

Journal of Mathematics Education at Teachers College

Fall – Winter 2011

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the
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Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select key words from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETS@tc.columbia.edu

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Integrating External Software into SMART Board™ Calculus Lessons

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Interactive Whiteboards (IWBs) are becoming commonplace throughout primary, secondary, and post-secondary classrooms. However, the focus of the associated lesson creation & management software tools delivered with IWBs has been the primary grades, while secondary and postsecondary mathematics lessons have requirements beyond what is delivered entirely within this software. This article describes how SMART Notebook™, the software delivered with SMART Technologies' SMART Board™, allows teachers to easily go beyond any perceived limitations by facilitating integration of a wide variety of other software packages to produce dynamic, visually appealing, and most important, effective calculus lessons. The content and visual requirements of such lessons are described and examples from actual lessons demonstrate how these requirements can be fulfilled.

Keywords: SMART Boards, Calculus Lessons, Graphics, Animations, High School Mathematics.

Interactive Whiteboards (IWBs), of which SMART Technologies' SMART Board™ is the dominant product in the market, are becoming commonplace throughout primary, secondary, and post-secondary classrooms. The SMART Board, along with its lesson creation/management software, SMART Notebook™, serves as a platform on which lessons can be created, delivered, accessed, managed, and continually enhanced.

SMART Notebook is delivered with a library of objects (the Gallery) that can be incorporated into a lesson. These objects include conventional “clip art” as well as audio enabled objects (e.g., a numeral 4 which, when touched, sounds out the word “four”), and animated clip art. The software also has a set of tools that allows objects to be imported or created, including animation/transition effects similar to those in PowerPoint.

Reviewing the SMART Notebook lessons which are available on the Internet^{i, ii, iii, iv} either from SMART Technologies or from the teachers who have created them and freely posted them online, reveals the following:

1. Virtually all Notebook files are not complete lessons, but rather are exercises, games, or question/response reviews.
2. The majority of Notebook files are for the primary grades (K-8), with fewer for secondary or postsecondary classes. Within the latter category, little is available for calculus.

These observations suggest that the educational community may not realize the functions to which a SMART Lesson file can and should be put. These functions derive from the very nature of a high school mathematics lesson as distinguished from other subjects such as English or History, and other grade levels. Entire

high school/college mathematics lessons can be contained in a SMART Notebook file. Typically, lessons consist of three distinct sections:

1. Concept Exposition
2. Procedural Instruction
3. Applications

These files can serve not only as the “classroom whiteboard lesson” for the teacher, but also as an asset for the students. For the students, the lesson files can be uploaded to the school's website, where they may be accessed for review and study. This benefits students who require clarification of the concepts taught in class, as well as students who may have been absent from the lesson and who wish to stay on schedule.

The design and content of SMART Classroom lesson files often exhibit the following:

- LARGE Fonts. A classroom lesson should be *VISUALLY* appealing. The content, especially details, must be sufficiently large in size and easy for students to see and decipher. However, this conflicts with the next requirement, Continuity of Procedure
- Continuity of Procedure. Mathematics is different from other school subjects in its heavy emphasis on procedure. In order for a procedure to be easier to comprehend, it should be explicated in one continuous segment, if possible. Hence, there is a tension between this requirement and the preceding one.
- Graphs. The importance of graphs in a calculus lesson is universally acknowledged. However, the details are also very important. The graphical component of mathematics pedagogy (along with numerical, analytical and tabular) can be used to illustrate the relationship between a function and its

first and second derivatives, to demonstrate the concept of a family of functions being the antiderivatives of a function, to illustrate the concepts of extrema or concavity, or to visualize a parametric surface in three dimensions.

- Animations. Animations can bring on that “ah-ha” moment for a student, such as when teaching the notion of the derivative as the limit of the difference quotient. One can use a set of commercially available animations, e.g. Calculus in Motion, © Audrey Weeks,^v which shows both the secant line and the tangent line to a function. Then, as we let “delta x” shrink to zero, the students can “see” the secant line approaching the tangent line, which makes it easier for them to accept the notion that the difference quotient, the slope of the secant line, becomes the slope of the tangent line when the limit is considered. (See Figure 1.)

The SMART Technologies’ recent release of Math Tools was a significant advance for the community of teachers wanting to use Notebook for mathematics. However, while Math Tools does, in fact, have a toolbar including many calculus symbols, its actual calculus functionality is limited. For example, while it can evaluate both definite and indefinite integrals in closed form for many simple functions such as polynomials and trigonometric functions, it does not have a mechanism to differentiate, either symbolically or numerically, any functions. Further, while it has excellent graphing functionality for explicit and some implicit functions of one variable, it can only graph two functions at a time in a coordinate plane. Further, it does not support polar coordinates, or functions of more than one variable. While these are significant for actually performing calculus or advanced algebra, these shortcomings are, with the exception of the graphing limitation, unimportant when it comes to constructing a lesson.

In addition, SMART Notebook treats text and mathematics similar to the way Power Point treats text. That is, it opens a “box,” either a math box or a text box, which you then fill with text characters or symbols **BUT NOT BOTH**. And that, it turns out, is a major problem in the context of creating a lesson. Given that a lesson will contain a continuous stream of mathematics, text, and

illustrations, it is cumbersome to produce a lesson when all three must be done separately. Further, switching back and forth between math and text in SMART Notebook requires an awkward keystroke/mouse click sequence. Fortunately, SMART Notebook provides tools for easy and effective integration of external software to achieve the following:

1. Graph more than two Functions on a set of axes
 - Graph 3D Surfaces
 - Provide Dynamic & Static Illustrations & Animations
 - Easily Integrated Text, Mathematics, and Illustrations, e.g. Graphs
2. Flexible Page Formatting
3. Inclusion of External Dynamic Illustrations

The following illustrates how a combination of Scientific Notebook®,^{vi} Adobe Acrobat®, Calculus in Motion™, Geometer’s Sketchpad®^{vii} and SMART Notebook allow us to achieve the above goals and produce effective calculus lessons. Space limitations require that these examples be brief.

Antiderivatives

One key concept in teaching antiderivatives is that all antiderivatives of a function have the same derivative. In graphical terms, that means that the slope of the tangent line to every member of the family of antiderivatives is the same for a given value of the independent variable. That’s easier to understand if seen visually, although making it visually explicit is challenging. Consider the function $f(x) = x^2$. The family of antiderivatives is, $F(x) = x^3/3 + C$. In Figure 2, produced in Scientific Notebook and imported into SMART Notebook, the red curve is the original function and the black curves are members of the family of antiderivatives. While it is clear that around $x=0$ all the black curves have the same slope, it is not so obvious farther away from the origin. To make it easier to see, one of the authors constructed a simple viewing device using very basic shape tools in SMART Notebook. This simple device allows students to focus their attention to a column. As the simple viewing device is moved across the graph, the students can easily see that the slopes in a column are all the same.

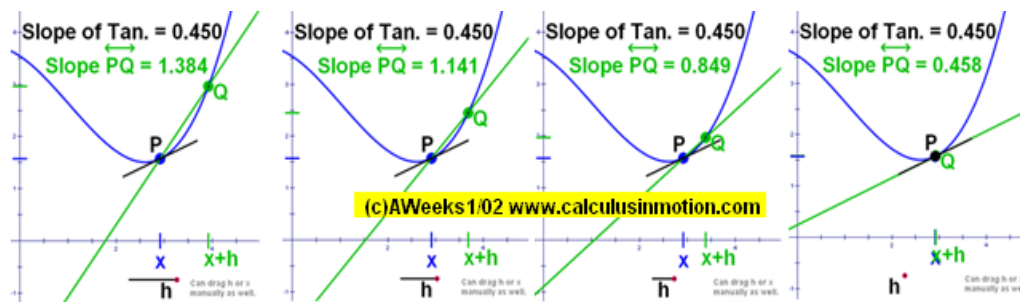


Figure 1. Animation demonstrating slope of tangent line as limit of difference quotient

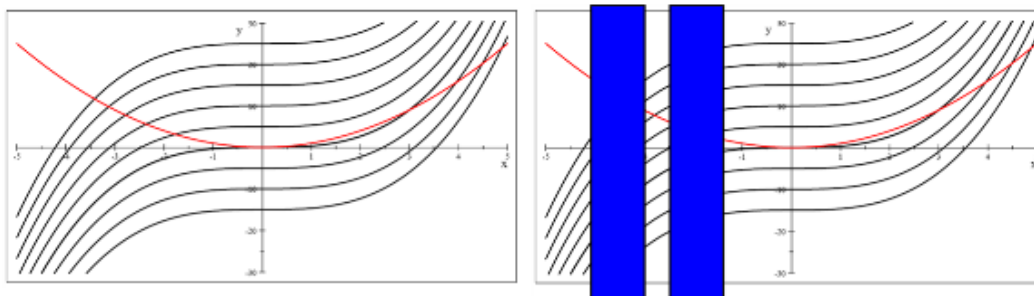


Figure 2. Antiderivatives with slope viewer

The Procedure is as follows:

Step	Find	How
1	x - int	"
2	y - int	"
3	VAs	"
4	HAs	"
5	CPs	"
6	INC	"
7	DEC	"
8	SCPs	"
9	CU	"
10	CD	"

The Procedure is as follows:

Step	Find	How
1	x - int	$f(x) = 0$
2	y - int	$f(0)$
3	VAs	$\frac{f(x)}{g(x)}; Q(x) = 0, P(x) \neq 0$
4	HAs	$\lim_{x \rightarrow \pm\infty} f(x)$
5	CPs	$f'(x) = 0, DNE$
6	INC	$f'(x) > 0$
7	DEC	$f'(x) < 0$
8	SCPs	$f''(x) = 0, DNE$
9	CU	$f''(x) > 0$
10	CD	$f''(x) < 0$

Figure 3. Table with cell shade "reveals" organizes procedural steps and promotes student engagement

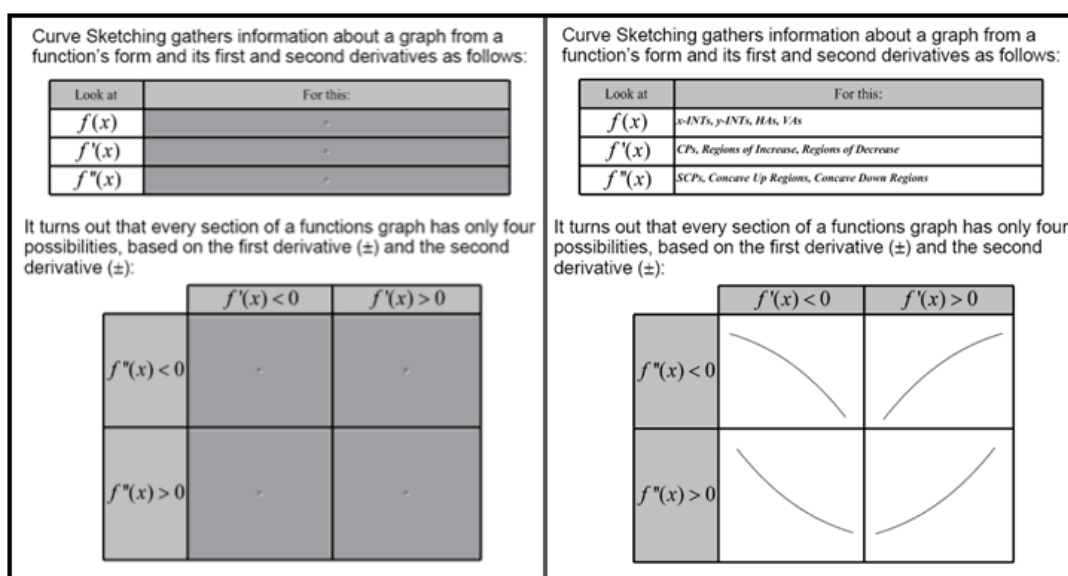


Figure 4. Table with cell shade "reveals" used to organize information and focus students' attention

Curve Sketching

In teaching curve sketching, the object is to understand how the essential characteristics of a graph, i.e. x and y intercepts, horizontal and vertical asymptotes, critical points and regions of increase or decrease, and “second critical points” (a term used in one of the author’s classrooms to identify points where the second derivative equals zero or is undefined), as well as regions of upward and downward concavity, fully determine the general shape of a curve. The purpose, of course, is to embed these concepts more deeply in students’ minds. In Figure 3, we present two tables with cell shades. The first describes what information is needed and where it will be found. The second bounds an initially intimidating problem for students by showing how every section of a graph must have one of only four appearances. The pictures of the curve were produced in Scientific Notebook and imported into the SMART Notebook table. The cell shades promote student engagement by allowing the teacher to elicit the answers before revealing them.

Figure 4 shows another table with all the specific characteristics of the graph that must be determined (the “what”). The shaded cells contain the “how,” and again promote student engagement as they are only revealed after the answers are solicited from students.

Finally, in Figure 5, all the information gleaned from analysis is gathered together and the curve is to be sketched based on it. A graph of the function, produced in Scientific Notebook, is hidden behind the blue reveal, which has the axes, intercepts, and asymptotes already drawn on it. In the second image, the bottom cell shades are removed one at a time to expose what the curve must look like based on its being increasing or decreasing and concave up or down. The curve is sketched based on all the information available. In the last image, the blue reveal fades out (a standard Notebook animation tool) to reveal the graph, usually to the students’ delight.

Related Rates

Related Rate problems require calculus, algebra, and geometry skills, of course. They also usually require the ability to visualize the dynamic nature of the problem at hand, whether it is the tank filling problem, the approaching vehicles problem, or in this case, the sliding ladder problem, where the bottom of a ladder resting against the side of a house is pulled out at a constant rate and it is desired to know the speed with which the top of the ladder is moving at any height above the ground. In this lesson, another of the Calculus in Motion animations is used to help the students visualize the dynamic nature of the problem. One additional feature of this particular animation which is instructive is the graph of velocity of the top of the ladder at various times. It allows discussion of what happens as the ladder approaches the ground and

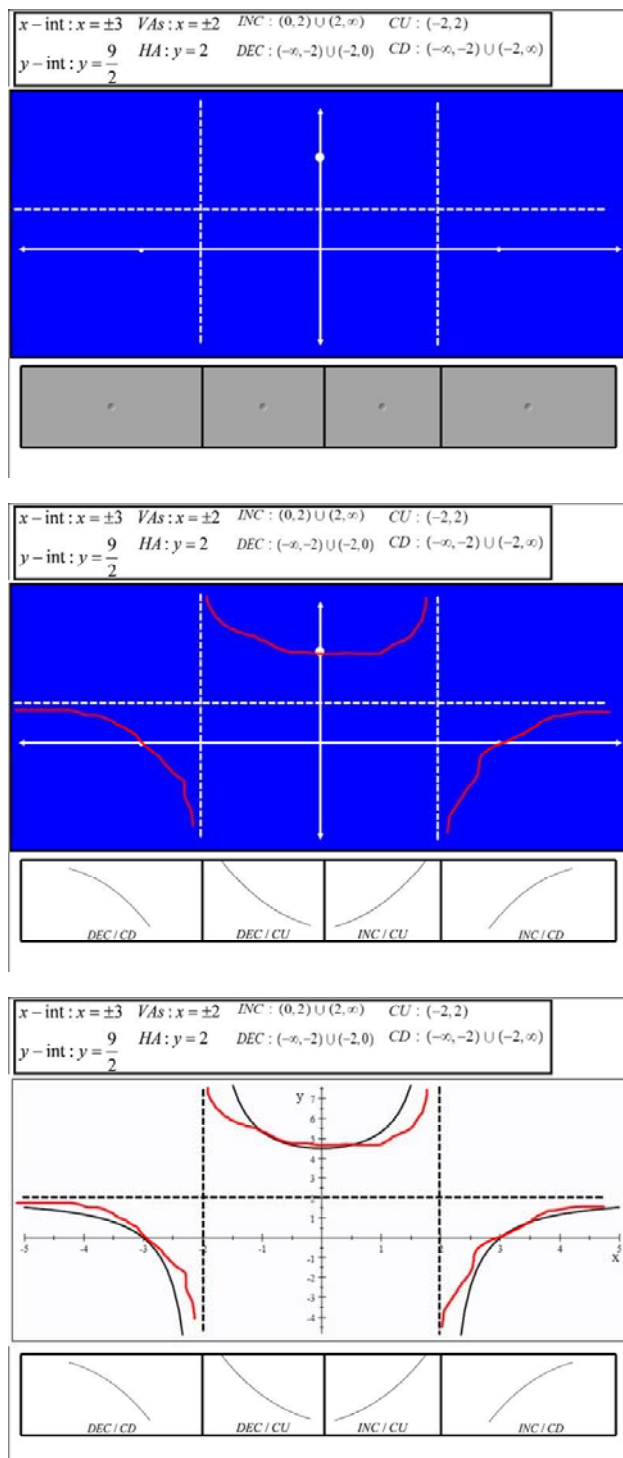


Figure 5. Custom reveals promote curve sketching and comparison to computer generated graph

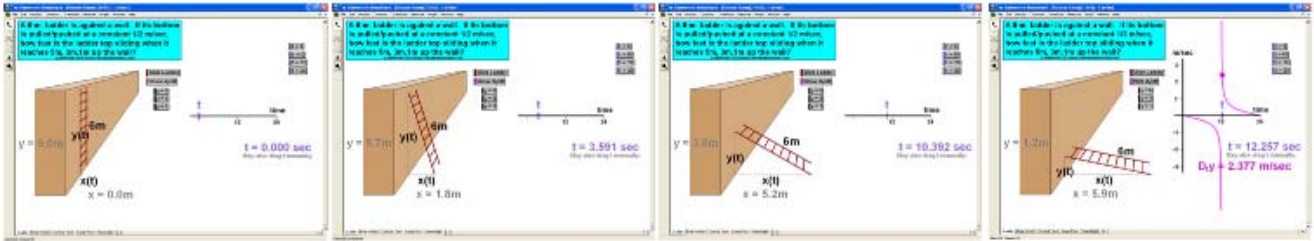


Figure 6. Animation depicting the Sliding Ladder Problem

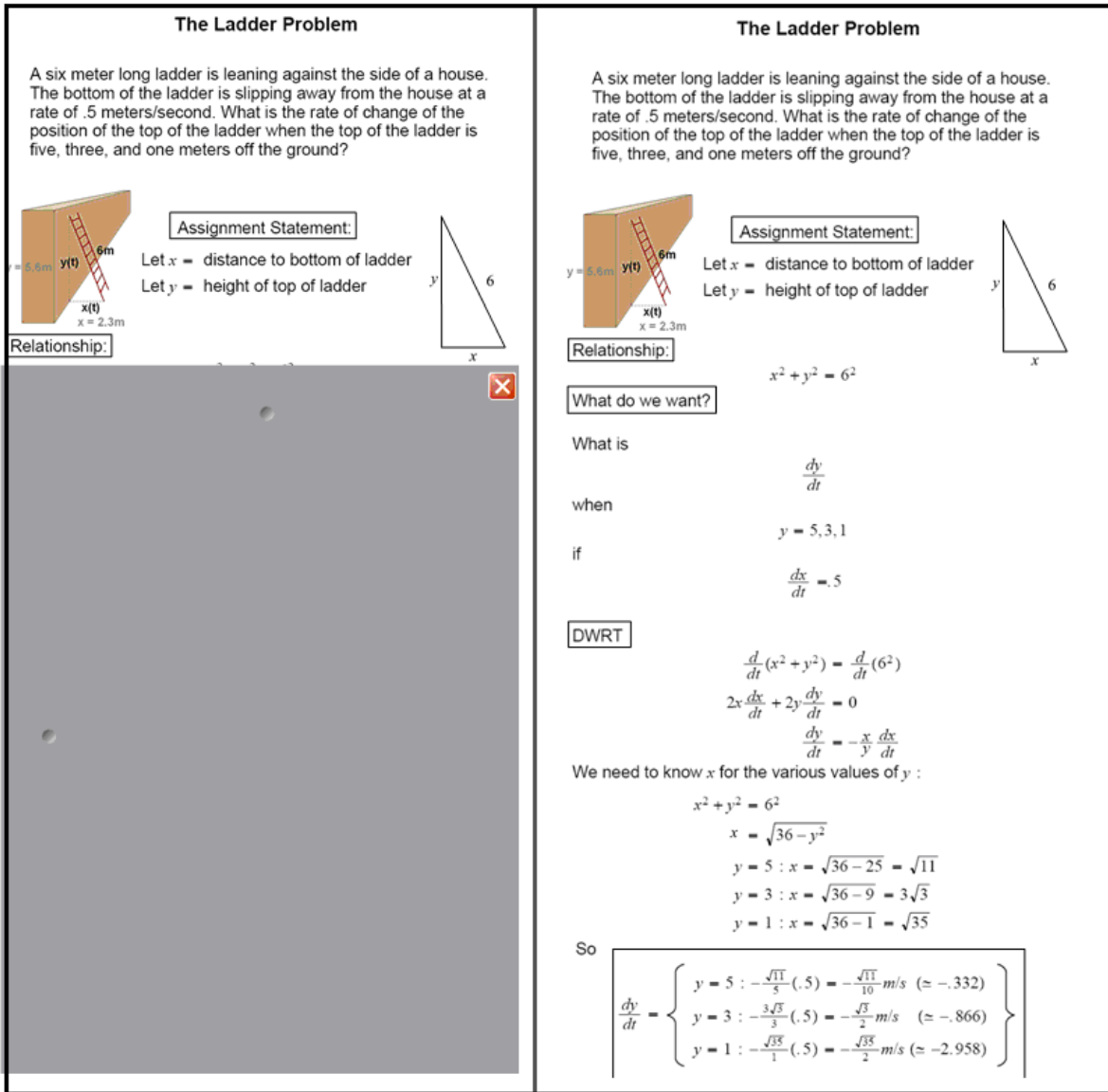


Figure 7. Solution to Sliding Ladder Problem, using the Screen Shade to reveal each step

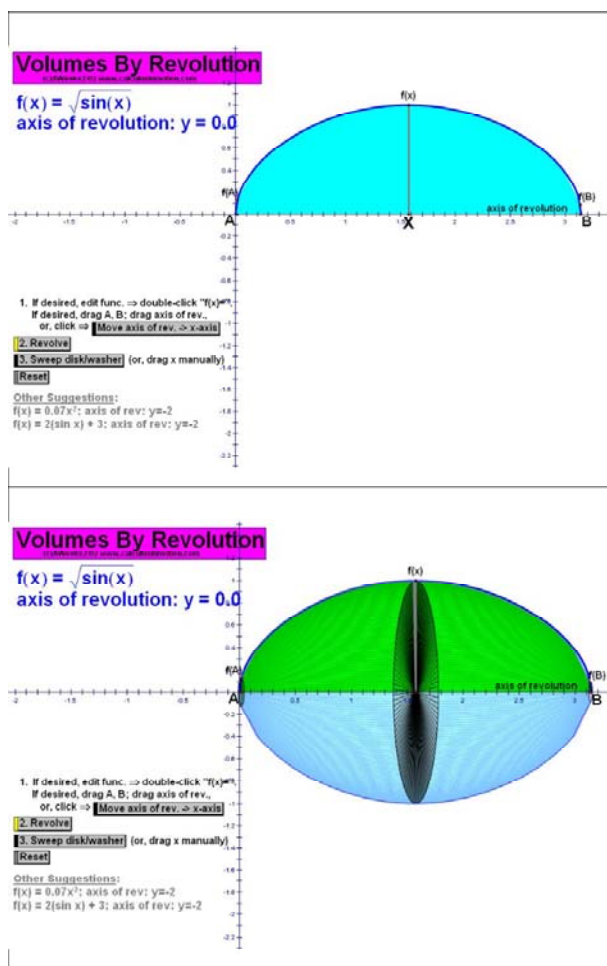


Figure 8. Animation showing differential disc formed by rotating function around x axis

its velocity increases without bound in the negative direction and then becomes unbounded in the positive direction after it hits the ground. (See Figure 6.) A good discussion about the nature of infinite discontinuities usually arises in this lesson, including the fidelity with which a mathematical model actually depicts a physical process.

Then the problem is worked out, using a standard SMART Notebook tool, the Screen Shade, revealing each step of the solution only as they are solicited from the students. (See Figure 7.)

Volumes of Revolution

Of all high school/college calculus problems, volumes of revolution are arguably the most difficult to visualize. Here again, integrating into SMART Notebook text, mathematics, and graphs from Scientific Notebook and animations from Calculus in Motion, one can produce an effective lesson. In this lesson, we are asked to find the volume of the solid formed by rotating one lobe of the

function $y = \sqrt{\sin(x)}$ around the x axis. By showing the animation first, the students are more easily able to grasp the image of the differential disc and work their way through to the final answer. (See Figure 8.)

The students are then led through to the solution, using the Screen Shade to reveal each step only as the students provide them. (See Figure 9.)

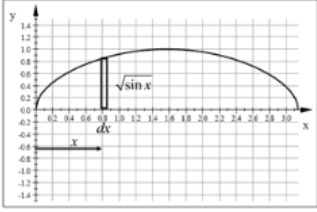
Concluding Remarks

We have presented just a few examples of how a variety of software packages can be integrated into SMART Notebook to create effective and engaging calculus lessons. These lessons have the following characteristics:

- Larger as opposed to smaller fonts and illustrations
- The use of animations as appropriate to demonstrate and illustrate difficult concepts
- The use of basic object tools in SMART Notebook to focus students' attention on a specific area on the screen

Interestingly enough, volumes of revolution are often easier to find in closed form than the corresponding area under the curve. For example:

Find the volume formed by revolving one lobe of the function $y = \sqrt{\sin x}$ around the x -axis

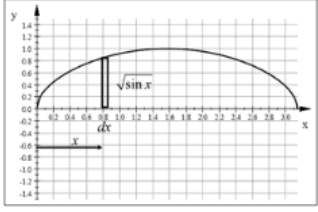


Finding the area under the curve would require numerical integration. But, finding the volume is simple:

1. dA :

Interestingly enough, volumes of revolution are often easier to find in closed form than the corresponding area under the curve. For example:

Find the volume formed by revolving one lobe of the function $y = \sqrt{\sin x}$ around the x -axis

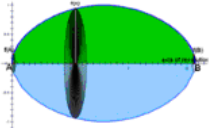


Finding the area under the curve would require numerical integration. But, finding the volume is simple:

1. dA :

$$dA = f(x)dx = \sqrt{\sin x} dx$$

2. dV :

$$dV = \pi [\sqrt{\sin x}]^2 dx = \pi \sin x dx$$


The region (limits) are obviously: $[0, \pi]$

So

$$V = \int dV = \pi \int_0^\pi \sin x dx = \pi(-\cos x)_0^\pi = \pi[(-\cos \pi) - (-\cos 0)] = \pi[(-(-1)) - (-1)] = \pi[(-(-1)) - (-1)] = 2\pi$$

Figure 9. Solution to Volume of Revolution Problem, using Screen Shade to reveal each step

- The use of reveals (Screen Shades, Table Cell Shades, and “fadeouts”) to pace a lesson and support an interactive dialog with students while exploring and problem solving

A similar approach can be adopted to design lessons in other areas of high school/college mathematics such as Precalculus, Discrete Mathematics, and Geometry. External software, if used creatively and discretely, can significantly enhance the quality of interactive whiteboard lessons and help teachers meet their goals for students. The readers are encouraged to explore opportunities for integrating external software into their own lessons, using the software described in this paper, or any other software that they may be familiar with and comfortable with.

As a cautionary note, the use of software in a lesson always should be dictated by the learning goals set for students. The instructors need to be very clear about what they are trying to accomplish using a specific pictorial representation or animation. Without a clear goal for students in mind, the incorporation of external software may turn into a flashy display of technical resources, without delivering any tangible benefits for students.

Endnotes

- ⁱ www.smarttech.com – SMART Technologies main company website
- ⁱⁱ www.exchange.smarttech.com – SMART Exchange. Repository of SMART lessons produced by SMART and by teachers
- ⁱⁱⁱ www.teacherspayteachers.com – Teachers Pay Teachers. An online marketplace for teacher developed resources, including SMART lessons
- ^{iv} www.legacyheritage.org/SJED/ – Legacy Heritage Fund SMART Board Project. An online repository for SMART lessons (General Studies and Judaics) produced in Jewish Day Schools
- ^v www.calculusinmotion.com – Calculus In Motion. Web site for Geometer's Sketchpad based Calculus and Algebra animations
- ^{vi} www.mackichan.com – MacKichan. Web site for publisher of Scientific Notebook software
- ^{vii} www.dynamicgeometry.com – Geometer's Sketchpad. Web site for GSP platform

Journal of Mathematics Education at Teachers College

Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Evaluation*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by January 21, 2012. The spring issue’s guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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