

# Journal of Mathematics Education at Teachers College

Fall – Winter 2011

A CENTURY OF LEADERSHIP IN  
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the  
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Dr. Robert Taylor was selected by the Teachers College sponsored Teachers for East Africa program to teach mathematics of Uganda's Makerere University. He returned to TC as an instructor in the Department of Mathematics, Statistics, and Computing in Education where he developed an innovative programming language (FPL) intended to introduce educators to the then-new field of computer programming. His seminal work entitled *Computers: Tutor, Tool, Tutee* led to leadership in the new field of computers in education. Dr. Taylor completed 33 years as a member of the Teachers College faculty in 2009.

Dr. Carl N. Shuster completed the doctorate at Teachers College in 1940 under the guidance of William David Reeve. Shuster joined the TC faculty at Reeve's invitation and soon was recognized as the nation's leading advocate of the use of traditional technology, especially measurement technology, in the mathematics classroom. Dr. Shuster served as President of the National Council of Mathematics from 1946 to 1948 and concluded his career as Distinguished Professor of Mathematics at Trenton State University.

**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

*JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

**Manuscript Submission**

*JMETC* seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at [www.tc.edu/jmetc](http://www.tc.edu/jmetc).

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select key words from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at [JMETS@tc.columbia.edu](mailto:JMETS@tc.columbia.edu)

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## Preserving Precious Instruments in Mathematics History: The Educational Museum of Teachers College and David Eugene Smith's Collection

Diane R. Murray  
Teachers College Columbia University

A history is given of the Educational Museum of Teachers College, which began in 1886, and David Eugene Smith's extensive collection of mathematical tools used in the Museum's exhibits is discussed. Historic mathematical instruments including, the astrolabe, abacus and counting rods, and the slide rule are examined. The author uses digitized versions of stereopticon slides from Smith's collection as images throughout the article.

*Keywords:* Educational museum, historic mathematical tools, history of mathematics, David Eugene Smith.

### Introduction

The fascinating thing about technology is the moment a new piece is released, it is almost immediately “out-of-date.” Consider Apple's iPhone, first introduced in 2007; yet, only four years later Apple will be releasing its 5th version. Will the first generation iPhone be a desired historical piece in museums one day? In one hundred years, will historians be studying the technology of our time? It is important for future generations to have access to and learn from current technologies. This appreciation for the past allows people to see the growth in our society. David Eugene Smith, honored professor at Teachers College and mathematics historian, had a great love and respect for historical books, manuscripts, and instruments of mathematics. Through Smith's lifetime, he collected numerous historical items that were made available to his students through the Educational Museum at Teachers College and the David Eugene Smith Mathematical Library of Columbia University. This article explores the Educational Museum of Teachers College with specific interest in the mathematical technology housed there.

### History of the Educational Museum of Teachers College

The Educational Museum of Teachers College started out as the “Children's Industrial Exhibition” given by the Industrial Education Association in 1886. The Industrial Education Association was “an organization for promoting manual and industrial education, the forerunner of the New York College for Training Teachers (1888), which in 1891 became Teachers College” (Andrews, 1908, p. 13). The Industrial Education Association, which was housed at 9 University Place, New York, had many exhibits scattered throughout the school's various departments of manual training, art, domestic science, domestic art, and natural science—with no real “home” (Andrews, 1908, p. 14).



**Educational Museum. Mathematical Exhibit. Teachers College. (Ca. 1903). Courtesy of Teachers College Columbia University.**

Finally in 1901, a few years after Teachers College moved to its present location at Morningside Heights, did the Educational Museum find a special room of its own on the second floor of the Main Hall—now called Zankel Hall—in room 215. The Educational Museum of Teachers College grew to be a specialized museum to showcase educational objects and historical artifacts in many subject areas, including mathematics.

In 1908, Benjamin R. Andrews, the former supervisor of the Educational Museum of Teachers College at the time, described the annual exhibition as follows:

During these days all departments and schoolrooms are open to public view; displays of pupils' work are arranged; demonstrations of actual teaching are sometimes given; and teachers and professors are on hand to explain the exhibits, the courses of

## EDUCATIONAL MUSEUM

study, and other matters to the visitors, who usually number several thousand. For the exhibition days the whole college might be termed an 'educational museum'. (p. 16)

Andrews also reported that there were more than 6851 visitors of the museum during the 1904-5 academic year.

### Mathematics Exhibits at the Educational Museum

During this time, the Department of Mathematics had an extensive collection of artifacts, historical documents and portraits, and mathematical tools. It was housed in 211 and 212 Main Hall. These collections were due to David Eugene Smith and his skillful acquisitions, which were "secured by him on numerous visits to various parts of the world, extending over a period of about forty years" (New York Museum of Science and Industry, 1930, p. 59). As the museum no longer exists today, current researchers have to rely on past descriptions of the museum. Andrews, in 1908, portrayed Smith's private collection as:

1600 portraits, 2000 autographs, and 150 medals of mathematicians; exhibits showing the development of mechanical mathematics from earliest forms to modern reckoning machines; mathematical manuscripts of the last 200 years; 500 mathematical books of earlier date than 1800; and a mathematical library of 7000 volumes and 5000 pamphlets. These private collections, together with the collection of apparatus, lantern slides, etc., form a unique museum of the history and teaching of mathematics. (p. 21)

It is obvious that the mathematics collections were very extensive. These collections were open to the Teachers College students and, upon requests, the general public.

This collection became very popular in other schools and colleges—so it became necessary for the Educational Museum to provide a way for people to view and use the instruments for educational purposes. The Museum found its solution in producing sets of stereopticon slides, which were sent out to requesting institutions. Due to the large collection of David Eugene Smith, the Educational Museum was confident in creating sets in "nearly every branch of the subject" (Smith, p. 1). The current Program in Mathematics Education at Teachers College is fortunate to have some of these stereopticon slides in its possession. The author has recently digitalized these slides and some of the images will be used later in this article.

### Instruments of Interest

David Eugene Smith's collection consists of more than 275 instruments. It is not feasible to appropriately describe each item. Smith depicted the most interesting pieces from his own collection of mathematical

instruments, used in his own teaching, organized in Table 1.

It is the purpose of the following section to give a brief history of a few items that were in Smith's collection and which are still available for viewing upon special request at the Rare Book and Manuscript Library (RBML) at Columbia University.

### *The Astrolabe*

Figures 1 & 2 are two stereopticon slides of astrolabes from Smith's collection. The astrolabe, an instrument of surveying and navigation, was invented in Greece and is credited to either Hipparchus, in 2nd century B.C., or Apollonius of Perga, in 3rd century B.C. It was used to observe stars and heavenly bodies, but also could solve problems related to distances, measuring heights of large buildings, trees, and mountains, as well as finding the depth of valleys and trenches.

The astrolabe was not used in Europe until the 12th century. Typical astrolabes were made of brass and were approximately 6 inches in diameter. The popularity of the astrolabe in Europe declined after the creation of the reflecting quadrant (Ginsburg, 1930). One of Smith's astrolabes was from Italy and it was signed by Bernard Sabeus in 1558. It is featured in the "Jewels in her Crown: Treasures of Columbia University Libraries Special Collections" online exhibit along with the famous "Plimpton 322" cuneiform tablet.



Figure 1. Astrolabes.  
(Top) Italian, 1599 and  
(Bottom) Arabic.



Figure 2. Astrolabes.  
(Top) Italian, 1450 and  
(Bottom) 1558.

**Table 1. Organized from Smith's writings in "The David Eugene Smith gift of historical-mathematical instruments to Columbia University." Science (New York, NY), 83, 2143, 79-80.**

Theme	Description of Items
Representing numbers	Tally sticks beginning with those of the thirteenth century; early Greek alphabetic numerals of the Ptolemaic period on an icosahedral die; several pieces of papyrus with coptic numerals; several hundred medieval parchments containing numerals; and a few knotted cords and prayer beads from the orient
Operations with numbers	Sets of counters used on a computing table; various forms of the abacus (Chinese, Japanese, Armenian, Russian and Western European); and a cast of an early Greek computing table now in Athens
Number games	Dice of various types and periods from Etruscan, Egyptian, Greek and Roman times through the medieval period
Measures	Length (rods showing the ell, cubit and other units); weight (numerous nests of brass weights some sealed officially and interesting as works of art; various forms of the steelyard; money-changers sets from various countries); angles (proportional compasses of the Roman period, ordinary compasses also Roman, and various types from the renaissance period, some with the names of prominent makers; various types of protractors)
Measures of time	Sundials of various countries and eras. Those of China are often works of art, being engraved with great care. Most of the various types of sundials are shown. These are in ivory, bronze, silver or wood. Two Japanese clocks of curious mechanism are included
Instruments of surveying and navigation	Several remarkable astrolabes beginning with one of the sixteenth century. There are also a number of early quadrants and other instruments in copper and in brass. In general they were secured in Italy, Germany, India, Austria, Iraq, Arabia and France
Astronomical instruments	Including those above mentioned and a number of brass armillary spheres (some of artistic merit) and celestial spheres of brass with the important stars inlaid in silver. These were secured in northern India about thirty years ago. One of them bears the inscription, dated 1645, stating that it is the work of the grandson of the emperor's chief astronomer. These instruments represent the elementary and utilitarian phase of mathematics

### *The Abacus and Counting Rods*

Figure 3 is a stereopticon slide that displays a Chinese abacus, called a suan-pan, a Russian abacus, called a s'choty, and Korean counting sticks. In China the abacus is used currently in banks and shops. Typically a long-time user of the abacus will work very rapidly, methodically clicking away the tiny beads on the instrument. The suan-pan appeared first in 12th century. Suan-pans can be used for operations other than counting, such as, multiplication and division, and even square root and cube root calculations (Smith, 1925). The suan-pan has a horizontal bar with vertical rows of 2 beads above the bar and 5 beads below. Values are read in relation to the horizontal bar—

beads below are ones and have the name "earth beads"; beads above are fives and have the name "heaven beads." For example, if there were a heaven bead and two earth beads at the horizontal bar, it would represent the number seven. The vertical rows represent place values as powers of 10. The Russian abacus, similar in design to the Chinese abacus except the wires were constructed horizontally, is called a s'choty. It was used in Russian schools up to the 1990s when the electronic handheld calculators took over (Bud & Warner, 1998, p. 7). This abacus consists of eleven wires with beads. It is slightly more complicated to use compared to the suan-pan. Starting at the bottom wire, wires 1-3 have ten beads, wire 4 has four beads, and wires 5-11 have ten beads. It is even further detailed by color variations; the ten beaded wires have the first four beads and last four beads in one color and the middle two are another color to aid in quick computing (Smith, 1925). The fourth wire, with only 4 beads, is a place holding wire—the wires above the fourth wire represent whole numbers. So for example, to represent 2514, push to the left edge two beads on the eighth wire, five beads on the seventh wire, one bead on the sixth wire, and four beads on the fifth wire.

There have been counting sticks or rods since before 500 B.C. The Korean counting sticks, called Ka-tji san, originated from China and Japan versions and are made of bone or possibly bamboo. The sticks remained in use long after they were abandoned in other countries due to the popularity of the abacus. About a hundred fifty were used for typical calculations. Figure 4 illustrates how the sticks were used to represent values 1-12. Eventually this counting method was replaced by the use of the abacus (Smith, 1925).

### *The Slide Rule*

In 1620, Edmund Gunter, a professor at Gresham College in London, published the first mentions of an instrument called "Gunter's Line of Numbers." Gunter had done extensive work with sectors, a variation of the instrument displayed in Figure 5 in one of Smith's stereopticon slides, which helped make it clear how to construct such a device that could add a pair of logarithms





Figure 3. Chinese suan-pan, Russian s'choty, and Korean counting rods.

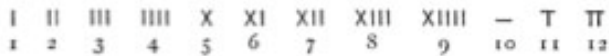


Figure 4. From Smith's *History of Mathematics Volume II* (1925) p. 173.



“Margarita Philosophica” (1503), showing the old (counter using an abacus) and new (algorism) reckoning.

together almost automatically (Williams, 1985). William Oughtred, a leading mathematician at the time, met Gunter and decided to improve on Gunter’s idea by constructing a circular version. Being such a dignified mathematician, Oughtred believed it beneath him to publish any work related to a “plaything” (Williams, 1985). Oughtred’s students thought otherwise, and in fact one of his students, Richard Delamain, is claimed to have used Oughtred’s ideas and published them as his own in 1630. Another student, William Forster, urged Oughtred to publish his work and in response to Delamain’s book, Oughtred finally agreed. Forster and Oughtred’s book, *Circles of Proportion*, was published in 1632 (Smith, 1925).

With all of this conspiracy and competition about the slide rule, unfortunately it did not reach full popularity until the 1850s, over 200 years later. This happened due to a 19 year-old French artillery officer, Amedee Mannheim, who created what became the modern version of a slide rule—displayed in Figure 6 in Smith’s stereopticon slide. It was almost instantly popular in Europe, but North America did not begin using it until 1888. The Mannheim slide rule was eventually manufactured in the United States due to the high demand for the instrument. Advancements to the Mannheim slide rule were the Thacher slide rule and the Fuller slide rule, both depicted in Smith’s stereopticon slide in Figure 7. The latter was

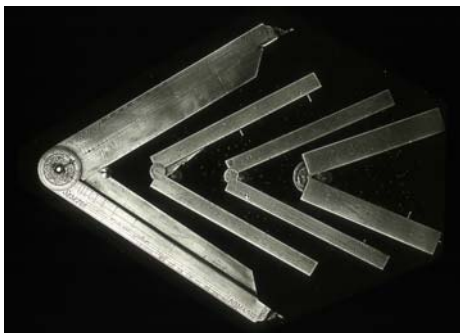


Figure 5. Sector compasses. Renaissance period.

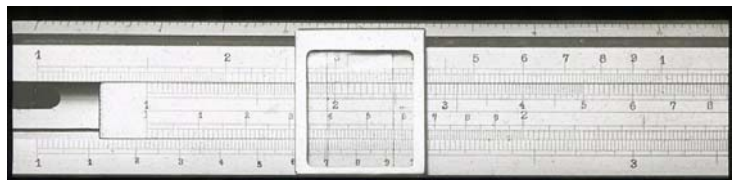


Figure 6. Mannheim slide rule.

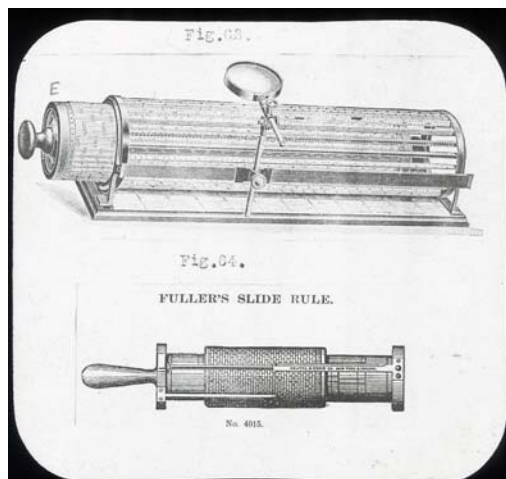


Figure 7. Thacher slide rule and Fuller slide rule.

equivalent to an 84-foot long standard slide rule (Williams, 1985). As knowledge of technology expanded, the slide rule would soon be used for demonstration purposes in classrooms. “It would remain a part of school and college mathematics teaching until the introduction of inexpensive handheld electronic calculators in the mid-1970s” (Kidwell, Ackerberg-Hastings, & Roberts, 2008, p. 122).

### Conclusion

Throughout this article some of the many stereopticon slides of mathematical instruments in possession of the Program of Mathematics at Teachers College were displayed. The wide variety of the slides in the collection opens the doors to many areas of research. Not only the history of the objects, but also the story behind the acquisition of the item through Smith’s perspective would be quite interesting. In 1931 David Eugene Smith donated his collection of historic mathematics materials, including his instruments, to the Columbia University Libraries. The collection is currently housed in the Rare Book and Manuscript Library at both on-site and off-site locations. Although Smith passed away in 1944, his love and appreciation of mathematics history, and even more precisely historic mathematical artifacts and instruments, lives on today. In 2002, an RBML exhibit entitled “*The Ground of Arts: Mathematical Instruments and Illustrated Books from the David Eugene Smith Collection*” showcased numerous books, manuscripts, and instruments from Smith’s collection. In 2012, a new museum named The Museum of Mathematics will open in New York City, which will advocate for the learning of mathematics and mathematics education history.

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## *Journal of Mathematics Education at Teachers College*

### **Call for Papers**

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Evaluation*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at [JMETC@tc.columbia.edu](mailto:JMETC@tc.columbia.edu) by January 21, 2012. The spring issue’s guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

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### **Looking Ahead**

Anticipated themes for future issues are:

Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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