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Hyper-acceleration of Algebra I: Diminishing Opportunities to Learn in Secondary Mathematics

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ABSTRACT An increasing number of students are *hyper-accelerating* their study of formal Algebra I to Grade 7 or earlier to maintain a competitive edge in the race to calculus. However, there is a lack of evidence that these students construct the conceptual foundations necessary for persistence in advanced mathematics. This paper maps the drive toward hyper-acceleration to the historical underpinnings of acceleration of Algebra I to Grade 8. We illuminate how acceleration can detract from opportunities for middle school students to engage in algebraic reasoning in preparation for advanced mathematics. We further describe how this pathway exacerbates persistent inequities in secondary mathematics education. Our synthesis of the literature on Algebra I acceleration, readiness for STEM undergraduate study, and equitable access is the basis for our argument for more research on hyper-acceleration.

KEYWORDS *algebra, calculus, hyper-acceleration, opportunity to learn, equity*

Students who take Algebra I in Grade 8 have more opportunities to take calculus in high school. The presumption is that they will have more access to rigorous mathematics experiences and more success in science, technology, engineering, and mathematics (STEM) in college. At the same time, *hyper-acceleration* of Algebra I to Grade 7 or earlier has emerged as a form of curricular intensification in secondary mathematics (Domina & Saldana, 2012). This further acceleration is in response to parent and administrator pressures for students to take calculus in their junior year of high school to improve competitiveness for college admissions (Bressoud, 2017). Hyper-acceleration is consistent with Lucas' (2001) theory of effectively maintained inequality in which "advantaged actors secure for themselves and their children some degree of advantage wherever advantages are commonly possible" (p. 1652). As an increasing number of students have access to Grade 8 Algebra I, privileged stakeholders will pursue further Algebra I acceleration as an educational advantage.

While school communities may idealize hyper-acceleration as an indicator of smartness and status, its implications for mathematics learning are frequently questioned in education blogs and editorials. For example, Kaplinsky (2017) and Pemantle (2016) challenged acceleration policies that compact middle school standards and diminish opportunities to learn foundational content for advanced mathematics. Picciotto (2014) argued that hyper-acceleration is motivated by the belief that "kids from certain families are just better at math and deserve the various advantages supposedly conferred by being 'ahead'" (para. 3). This pathway is inconsistent, however, with expert recommendations about the judicious acceleration of algebra and deep mathematics learning. The National Council of Teachers of Mathematics (2016) cautioned that students who are talented or express strong interests in mathematics should not rush through critical concepts. Sheffield (2017) argued similarly that the appropriateness of acceleration of secondary mathematics by more than one year for gifted students is a "dangerous myth" (p. 21) and "not beneficial for a majority of top students" (p. 22).

Despite these admonitions, the pervasive drive to complete more Advanced Placement mathematics courses makes it unlikely that students, parents, and teachers will support a reversal of this trend toward taking Algebra I at younger ages (Bressoud, 2020; NCTM, 2018). In addition to enrolling in higher course levels, students on hyper-accelerated Algebra I pathways are presumed to have access to more rigorous content and more qualified teachers. According to Tate (2004), quality of instructional delivery, content emphasis, content exposure, and coverage are measures of opportunity to learn (OTL) in mathematics. Using these metrics, hyper-accelerated students may appear to have a greater OTL because they enroll in and complete more secondary mathematics courses. However, lost opportunities for meaningful mathematical reasoning can be hidden behind the presumed advantages of hyper-acceleration. Hyper-accelerated students may experience mathematics as a competitive hierarchy instead of as a creative sense-making endeavor (Galanti, 2021).

Because the empirical research on hyper-acceleration is limited, we synthesize literature on Grade 8 Algebra I and readiness for STEM undergraduate study to contextualize this phenomenon. We also advocate for additional research on OTL with hyper-acceleration at both individual and systemic levels. In what follows, we first present a brief history of Algebra I acceleration. Then, we discuss the evidence that hyper-acceleration may not adequately prepare middle school students for advanced mathematics. Next, we argue that acceleration can detract from opportunities for students to engage in algebraic reasoning as a sense-making endeavor in the early grades. Finally, we describe how this pathway exacerbates persistent inequities in access to high-quality secondary mathematics education.

History of Algebra I Acceleration

Algebra I in Grade 8 or Earlier

Over the last 30 years, access to Algebra I in Grade 8 has increased with the goal of improved student achievement and college readiness (Clotfelter et al., 2015). The enrollment of U.S. students in Algebra I in middle school grew from 16% in 1990 to 47% in 2011 (Loveless, 2013). We argue that this increase explains the sociological motivations for hyper-acceleration. These motivations are also at the root of issues of equity and access in secondary mathematics education. Before the early 1990's, access to Algebra I in Grade 8 was reserved for a small percentage of students who demonstrated strong aptitude in pre-algebra. Moses (1995), however, challenged mathematics education researchers at the Algebra Initiative Colloquium to view algebra as the new civil right for students from all backgrounds. Participants debated the assumed challenge of reforming curriculum and pedagogy with this increased access. They wondered whether "algebra for all" would lessen the rigor of Algebra I content for an increasingly diverse community of learners. Lacampagne (1995) remarks:

> A question that plagued Colloquium participants was, "How do we ensure that 'algebra for all' is not 'dumbing down' algebra?" The mathematical community as well as parents of college-bound students will and should demand sound preparation in algebra for the college bound. We will be faced with building an algebra curriculum and pedagogy that will support the needs of all students. (p. 4)

This statement suggests that broader access to Algebra I could threaten the privileged role of formal mathematics education in identifying the elite students of the future. These concerns foreshadowed the emergence of hyper-acceleration of Algebra I to earlier grades.

Empirical studies broadly define Algebra I acceleration for Grade 8 or earlier. There is no direct focus on hyper-acceleration; however, inferences about the expansion of Algebra I in Grade 7 can be made from a series of relevant studies. Many school divisions offer Algebra I, Geometry, and Algebra II in a linear sequence between Grades 7 and 9. Domina et al. (2016) found that between 2003 and 2013, the percentage of students enrolled in Grade 8 Geometry in California more than tripled from 2% to 7% in parallel with "algebra for all" initiatives. This indicates that the number of students taking Algebra I in Grade 7 increased as more students gained access to Algebra I in Grade 8. Moreover, our analysis of course-taking data from the Characteristics of Successful Programs in College Calculus (CSPCC) study (Mathematical Association of America [MAA], 2017) shows that 12.8% of freshman-level calculus students had completed both Geometry and Algebra II by the end of Grade 9. Based on this finding, we inferred that these college freshmen took Algebra I in Grade 7.

Algebra I Acceleration Outcomes

The acceleration of Algebra I to Grade 8 has a history of mixed outcomes for students. For instance, accelerated Algebra I has been associated with higher standardized test performance, grades, and college enrollment (Gamoran & Hannigan, 2000; Smith, 1996; Spielhagen, 2006; Stein et al., 2011). However, other factors such as gifted status, school context, school attendance, and parents' education levels contributed substantially to differential outcomes in achievement between non-accelerated and accelerated students (Rickles, 2013). Furthermore, universal Grade 8 Algebra I policies in states like California and North Carolina had adverse effects (Clotfelter et al., 2015; Domina et al., 2015; Finkelstein et al., 2013; Remillard et al., 2017). For example, Clotfelter and colleagues (2015) reported that acceleration has "statistically significant harmful effects" (p. 180) based on end-of-course test scores in Algebra I, Geometry, and Algebra II. Adverse effects were reported for up to the 60th percentile of the Grade 6 and 7 mathematics achievement distribution. Broad Algebra I acceleration has also resulted in students repeating mathematics courses (Finkelstein et al., 2014; Lee & Mao, 2020) and students exiting the mathematics pipeline before their senior year of high school (Finkelstein et al., 2014). Our descriptive analysis of transcript data presented in the Finkelstein et al. (2014) study revealed that nearly half of the students who studied Algebra I in Grade 7 repeated the course in Grade 8.

These findings challenge the assumptions that Algebra I acceleration equalizes outcomes and increases access to advanced mathematics courses. They also suggest that such outcomes could be worsened by further accelerating Algebra I for students who have not yet built strong pre-algebraic foundations. Hyper-acceleration may not be fostering the mathematical understanding and confidence that many students need to be successful on advanced secondary mathematics pathways.

Early Algebra vs. Algebra Early

The shift of Algebra I to the middle grades is part of a continuing conversation in mathematics education about early algebra. Contextual sense-making grounded in algebraic reasoning and generalization from arithmetic is often referred to as early algebra (Carraher et al., 2006; Stephens et al., 2017). Kaput (2008) argued that early algebra could "democratize access to powerful ideas by transforming algebra from an inadvertent engine of inequity to a deliberate engine of mathematical power" (p. 6). Yet experts in this field of study distinguish early algebra from Algebra I studied in the early grades. Early algebra emphasizes background contexts in problems while gradually introducing formal notation (Carraher et al., 2008; Mason, 2017). These experiences are fundamental to understanding abstraction and structure within Algebra I as variables and symbolic notation are formally introduced (Driscoll, 1999). The construction of deeper understandings of rational numbers and proportional reasoning in the middle grades before formal algebra courses at the high school level is also crucial (Sheffield, 2017). Taking Algebra I in Grade 8 or earlier may adversely impact students' productive experiences with early algebra and their conceptions of ratio and proportion. Because of societal beliefs that early Algebra I provides a competitive advantage in college admissions, many students may accelerate toward advanced mathematics at the expense of meaningful algebraic reasoning in the elementary and middle grades.

Accelerated Algebra I and STEM Readiness

A common rationale for Algebra I acceleration is the perception of improved readiness for undergraduate STEM study. Taking Algebra I in Grade 8 has also been associated with taking calculus in high school and increasing the likelihood of STEM undergraduate study (Lee & Mao, 2020; Rickles, 2013). Many students believe that they will be at a disadvantage in college if they have not studied calculus in high school (Bressoud, 2020). Despite the increase in the number of students who are accelerating their study of Algebra I as a pathway to high school calculus, research has shown that many high school graduates are not prepared to succeed in undergraduate mathematics. Only 20% of the 2019 high school graduating class met the American College Testing (ACT) STEM readiness benchmark (ACT, 2019). This benchmark is derived from both mathematics and science subscores and college student performance data. MAA and NCTM warned that the increasing acceleration of traditional secondary mathematics courses is not only ineffective but counterproductive in building foundational mathematical knowledge for a STEM career (Bressoud et al., 2012). According to Stewart and Reeder (2017), many students struggle in college-level mathematics because of incomplete or insecure understandings of algebraic topics situated within middle school and high school curricula.

Multiple studies quantify the impact of Algebra I acceleration when students do not build the requisite confidence and conceptual understandings necessary for success in college STEM courses. Sadler and Sonnert (2018) reported that end-of-course grades in high school precalculus courses along with SAT/ACT scores explained more than twice the variability in college calculus performance than grades in high school calculus courses. The National Council for Education Statistics (2013) reported, meanwhile, that 13.5% of students who completed high school calculus enrolled in remedial mathematics in college. Additionally, the Factors Influencing Calculus Success in Mathematics (FICSMath) study showed that 20% of calculus students who first enrolled in a college precalculus course had already completed high school calculus (Sonnert & Sadler, 2014). In the CSPCC study of over 14,000 students who enrolled in an entry-level calculus course required for STEM majors, 67% had studied calculus in high school. 36% of students who earned a three or higher on their AP Calculus exams earned a "C" or lower in college calculus. The grade distribution for students who had earned less than three on their AP Calculus exams was comparable to that of students who did not take calculus in high school (Bressoud, 2015). The FICSMath and CSPCC studies did not segregate the data on Algebra I course-taking by grade level; however, the findings should raise questions about the consequences of hyper-acceleration and the loss of not one but two years to build foundational mathematical understandings.

Algebra I Acceleration as a Matter of Equity

The negative consequences of Algebra I in Grade 8 or earlier for individual students must also be accompanied by a broader critique of acceleration as a form of tracking that exacerbates systemic inequities in middle school mathematics. Schmidt (2009) examined OTL as content coverage in Grade 8 mathematics courses using teacher survey data on the Trends in International Mathematics and Science Study (TIMSS). He attributed 40% of the variation in mathematics achievement to differences in content coverage as a result of tracking. Stein and colleagues (2011) analyzed both universal and selective Algebra I acceleration policies. They suggested that students who would have been excluded from algebra under the "old rules" might be grouped and taught a less rigorous version of algebra. Open-enrollment policies led many middle schools to create multiple algebra courses. More recently, analysis of OTL using student reports of mathematical content coverage within the 2012 Programme for International Student Assessment (PISA) data showed that the greatest differences in OTL in the United States occurred *within* schools and not *between* schools (Schmidt et al., 2015). These persistent educational inequities affirm Schmidt's (2009) earlier study of OTL in Grade 8 mathematics in which he concluded that only the highest-achieving students benefit in tracked schools. As we look across these findings, we can infer that hyper-acceleration intensifies the stratification of middle school mathematics courses and lowers content expectations for students who struggle.

Hyper-acceleration has moved the historical gatekeeping role of Algebra I to an even younger age with no empirical justification. This has contributed to further stratification along race and class lines, as narrow constructions of mathematics ability and achievement are often defined by race and class (Boaler, 1997; Boaler & Greeno, 2000; Gutiérrez, 2012; Louie, 2017). These biases continue to be reflected in the overrepresentation of White and Asian students in accelerated Algebra I classrooms (Education Trust, 2020; Grissom & Redding, 2016). Stinson (2004) asked, "How might mathematics educators ensure that gatekeeping mathematics becomes an inclusive instrument for empowerment rather than an exclusive instrument for stratification?" (p. 8). We must interrogate hyper-acceleration as a structure that undermines student learning and exacerbates the underrepresentation of other racial and ethnic groups in accelerated secondary mathematics (Irizarry, 2020; McCallum & Novak, 2020; Morton & Riegle-Crumb, 2020). Hyper-acceleration diminishes meaningful learning when mathematical success is defined by faster course completion. Secondary mathematics experiences should instead foster creative problem solving and persistence in mathematics for all students.

Directions for Future Research

Research on the acceleration of Algebra I is often framed in terms of increased access to rigorous mathematics in high school. However, Algebra I acceleration as policy cannot be separated from the equally important obligation to build a foundation for continued course taking and productive dispositions in mathematics. There are empirical contradictions between studies that relate improved secondary outcomes with the increased acceleration of Algebra I and studies that challenge acceleration as detrimental to building strong precalculus understandings. The mathematics education community needs to unpack these contradictions as we strive for more heterogeneous learning environments and position more students to succeed in advanced mathematics courses. Ideally, hyper-acceleration should motivate and empower students with mathematical talent, creativity, and passion irrespective of race, class, or economic status. Instead, it introduces the potential to devalue early algebra experiences and reify existing societal power structures.

Our synthesis of research on Grade 8 Algebra allows us to make inferences about the risks of hyper-acceleration. Still, there is a need for further research to understand this phenomenon. It is concerning that the presumed educational advantage of offering hyper-accelerated Algebra I within a school might become more important than increasing opportunities for meaningful algebraic reasoning for every student. Large-scale research using test scores, course completion, and advancement toward college can hide the individual and systemic implications of hyper-acceleration. This research must be accompanied by an examination of the quality of learning within these contexts. The increasing stratification of secondary mathematics courses demands critical questions about who has access to this social capital and how student identities and backgrounds predict participation in these courses.

The following research questions can quantify structural disparities and student outcomes from a content perspective on OTL:

- How are students identified and selected for hyper-accelerated Algebra I?
- How does hyper-acceleration relate to further mathematics course taking and undergraduate STEM participation?

There is also a need to look beyond traditional OTL metrics of content coverage and delivery of instruction. A situative perspective on OTL (Greeno & Gresalfi, 2008) captures the complexities of interactions amongst students, parents, teachers, administrators, and curriculum. OTL can thus be described as a longitudinal trajectory of mathematics participation with a past, a present, and a future. This situative perspective can illuminate how differential social status associated with hyper-acceleration relates to classroom participation, identities of competence, and persistence in mathematics. The following additional research questions can elicit the contextualized and affective aspects of hyper-acceleration from a situative perspective on OTL:

- How do community stakeholders (e.g., students, parents, teachers, and administrators) describe the appropriate acceleration of Algebra I?
- What opportunities do students have to make sense of algebraic content in hyper-accelerated Algebra I classrooms?
- How does hyper-acceleration relate to a student's evolving sense of mathematical competence?

Conclusion

In this paper, we presented literature and data about the acceleration of Algebra I and the related adverse outcomes. The contradictions raised in this article should motivate the empirical investigation of hyper-acceleration as both policy and practice. Building on these contradictions, we offered potential paths for future research. Suppose we continue to encourage hyper-acceleration as a social marker of distinction, without evidence of its individual and systemic impacts. In that case, we will lose ground on building a more diverse community of mathematically promising students (NCTM, 2016). It is time for the field to engage in a scholarly examination of hyper-acceleration related to conceptual understanding, persistence in advanced mathematics, and more equitable ideas about what it means to be "good" at mathematics.

Questions about hyper-acceleration are timely, and they relate to opportunities for all students to engage in rigorous secondary mathematics. These questions will become more critical as we emerge from the coronavirus pandemic and its legacy of differential access to high-quality mathematics teaching and learning. The June 2020 joint position statement from NCTM and the National Council of Supervisors of Mathematics (NCSM), *Moving Forward: Mathematics Learning in the Era of COVID-19*, reiterated earlier calls for detracking in the form of heterogeneous groupings in middle school mathematics classrooms (NCTM, 2018). These detracking efforts have become even more crucial in the COVID-19 era as school closures, absenteeism, and unequal access to technology exacerbate long-standing inequities and biases in assessing mathematical readiness (NCTM & NCSM, 2020; TODOS: Mathematics for All, 2020). Efforts to challenge and dismantle tracking (Berry, 2018; NCSM, 2019) cannot be successful without a critical examination of hyper-acceleration.

The perception that faster is better will continue to drive the political discourse in high-achieving school districts in the absence of new knowledge about the unintended consequences of hyper-acceleration. By arguably narrow processes for identifying students, hyper-acceleration can create structural barriers to learning not only for those who struggle within these new tracks but also for those who operate outside of these tracks. School stakeholders need empirical evidence to make informed decisions when faced with community pressures for this further acceleration of Algebra I. If offering increasingly advanced mathematics courses in middle school remains grounded in the desire of students, parents, and teachers to gain a competitive advantage in college admissions, we will see the continued growth in this phenomenon and, in turn, the growth of structural inequality in mathematics education. As we build an empirical basis for hyper-acceleration, we will move our field forward in new ways that inspire more meaningful participation in advanced mathematics for all students.

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