all three pairs of opposite sides of the cube are stretched around and joined together. Hence to visualize the space, we are limited to imagining what it would look like from the inside.

If you stand in the middle of the 3-torus and look straight ahead, your line of sight will pass through the front wall and come out of the back. The result is that you will see yourself from behind, in what appears to be a second copy of the room. (However, it is in fact the same room!) Similarly, if you look to the right, left, down, up, or at any other angle, you will see yourself in the room, but from different angles. It will appear as though you are surrounded by cubes stretching off forever in every direction, despite the fact that your entire "universe" consists of the one cubical room with finite volume.

Understanding what the inside of a 3-manifold looks like is a difficult proposition indeed. Fortunately a mathematician named Jeffrey Weeks has created a computer program called *Curved Spaces* which allows users to fly through 3-manifolds to analyze their shape; the software is freely available to download for multiple platforms at http://www.geometrygames.org. (The website also has programs to help visualize the two-dimensional example described above.) Figure 3(a) shows what it would look like to stand inside a 3-torus which contained a large Earth in the middle. Notice how the multiple Earths seem to appear in every direction; the 3-torus appears to be infinitely large, but it is in fact finite—the size of one cube, whose opposite sides are glued together in a special way. *Curved Spaces* includes a variety of 3-manifolds for users to explore interactively, many of which are created with non-cubical rooms. Figure 3(b), for example, shows *Poincaré Dodecahedral Space*, in which a spaceship flying through a pentagonal face of a dodecahedron reappears on the opposite side of the polyhedron.

Curved Spaces is a fantastic piece of software, and perhaps the single most effective mathematical visualization tool this author has encountered. It makes complex, graduate-level mathematics accessible to school-age children; at the University of Minnesota it is regularly used in enrichment programs for students as young as fourth grade.

Readers who are interested in further exploration of 3-manifolds are highly encouraged to read (Weeks, 2001). Weeks is a masterful writer and received a MacArthur Fellowship in recognition of his work helping students understand and visualize manifolds.

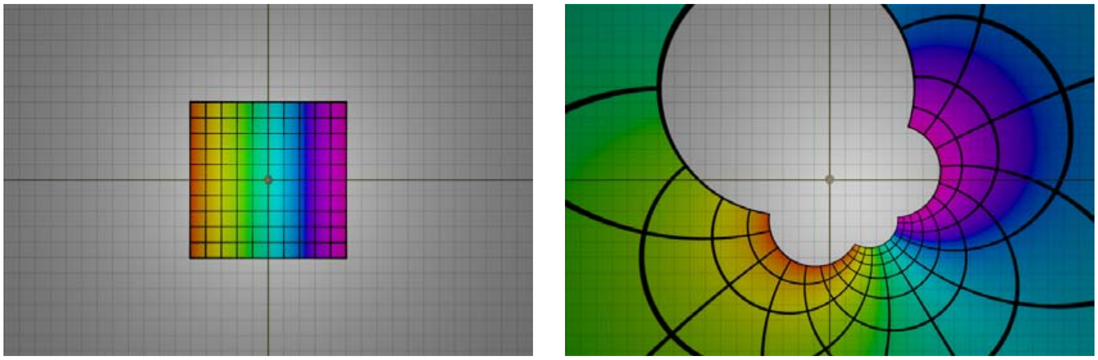

Figure 4(a) and 4(b).

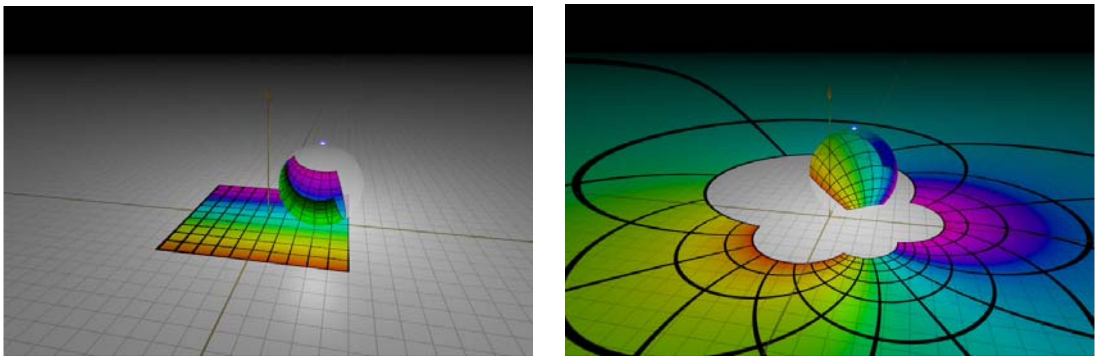

Figure 5(a) and 5(b).

Example 3: Möbius Transformations Revealed. A *Möbius Transformation* is a function of the form $f(z)=(az+b)/(cz+d)$ where *a*, *b*, *c*, *d*, and *z* are all complex numbers, and $ad-bc\neq 0$. They are well known in complex analysis as functions which are conformal and which send circles and lines on the complex plane to circles and lines. See Figures 4(a) and 4(b) for the effect on one particular Möbius Transformation; the points in the colored grid in Figure 4(a) are sent to the corresponding points in Figure 4(b). Notice that the straight line segments were sent to arcs (i.e. segments) of circles, and the arcs still meet at right angles. Surprisingly, the colored grid has been turned inside out, so that the gray region of the plane is now bounded, and the colored areas stretch out to infinity.

Möbius Transformations Revealed is a short film created at the University of Minnesota by Douglas Arnold and this author for the 2007 International Science and Engineering Visualization Challenge, in which it won Honorable Mention. A primary goal of the film was to show non-mathematicians how beautiful mathematics can be, but the video is more than random pretty pictures. It illustrates a theorem which states Möbius Transformations can be constructed by copying the original points onto a sphere via inverse stereographic projection, moving and rotating the sphere as necessary, and then projecting the points on the sphere back to the plane. Figures 5(a) and 5(b), for example, show how to represent the Möbius Transformation used in Figure 4 using a sphere.

Anecdotal reports from students and instructors around the country suggest that, for large numbers of students, this representation of Möbius Transformations has proven easier to understand than the algebraic descriptions contained in typical complex analysis books. This alone would make the video a highly successful visualization. However, *Möbius Transformations Revealed* has an interesting follow-up story. A low resolution version of the was uploaded to YouTube.com so that the creators could easily share it with others. Much to their surprise, it appeared on the YouTube home page as a featured video, eventually garnering nearly two millions views.

With so much content online, it is rare for any video to go viral, let alone a video illustrating high level mathematics. News outlets such as Minnesota Public Radio and the Associated Press took notice and further publicized the film. While the general public may not have known—or cared—about the theorem illustrated in the video, they could enjoy the mesmerizing images and colors in the movie and get a glimpse of the world of mathematics beyond simple arithmetic and algebra. Hence *Möbius Transformations Revealed* became an ambassador of sorts for mathematics as a whole.

For more information about the mathematics behind *Möbius Transformations Revealed*, see (Arnold & Rogness, 2008). The film is available for download at http://www.ima.umn.edu/~arnold/moebius/.

What Makes a Visualization Good?

In an ideal world this article would list twelve steps that every reader could follow to decide if a certain visualization would be effective in the classroom. Unfortunately the situation is not so clear. Indeed, it is not certain what makes a visualization good or bad, or even what the definition of *visualization* should be. After surveying 247 articles related to visualization in science and mathematics education, (Phillips, Norris & Macnab, 2010) states: “Perhaps the most defining feature of the current state of empirical research on visualization is the lack of consensus about the most elemental issues that surround it, including settling on a definition for visualization… and deciding how to document both short-term and long-term effectiveness.”

After evaluating the literature, (Phillips, et al., 2010) do identify five characteristics of effective visualizations. Their descriptions are summarized here; for full references for their conclusions, see the original source.

1. *Color*. There is some evidence to suggest colorful images may be more effective in triggering student learning than simple black and white images.
2. *Realism- or lack thereof*. Abstracts line diagrams that focus on the essential details of a concept may be more effective than overly complicated images which include unnecessary detail.
3. *Relevance*. This can refer to cultural relevance, for example of geometric designs in art, or in the relevance of the images to the problem at hand—is the visualization really necessary to help solve the problem, or does it serve as a distraction?
4. *Interactivity*. The ability to control and interact with the visualization seems to be an effective way to stimulate student learning; this is similar to the use of physical manipulatives in a classroom.
5. *Animation.* Many mathematical concepts depend on a changing parameter which can be represented as time. Animations can provide a more accurate representation of such ideas than a static image.

Having identified these characteristics, one might hope we have an algorithm for making an effective visualization: create a colorful image, relevant to the problem at hand, which includes only the essential details, and allow the image to vary with time as appropriate, perhaps through the control of the student. Unfortunately, these guidelines might be helpful, but they provide no guarantee about the effectiveness of the resulting image.

The problem is that incorporating these five features of effective characterizations is not a simple yes or no proposition. All of these characteristics live on a spectrum, and there are choices to be made. For example, color can certainly make an image more visually arresting, but too many colors, or clashing color combinations, could be detrimental. Certain color combinations might be highly effective for some students, but useless to those who are color blind. All of these issues must be balanced, and the right balance depends not only on the subject matter but on the viewer—and no two viewers are exactly the same.

To further illustrate the difficulties in identifying, let alone creating, good visualizations, the reader is encouraged to think about the three examples described earlier. What makes them effective? How do they make use of the five features of visualizations described in this section? Certainly all three use visually arresting, colorful images. They all describe complex ideas, but the pictures have been simplified to show only essential features. Two of them are animated movies while the third, *Curved Spaces*, is a highly interactive visualization.

Yet it is impossible to attribute the success of these visualizations to the specific color, layout, or interactive controls chosen in their design. Would the geodesic video be any less instructive if the figure eight surface were a different color? If the colors in *Möbius Transformations Revealed* were arranged differently, would student learning be diminished? While these questions may seem silly, they reflect the fact that there are no definitive answers. In the end, to paraphrase Potter Stewart, "We know a good visualization when we see it," but it is difficult to be more precise.

In the end, trying to write out specific guidelines for creating visualizations is as fruitless as describing how to write the perfect novel. In both fields we can describe aspects of successful creations, but there will always be an intangible quality that defies quantitative evaluation. It is interesting to note that in a (highly unscientific) survey conducted by this author of post-secondary mathematics faculty who are heavily invested in creating and using visualizations—including some of the creators of the three examples above, and members of a Mathematical Association of America task force which was evaluating online resources for multivariable calculus—none of them had surveyed the educational literature to help guide their decisions about color, interactivity, or the design of their images. In short, they are "winging it," trusting their own pedagogical, mathematical and artistic instincts to decide what would be most effective in facilitating student learning.

The Need for Visualization

After reading the previous section one might be tempted to give up on the creation and evaluation of visualizations for the mathematics classroom, but this would not be a responsible approach to dealing with our students. The current generation of students has been raised with constant visual stimuli, whether television, computer graphics, or other media, and have come to expect such presentations. It is important not to pander to this view and replace the symbolic calculations and logical arguments of mathematics courses with entertaining but uninformative visual images. However, as educators we must find the best way to engage our students in the material. If that includes flashy pictures, so be it, as long as we then begin the process of training students to back up their ideas with proper mathematical reasoning and writing.

It is also worth noting that visualization is not just useful for education, but has become an important tool in mathematical research. The proof that the Costa surface is a complete minimal embedded surface of finite topology, for example, was driven by the analysis of computer generated images. One of the mathematicians involved in the proof, David Hoffman, wrote an excellent article (Hoffman, 1987) describing the process, and concluded: "The computer-created model is not restricted to the role of illustrating the end product of mathematical understanding, as the plaster models are. They can be part of the process of doing mathematics." Similarly, Jeffrey Weeks, creator of *Curved Spaces*, wrote "It began with a desire to show people the images that mathematicians already had in their heads. But then... I found that the interactive visualizations not only allowed me to communicate mental images I already had, but actually gave me new mental images, in effect deeper understandings of things I had thought I had already understood." (Weeks, personal communication, 2011) These mathematicians are just two of many whose research has been heavily influenced by computer graphics. Hence the use of visualizations in mathematics classrooms does not just help students understand concepts; their use provides important training for students to *do* mathematics in the classroom and beyond.

Walter Whiteley, a mathematician at York University, makes the further point that using visualization can help develop visual reasoning skills, an area often overlooked in our curriculum. When students struggle with algebra skills, it is generally expected that they can improve with practice and good teaching. Yet students who say they cannot think visually are often told not to worry; some people just are not "wired" to think that way. Whiteley has stated a number of claims related to the importance of visualization in both mathematics and mathematics education, including the following; for full details and citations related to his claims, see (Whiteley, 2004):

- Visuals are widely used, in diverse ways, by practicing mathematicians.
- Visual reasoning in solving problems is central to numerous other fields: engineering, computer science, chemistry, biology, applied statistics.
- We create what we see. Visual reasoning or "seeing to think" is learned. It can also be taught and it is important to teach it.
- Children begin school with relevant visual abilities, including 3D. In North America, this declines through school.
- Visually based pedagogy opens mathematics to students who are otherwise excluded, such as those with special needs or learning styles.

As educators this leaves us in an uncomfortable position. Visualization in the mathematics classroom is important for a variety of reasons: initial engagement of students' interest; improving student understanding of a concept, particularly for visual learners; development of visual reasoning skills; and as an important tool for mathematical exploration and research. At the same time, we are unable to identify precisely what makes any particular visualization effective or not. Where do we go from here?

Future Directions

Clearly there is no shortage of open questions for research related to visualization in mathematics education. While we may never find definitive answers, the more data we have about how students learn from visual stimuli, the more effective we can be in the classroom. Many of the most basic questions are too deep to answer in the near future; it will be many years, for example, before cognitive psychologists fully understand the process of how the viewing of a visual image or animation produces learning. However, there are many questions which could be addressed by current researchers. (Whiteley, 2004) suggests a number of research questions, including:

1. Which visuals are used by experts when solving mathematical problems? How are they used?
2. How do the visual practices of experts compare with the visual representations and processes supported by our teaching materials and pedagogies?
3. How does a teacher recognize the mis-seeing and misinterpretation [of mathematical concepts] and support change?
4. If a person does not use visual reasoning, is some portion of that skill set lost? At what ages?
5. What proportion of our students would engage mathematics more effectively and more enthusiastically through visual processes?

This author further suggests the following questions as possible avenues of research.

1. When should a visualization be used in the classroom? In some cases this may be a simple pedagogical choice, but with some concepts this may be critically important. Showing students a visualization at the beginning of a lesson may help engage students and motivate the rest of the lesson. In other cases it may intimidate and confuse students, and they will tune out before learning the theoretical concept behind the image.
2. Many geology departments around the country have used a so-called *GeoWall* system to display images in three dimensions, similar to the systems used to show three-dimensional Hollywood movies in multiplex theaters. Would the use of this system in multivariable calculus and other courses improve student learning of concepts in three dimensions?

Answering one of these questions, or any of the others which may have occurred to readers, would provide useful information for educations. The creation of visualizations will always remain an inexact science, but given their importance in mathematics, we have a duty to learn as much about their use as possible.

Practical Advice for Creating and Using Visualizations.

Some people have the patience and skills to write computer programs from scratch to display the precise visual images they desire; *Curved Spaces*, for example, is written in C++ using OpenGL routines. Computer programming is not for the uninitiated, however. Fortunately there exist many higher level tools to create visualizations, as well as collections of pre-existing materials which are appropriate for classroom use.

A word of advice: do not attempt to reinvent the wheel. Many "standard" classroom demonstrations and visualizations have already been done, and done well. There is no need to write a program which will accept a function $f(x)$ and graph its derivative $f'(x)$. There are a multitude of programs and online applets which have this functionality, and your time would be better spent on other endeavors unless you have a very specific need which they do not implement. Finding these programs can sometimes be difficult, however; an online search for calculus applets returns millions of results, many of which are out of date and of low quality. One useful resource is the Course Communities section of the Math Digital Library, run by the Mathematics Association of America (MAA). On this site faculty from around the country have evaluated visualizations and other online resources for use in the classroom: http://mathdl.maa.org/mathDL/61/.

Other useful tools include:

GeoGebra. This dynamic geometry software allows you to create highly interactive demonstrations, and automatically export them to web pages which use a java applet. The GeoGebra website includes hundreds of visualizations created by users around the world. (Available at http://www.geogebra.org/.)

Mathematica, Maple and Sage. These applications are powerful computer algebra systems with extensive graphing capabilities. Mathematica and Maple are expensive commercial packages, whereas Sage is free and is developed by a network of mathematicians around the world. The use of a computer algebra system (CAS) can greatly simplify the creation of visualizations; the CAS can handle the computations of derivatives, locations of objects and other details, all of which can be very tedious otherwise. The movie of a spaceship on a torus (see Figure 2) was created wholly within Mathematica. (Available at http://www.wolfram.com, http://www.maplesoft.com, and http://www.sagemath.org.)

POV-Ray (Persistence of Vision Raytracer). Raytracing software allows a user to give a simple description of the mathematical objects in a scene—a sphere, a plane, and so on—along with the placement of light sources and then generates an image of the scene; this is the type of software used by computer animation studios. POV-Ray is freely available and has extensive documentation online. The individual frames of *Möbius Transformations Revealed* was rendered using POV-Ray, although some of the mathematical computations used to describe the scene were done with Mathematica. (Available at http://www.povray.org. Sage also has a built in raytracer called *Tachyon*.)

References

Arnold, D., & Rogness, J. (2008). Möbius Transformations Revealed. *Notices of the AMS*, 55, no. 10, 1226–1231.

Hoffman, D. (1987). The computer-aided discovery of new embedded minimal surfaces. *The Mathematical Intelligencer*, 9, no. 3, 8–21.

Phillips, Linda M., Norris, Stephen P., & Macnab, John S. (2010). Visualization in Mathematics, Reading and Science Education: Vol. 5. Models and Modeling in Science Education. Springer.

Polthier, K., Schmies, M., Steffens, M., & Teitzel, C. (1997). *Geodesics and Waves.* Retrieved from http://page.mi.fu-berlin.de/polthier/video/Geodesics/index.html.

Weeks, J. (2001) *The Shape of Space*. New York, NY: Marcel Dekker, Inc.

Whiteley, W. (2004). *Visualization in Mathematics: Claims and Questions towards a Research Program.* Retrieved from http://www.math.yorku.ca/Who/Faculty/Whiteley/menu.html. (Direct link: http://www.math.yorku.ca/Who/Faculty/Whiteley/Visualization.pdf)

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Evaluation*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by January 21, 2012. The spring issue's guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

TO OBTAIN COPIES OF *JMETC*

To obtain additional copies of *JMETC*, please visit the *Journal*'s website www.tc.edu/jmetc. The cost per copy delivered nationally by first class mail is $5.00. Payment should be sent by check to *JMETC*, Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.