Journal of Mathematics Education at Teachers College

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The Journal of Mathematics Education at Teachers College is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

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Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Math ematics. As a peer-rev iewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spr ing-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job position ns—teachers, principals, superintendents, professors of educ ation, and other leaders in education. Articles to app ear in the *JMETC* include r esearch reports, commentaries on practice, historical an alyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathemat ics educators. Artic les should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been a ccepted for publication elsewhere. To keep the submission and review pro cess as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include a n abstract with keywords. Abstracts d escribing the essence of the manuscript should not exceed 150 words. Authors sho uld select key words from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be sub mitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Tech@MoMath: Technology Use in the Forthcoming Museum of Mathematics

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In the late Fall of 2012, the Museum of Mathematics ("MoMath") will open in New York City and will be the only museum dedicated solely to mathematics in the United States. The content and design of most of the exhibits are not yet available to the public but some information about how the Museum will use technology has been released. The ways that the Museum makes use of technology can be classified into two categories: those that increase accessibility and those that increase exposure. The specific technologies used within both categories are discussed in the context of their pedagogical value. Teachers then are introduced to some ways that they can adapt MoMath's technology use to their own classrooms.

Keywords: Museum of Mathematics, MoMath, Teaching with Technology.

Introduction

Around the United States, various museums of science exist, many of which house mathematics exhibits, either permanent or temporary. There is no museum; however, that focuses only on mathematics (Chang, 2011). In late Fall 2012, that will change: the Museum of Mathematics ("MoMath") will open its doors in New York City's Flatiron District¹ (Shaughnessy, 2011). Its mission is simply to "enhance public understanding and perception of mathematics...[and it] will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics" ("Mission," n.d.).

According to Cindy Lawrence, the Museum's Chief of Operations, MoMath will feature "hands-on, interactive, engaging exhibits highlighting the beauty of mathematics" (personal communication, August 9, 2011). MoMath's exhibits will be accessible to all audiences with a particular focus on students in the middle grades, 4-8 (C. Lawrence, personal communication, August 9, 2011). Despite the fact that the exact topics the exhibits will feature are not yet public, it is very likely that mathematics teachers around the tri-state area, and perhaps beyond, soon will be clamoring to arrange field trips to this exciting museum.

What, though, does this museum have to do with technology in relation to mathematics education? Quite simply, it serves as an example of how mathematics educators can and must use technology to enhance their craft. Of those technologies at MoMath that may be discussed as of this moment, there are two main uses that teachers can imitate: technology to increase accessibility and technology to increase exposure.

¹ The Museum will be located at 11 E. 26th Street, New York, NY.

Technology to Increase Accessibility

"Technology to increase accessibility" can be understood as any use of technology that fosters increased understanding of the subject, specifically with reference to access in terms of language use and ability level. The exhibits at MoMath will use a feature that adapts signage to visitors' ages, mathematical ability, and language spoken, a feature that is believed to be the first one of its kind used in any museum (C. Lawrence, personal communication, August 9, 2011). The pedagogical implications of this piece of technology are obvious, particularly when focusing on the visitor's ability level: common sense would indicate that an effective presentation must not be below the student's ability as this will cause attention loss, and it must not be above the student's ability as this will cause students to be confused and to "tune out." The pedagogical achievement of this technology is that the Museum developers will have found a way to make the same bit of mathematics accessible and interesting to most visitors, from children in elementary school to adult mathematicians (C. Lawrence, personal communication, August 24, 2011).

Technology to Increase Exposure

"Technology to increase exposure" can be understood as any use of technology to promote awareness of a subject. MoMath currently does this using a social media platform; Facebook² and Twitter³ posts lead followers to

² To follow, go to facebook.com/MoMath1.

³ To follow, search the Twitter username @MoMath1.

various stories, articles, and activities that are related to mathematics and mathematics education. These posts remind followers not only of the Museum's existence, but also that mathematics is a dynamic, ever-growing field. Therefore, these posts have a twofold purpose, advertisement and promotion of the MoMath mission.

Furthermore, the Museum features a presentation series, *Math Encounters*, that "[celebrates] the spectacular world of mathematics" ("Math Encounters," n.d.). Some of these lectures can be found in part or in whole on the Museum's YouTube channel⁴. The lectures in this series highlight the underlying mathematics in a wide range of areas such as soap bubbles (Frank Morgan's presentation "Soap Bubbles and Mathematics"), origami (Eric Demaine's "The Geometry of Origami"), and dancing (Scott Kim's "Symmetry, Art, & Illusion"), among others (ibid). Because of the online availability of some of the lectures, viewers from around the world can learn about the vast array of topics presented.

Finally, in the future, the MoMath developers plan to have online versions of their exhibits. According to Lawrence, these online recreations of the physical exhibits not only will allow people all around the world to access the content of the Museum, but they will "allow [online] visitors from afar and visitors in the Museum to interact with each other" (C. Lawrence, personal communication, August 9, 2011). Online visits to these exhibits will help people from all around the world become aware of the inherent mathematics in the subjects of the exhibits themselves and, hopefully, in the world around them.

Adapting MoMath's Technology Use to Your Classroom

Teachers at most levels should be able to benefit from MoMath's technology use. They may do this by following the Museum developers' pedagogical examples, by imitating their practices, or by using the online materials provided by MoMath.

Following Pedagogical Examples

The main pedagogical example produced by MoMath's use of technology—adjusting signs to each visitor's language, age, and mathematical ability—is one that teachers usually follow intuitively and without the use of technology. (While doing this without technology is feasible in a classroom setting, it is not feasible in a museum setting.) The appropriate language and ability levels must be considered when interacting with students. Clearly, students cannot learn if they cannot understand the language level (or, in the case of MoMath, the actual language), so language use must be adjusted based on the needs of the students in the classroom. If, for example, there are many English language learners (ELLs) in a mathematics classroom, Winsor (2007) suggests that teachers might find it helpful to set up collaborative communication groups of ELL students with at least one student who is advanced enough in English in order to understand the teacher well. The more advanced student can be assigned a leadership role within the group, helping their peers understand the lesson without translating word-for-word. Winsor (2007) also suggests allowing ELLs to access bilingual mathematical dictionaries or glossaries to facilitate better communication of ideas.

Imitating Technological Practices

MoMath makes extensive use of social media to "advertise" to their followers. In a single post, they advertise both for the Museum and its "personality," as well as interesting mathematical ideas. Teachers of high school and college students⁵ can imitate this practice themselves. For example, a teacher can set up a Twitter account for their students to follow and use it to post links to a class website that lists the day's homework assignment, podcasts created by the teacher⁶, a website that helps clarify a tough topic, a video that describes an application of the mathematics covered in the day's lesson, and so on. Sharing via social media, as in the example, can help teachers keep in touch with students when they are not in school. Posts that help students understand the underlying mathematics in the things around them, that show how the mathematics they are currently learning can be used in the real world, or that simply describe the beauty of mathematics in a way that the students can relate to will likely increase overall interest in the subject.

Using Online Materials Created by MoMath

The MoMath website will be featuring online versions of the exhibits found within the actual Museum. This means that visitors from all over the world can "visit" the Museum in some capacity, although the specifics of how this will be done have not yet been released to the public.

⁴ To access, go to youtube.com/user/MuseumOfMathematics.

⁵ Facebook's Statement of Rights and Responsibilities requires that the user be at least 13 years old ("Registration and Account Security," *Section 4-5*, retrieved 8.18.2011 from http://www.facebook.com/ terms.php?ref=pf), and Twitter, while not explicitly stating that users must be a certain age in its Terms of Service, indicates that "Twitter is not for children under the age of 13" in its Safety tips section ("How Do I Protect my Kids on Twitter?," *Safety: Parent and Teen Tips*, retrieved 8.18.2011 from https://support.twitter.com/groups/33-report-a-violation/topics/166-safety-center/articles/470968-safety-parent-and-teen-tips). Teachers of students younger than 13 should not participate in these social media activities because of the conditions of use set forth by these social media services. ⁶ For example: "Author," (2010).

However, Lawrence has indicated that the online exhibits will exist in some sense in a Web 2.0 platform-that is, online visitors will be able to interact with visitors in the Museum (C. Lawrence, personal communication, August 9, 2011). One might also presume that they will be able to interact with other online visitors, as well, although this is not confirmed. It is easy to see how these materials can be used in the mathematics classroom. Teachers at all levels can access the exhibits created by MoMath, use them to help students explore the relevant mathematical ideas, and, supposedly, to help students learn about the value of communication and collaboration in mathematics. (One need only look at the Polymath Project⁷, a "massively collaborative mathematics" project, to confirm the value of collaboration in mathematics ["Timothy Gowers," 2011].)

The Math Encounters series also can be used by teachers in the mathematics classroom, particularly those teaching ages 12 and older. The Museum features some videos from the series on its YouTube channel, and for those who cannot access videos online, DVDs of some of the lectures are also available to purchase in the Museum's online shop ("Shop," n.d.). The lectures in this series are varied both in terms of the mathematical and the nonmathematical topics on which they focus. The one thing that has remained constant, however, is that the topics are interesting and bring to light mathematics where it might otherwise go unseen. Teachers can present these videos to their students in order to promote their interest in and curiosity about mathematics. Beyond using these videos to develop appreciation of the subject matter, teachers can also use them as a precursor to classroom discussions. Students may offer ideas and questions about a given presentation and collaborate with their peers to gain a deeper understanding of the material. In this type of activity, students will learn to speak the language of mathematics and develop the thinking process needed to promote mathematical understanding (Barnett-Clarke & Ramirez, 2004).

Directions for Future Analysis of Technology

Unfortunately, the technology issue of this Journal will be published about one year too soon in order for more detailed analysis of the Museum of Mathematics' technology use. Because many of the details about MoMath's exhibits are not yet released to the public—and probably won't be until the Museum opens its doors in late Fall 2012—more in-depth analysis cannot be done. Once MoMath opens, though, mathematics educators and those involved in instructional technology will be interested in how technology within the exhibits is used in a

pedagogical sense. Lawrence has confirmed that many exhibits will rely on technology and even that "many of them will be viewed as high tech" (C. Lawrence, personal communication, August 9, 2011).

Upon the Museum's opening, one of the more important questions that should be asked is "how does the technology used in the exhibits facilitate learning where it would otherwise be much more difficult or even impossible?" An example of the use of technology in such a way is found in the Math Midway⁸, the Museum's traveling exhibition. Math Midwav uses are carnival-like atmosphere to promote the same mission as the Museum itself ("Learn More," n.d.). In the Midway, there is an exhibit called "Ring of Fire" that features a large ring with lasers that project a plane surface along the Ring's interior. Three-dimensional shapes with clear faces are provided with the Ring. When these shapes are held inside the Ring. the intersection of the laser plane and the threedimensional shape's boundaries are illuminated, marking the cross section. Visitors are challenged to find, for example, the hexagonal cross section of a cube ("Activities," n.d.). Without this piece of technology, teaching about cross sections requires that students have the geometric imaginations required to "see" them or it requires that three-dimensional shapes actually be cut to see the desired cross section. Both of these nontechnological alternatives can be difficult and timeconsuming. This is a prime example of how technology can be used pedagogically to overcome problems in teaching and learning that are difficult or impossible to overcome otherwise.

Conclusion

The Museum of Mathematics provides many opportunities to examine technology use in education. Its varied uses of technology for varied purposes allow educators to improve their craft by learning through example. Museums and classrooms alike are educational institutions and so the best practices from both should influence the other. Increasing accessibility and exposure to mathematics are two methods employed by MoMath that teachers can use to improve learning and engage students. Because of the supposed "high tech" nature of the Museum's planned exhibits, it is unlikely that most teachers will be able to emulate the use of this technology in their own classrooms, so future researchers will be tasked with determining how teachers can benefit from technology use and the pedagogical practices associated with it in MoMath's exhibits.

⁷ See polymathprojects.org for the Polymath blog.

⁸ For more information, visit mathmidway.org.

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Journal of Mathematics Education at Teachers College

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Evaluation*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by January 21, 2012. The spring issue's guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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