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**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Fall-Winter and 2013 Spring-Summer issues are *Equity* and *Leadership*, respectively.

*JMETC* readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

**Manuscript Submission**

*JMETC* seeks conversational manuscripts (2,500-3,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at [www.tc.edu/jmetc](http://www.tc.edu/jmetc).

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at [JMETS@tc.columbia.edu](mailto:JMETS@tc.columbia.edu).

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## *Journal of Mathematics Education at Teachers College*

### **Call for Papers**

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Equity*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3500 words describing research, experiments, projects, innovations, or practices related to equity in mathematics education. Articles should be submitted to Ms. Krystle Hecker at [JMETC@tc.columbia.edu](mailto:JMETC@tc.columbia.edu) by September 1, 2012. The fall issue’s guest editor, Mr. Nathan N. Alexander, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2012, and final manuscripts of selected papers are to be submitted by October 15, 2012. Publication is expected by November 15, 2012.

### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at [hecker@tc.edu](mailto:hecker@tc.edu) or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### **Looking Ahead**

Anticipated themes for future issues are:

Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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# Journal of Mathematics Education at Teachers College

Spring – Summer 2012

A CENTURY OF LEADERSHIP IN  
MATHEMATICS AND ITS TEACHING

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## ASSESSMENT NOTES FROM THE FIELD

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### Will the CCSSM Have Staying Power?

Matthew R. Larson

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The past quarter-century has witnessed several mathematics standard reform efforts. For the *Common Core State Standards for Mathematics* to result in significant and lasting change, and not simply become another footnote in the history of mathematics education, the implementation of the *Common Core State Standards for Mathematics* will need to address five fundamental paradigm shifts.

*Keywords:* CCSSM, national standards initiatives, paradigm shifts.

#### A Brief History of Mathematics Standards

The last quarter-century has witnessed several mathematics standards reform efforts. In 1989, NCTM initiated the national standards movement with the release of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). Followed by the release of *Principles and Standards for School Mathematics* [PSSM] in 2000, these two standards documents introduced the importance of mathematical processes (called *mathematical practices* in the CCSSM) in addition to content strands (called *domains* in the CCSSM), and significantly influenced the development of states' mathematics standards and frameworks in the 1990s and the first two decades of the 21st century. In 2006, NCTM released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, which identified areas for grade level focus, or focal points, in the standards. The focal points are, with some modification, essentially the critical areas for focus in the CCSSM.

A decade after the release of PSSM, the Council of Chief State School Officers and the National Governors Association released the *Common Core State Standards for Mathematics* (NGACBP & CCSSO, 2010) and as of this writing the CCSSM has been adopted by 45 states. It is worth pausing to ask the question whether the CCSSM will still be the focus of mathematics education a decade from now, or whether it will become another historical footnote in the list of mathematics standards documents that preceded it? And more important, what will it take for the CCSSM to have staying power and significantly influence mathematics education and improve student learning?

#### Making the CCSSM Last

There is evidence that previous standards and frameworks implementations have had an overall positive impact on student achievement in the United States. For example, in 2011 fourth graders posted their highest score to date on the NAEP (NCES, 2011). While all disaggregated groups of students have generally shown steady improvement on the NAEP over the past two decades, significant achievement differentials persist. For example, the 20 point score differential between white and Hispanic students at the fourth grade level in 2011 remained the same as it was in 1990. The potential for the *Common Core State Standards* to reduce these achievement differentials has recently been called into question because most of the variation in achievement on NAEP occurs within states and not between states, as each state already has its own specific common state standards (Loveless, 2012).

Successful implementation of the CCSSM, where success is defined as improving the achievement of *all* students while simultaneously reducing current achievement differentials, lies in addressing five paradigm shifts in fundamental beliefs and behaviors about the teaching and learning of mathematics (Kanold & Larson, in press)—paradigm shifts that have not all been addressed in previous standards reform efforts.

Implementation of previous mathematics standards was limited by the notion of invitation. With the exception of implementing content standards, which in too many cases devolved into mere correlation mappings and the posting of standards in classrooms, school districts were *invited* to implement research-based instructional practices, assessment processes, effective professional

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development structures to support teachers, and mathematics intervention programs to support all students in meeting the standards. The authors of the CCSSM wrote,

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep (NGACBP & CCSSO, 2010, p. 5).

Implementation of the CCSSM presents an unprecedented opportunity to change the system of mathematics education truly—to focus on second order change, change that requires working outside of existing paradigms by embracing new paradigms of thought and work (Waters, Marzano, & McNulty, 2003). The five paradigm shifts required for successful implementation of the CCSSM include (Kanold & Larson, in press):

- Mathematics Curriculum
- Mathematics Instruction
- Mathematics Assessment
- Mathematics Intervention
- Professional Development

### *Mathematics Curriculum*

The CCSSM requires a paradigm shift to a “less is more” orientation with respect to curriculum. Specifically, it requires that fewer standards be taught at a higher level of cognitive demand at every grade level than currently characterizes most state standards documents (Porter, McMaken, Hwang, & Yang, 2011). The focused nature of the CCSSM, and the careful attention paid to students’ developmental learning progressions (Confrey, Maloney, & Nguyen, 2010), means that some of the topics traditionally taught in certain grades have been moved to other grades, and some topics have simply been eliminated from the curriculum. For example, the CCSSM emphasizes fractions beginning at the third-grade level and delays probability until the middle grades. Traditionally, both of these topics were introduced in the primary grades and remained topics in each elementary school grade. The purpose of this more focused curriculum is to provide time to teach fewer critical topics in greater depth. Failure to follow the grade-level content standards will not provide the instructional space necessary to teach for depth of student understanding.

### *Mathematics Instruction*

The implementation of the CCSSM will require a paradigm shift to lesson designs that embed the Standards for Mathematical Practice into teachers’ daily lesson plans

so that students develop not only procedural fluency, but also deep conceptual understanding of the content, and the ability to draw on both to persevere and solve problems. The CCSSM calls for a distinctive, in some cases significantly different, way of approaching the content, embodied in part by the Standards for Mathematical Practice. “The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGACBP & CCSSO, 2010, p. 6). Ultimately any improvement in student learning will have to emphasize a strategy designed to improve instruction (Noguera, 2004; Schmoker, 2006). Implementation of the CCSSM presents an opportunity to broaden this standards reform effort beyond mere content correlation mappings to include this critical and necessary focus on instruction that seeks to develop student understanding and engage students in the Standards for Mathematical Practice.

### *Mathematics Assessment*

Implementation of the CCSSM will require a paradigm shift toward formative assessment as a multifaceted and continuous process to guide instruction and support students, and away from reliance on periodic benchmarking assessments instruments and a narrow focus on summative assessments. Much of the assessment focus during the implementation of the CCSSM will likely be on preparation for one of the two new national assessments being designed for implementation during 2014–2015 (Program for the Assessment of Readiness for College and Careers [PARCC] and Smarter Balanced Assessment Consortia [SBAC]). Both assessment consortia are designing interim assessments to provide data during the academic year to inform instruction, interventions, and professional development. Over-reliance on “benchmarking” assessments as a strategy to increase student performance is questionable. There is little research on the effectiveness of interim assessment to improve student achievement, and the research that has been conducted does not demonstrate significant improvement in student achievement (Goertz, Olah, & Riggan, 2009; Henderson, Petrosino, Guckenber, & Hamilton, 2007). To be effective as a tool to improve student achievement, formative assessment can no longer be viewed simply as a periodic benchmarking instrument, but rather as “a process that is fundamental and indigenous to the practice of teaching and learning” (Heritage, 2010, p. 1). This formative process allows assessment to serve as a “means” and not an “ends” to teaching and learning.

### *Mathematics Intervention*

Implementation of the CCSSM requires a paradigm shift toward grade level or subject-based teams of teachers implementing *required* responses to intervention for all

## ASSESSMENT NOTES FROM THE FIELD

students. Response to Intervention (RtI) can no longer be invitational as it currently is in too many schools. Research indicates that formative assessment processes, if used to guide *targeted additional instruction*, can support students in achieving grade level standards (Baker, Gersten, & Lee, 2002; NMAP, 2008; Popham, 2008; Wiliam, 2007; Wiliam & Thompson, 2007). For all students to acquire the more rigorous CCSSM, the time devoted to mathematics instruction, and the level and intensity of support, must become more variable in school than they typically are. Acknowledging this reality and prioritizing instructional time within the school day to provide all students with the instructional support they require will be a critical test of school leadership.

### *Professional Development*

Implementation of the CCSSM requires a paradigm shift to move the grain size of professional development beyond the individual teacher working in isolation, who receives periodic one-shot professional development, to teachers working in grade level or course-based collaborative learning teams. Working within collaborative teams professional development becomes a continuous process, not an event, focused on developing teachers' understanding of the CCSSM, collaborative development of lessons that embed and evaluate the effectiveness of the Standards for Mathematical Practice in their daily lessons, the development of collaboratively designed and scored assessments, and a collective sense of responsibility for students when they have not mastered content and a collaboratively designed response to support students until they meet the standards.

### Looking Ahead

The implementation of the *Common Core State Standards for Mathematics* presents the nation with the opportunity to press the "reset button" on mathematics education and rededicate itself to offering all students high quality mathematics instruction and the support necessary to guarantee all students have the opportunity to acquire the mathematics outlined in the CCSSM (Larson, 2011). Standards reform efforts to date have made progress toward this vision, but have fallen short of making this vision a reality for all students. In order to turn vision into reality, the implementation efforts surrounding the CCSSM must address five fundamental paradigm shifts, including instruction, assessment, intervention, and professional development, in addition to curriculum, which has been the near exclusive focus of previous standards reform efforts. If all five paradigm shifts are addressed, then the CCSSM is likely to remain a significant force in mathematics education into the foreseeable future and positively impact the learning of *all* students.

### Acknowledgements

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### Notes

This paper is based in part on a colloquium presentation made by the author at Teachers College, December 2011, and the forthcoming book, *Common Core Mathematics in a PLC at Work<sup>TM</sup>: Leader's Guide*, by Timothy D. Kanold (tkanold.blogspot.com) and Matthew R. Larson, published jointly by Solution Tree Press and NCTM.

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### Using Item Analysis Data as a Tool to Inform Instruction in the Mathematics Classroom: A Model of Data-Driven Instruction

William Farber  
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This article presents a method of using assessment data to help increase student learning and promote a variety of instructional practices in mathematics. The specific model being introduced links the assessment data from the New York State testing program in mathematics to instructional strategies applied in the mathematics classroom. Moreover, this model incorporates the application of an item analysis of the New York State Grades 3–8 Mathematics tests. This item analysis, developed by the New York City Department of Education, provides test data information which will help inform instruction by connecting assessment data to mathematics instructional approaches.

*Keywords:* item analysis, differentiated instruction, assessment, test-taking skills, exemplar, performance indicator, tiered learning.

#### The Need to Examine the Connections Between Instruction and Assessment

Recent declines in scores on tests of achievement in mathematics have become a focus of steadily increasing concern to mathematics educators. In fact, according to The University of the State of New York Office of Communications, “Nationally, about a third of fourth and eighth graders met the cutoff to be assessed as ‘proficient’ in math this year. New York’s students performed a little lower than that” (NYSESED, 2011). The achievement tests in

mathematics are deemed as “high-stakes” tests and generally consist of traditional paper-and-pencil assessments (McMillan, 2011, p.172). These types of assessments have been used for decades for measuring student achievement. In addition, school districts across the United States that use a “high-stakes” testing program have recently changed focus from monitoring student achievement to an accountability system, i.e., monitoring accountability of students, teachers, and school and district supervisors. In the past twenty years, a multitude of reform efforts connecting assessment data to instruction have taken place, which includes professional development for