The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

Dr. Elizabeth Hagan was discharged from the US Navy as a Lieutenant in 1946 and completed the PhD in Measurement and Evaluation at Teachers College in 1952. Her collaboration with Robert Thorndike resulted in many papers and books including the influential 1961 John Wiley publication, *Measurement and Evaluation in Psychology and Education*. Dr. Hagan concluded her career at Teachers College in 1976 as Acting Dean of Academic Affairs.

Dr. Stuart Weinberg was the Mathematics Department Chairman at Stuyvesant High School before joining the Teachers College faculty as Director of Student Teaching for the Program in Mathematics. Dr. Weinberg has applied his extensive classroom experience to the development of methods of assessing teachers' classroom performance utilizing belief systems and attitudes.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Fall-Winter and 2013 Spring-Summer issues are *Equity* and *Leadership*, respectively.

JMETC readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu.

Copyrights and Permissions

Library of Congress Cataloging-in-Publication Data

Journal of mathematics education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and teaching—United States—Periodicals
QA11.A1 J963

More Information is available online: www.tc.edu/jmetc

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Equity*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3500 words describing research, experiments, projects, innovations, or practices related to equity in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2012. The fall issue's guest editor, Mr. Nathan N. Alexander, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2012, and final manuscripts of selected papers are to be submitted by October 15, 2012. Publication is expected by November 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

TO OBTAIN COPIES OF *JMETC*

To obtain additional copies of *JMETC*, please visit the *Journal*'s website www.tc.edu/jmetc. The cost per copy delivered nationally by first class mail is $5.00. Payment should be sent by check to *JMETC*, Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Journal of Mathematics Education at Teachers College

Spring – Summer 2012

A Century of Leadership in Mathematics and its Teaching

TABLE OF CONTENTS

Preface

Articles

Assessment Notes from the Field

TABLE OF CONTENTS, continued

Journal of Mathematics Education at Teachers College
Spring–Summer 2012, Volume 3

The Russian Uniform State Examination in Mathematics: The Latest Version

Albina Marushina
Teachers College Columbia University

This paper aims to tell how the Russian national examination in mathematics (the Uniform State Examination or USE) has been conducted most recently. The author must say at once that the history of the system of secondary school graduation examinations or even the history of the USE will be covered only to the small degree that is necessary for understanding the current situation (more information can be found in Karp & Zvavich, 2011; Karp, 2007; Karp, 2003; these papers also served as a resource for writing the introductory sections of this paper). The goal of this paper looks modest, particularly given that the format of the USE obviously is subject to change; the USE, however, is so important for the Russian mathematics education, and the current situation is so typical of the challenges that are confronted in conducting assessment on the national level, that this situation deserves to be analyzed in considerable detail.

Keywords: Russian education, national examinations, graduation examinations, mathematics assessment items.

Secondary School Graduation Examinations in the USSR and Russia

There has never been a national examination in the United States and introducing such an examination would hardly be possible without a major change in the laws and (probably more importantly) in national psychology. The case is very different in the USSR, where such an examination emerged in the 1930s, when fundamental counter-reforms in education were conducted. These counter-reforms cancelled all of the innovations that had appeared after the Revolution of 1917 and effectively, at least partly, brought the country's education back to pre-Revolutionary approaches (Karp, 2010). At first, the problems for this examination were written in each of the country's regions, but starting in the mid-1940s, all problems began to be written in Moscow for the entire country. This national examination for graduating students co-existed with entrance examinations to institutions of higher education. These entrance examinations were conducted by each institution separately (although each of them had to report the results of the examinations to the Ministry to which it belonged).

Typically, there was a small number of problems—4, 5, or 6—in the written mathematics graduation examination (and usually in the written entrance examination as well). However, students were required to provide not only answers, but also extensive and theoretically and logically sound explanations. Teachers specifically trained students in giving such explanations instead of simply giving computations only.

Quickly (even if not immediately), a multitude of violations of the examination procedures or, to put it simply, cheating, emerged in the examinations. By the 1970s in some regions these "violations of the procedure" became truly enormous: the problems on the examination could become known to students a few days before the examination. These problems were discussed by students in detail; they could receive any consultation, etc. Correspondingly, the role of examination hardly resulted in real student assessment. It was limited to providing a kind of a short form of the standards—tasks and topics that were included in examinations were considered the most important.

Eventually, in addition to the "regular" version of the examination, a version for schools with an advanced course of study in mathematics emerged (these schools have a much more enriched and deep curriculum in mathematics and attract students who are more motivated and talented in mathematics—Karp, 2011). When Gorbachev's *perestroika* started in 1985, the process of changing the style and organization of the examinations became more rapid and effective. In addition to two versions of the mathematics examination mentioned above, a new version for schools with an advanced course of study in the humanities appeared. Even more importantly, some regions received the right to develop their own versions of problems, employing other formats than those traditionally employed (Karp, 2003). Educators in these regions often reached agreements with higher education institutions that provided that the graduation examination would also be counted as an entrance examination, so that students did not have to take two rounds of examinations. All these experiments were, however, terminated: by the end of 1990s, the country's government approved the idea of the Uniform State Examination (or, to be precise, examinations, because this system was introduced not only for mathematics, but for other school subjects as well). These Uniform State Examinations were supposed to replace both graduation and entrance examinations.

The USE: A Brief History

There were many different attempts to validate the need for these examinations. Probably, the most frequent explanation was based on the need to fight against the corruption from which entrance examinations seriously suffered. Another popular argument invoked the obvious need to give gifted students from small provincial towns and villages the possibility of applying to the best central universities in the country (one can note here, however, that the mobility of the population in Russia is substantially lower than in the United States, and it is not simply because they cannot come to Moscow to take entrance examinations—which were supposed to be replaced by the USE—that students from provincial towns and villages do not attend universities in Moscow. Rather, it is because their families cannot afford their living expenses in Moscow).

Experiments in conducting the USE started in the early 2000s, and since 2009 the USE has become the only form of graduation examination. Regarding entrance examinations, some reservations emerged because a number of universities—the very best ones—received the right to conduct additional entrance examinations on their own. Other institutions of higher education, however, admit students based on their USE results only. Technically, they admit students who received the required number of points in all required examinations (again, a USE exists in basically every subject).

The written USE in mathematics lasts four hours (240 minutes). Originally, the examination contained three parts. Problems A1 through A10 were multiple choice assignments. Problems B1 through B11 required a short answer. Finally, problems C1 through C5 called for a long answer with all explanations and all work demonstrated (Karp & Zvavich, 2011). The following problem can serve as an example of a group A assignment (Nekrasov, Guschin, & Zhigulev, 2007, p. 5):

Simplify the expression $\dfrac{\sqrt[3]{a^2}-16}{\sqrt[3]{a}-4}-\sqrt[3]{a}$.

1) -4 2) 4 3) $-2\sqrt[3]{a}$ 4) 0

Multiple choice tasks, however, met with a very negative reaction from many mathematics educators. Indeed, this form had been criticized for many years in the Soviet period and even later was never popular. Correspondingly, there are no multiple choice assignments on the 2011 examination, and, although it is somewhat strange, these exams contain only problems of type B and C.

The 2011 Version: Mathematical Content

There are 12 short-answer assignments (B) in the 2011 version of the examination. As a kind of innovation, this year one can see the appearance of assignments that to some extent can count as real-world problems. The following assignment is an example of such a problem (here and below, the *version for display* from the site ege.edu.ru is cited):

> The price of a bus ticket was 15 rubles. What is the greatest number of tickets that can be bought with 100 rubles after a 20% increase in the price of a ticket?

The version includes a traditional word problem on "working together" and a far less traditional problem on finding the optimal cost of the purchase. There is also a problem that requires an analysis of a graph and a problem that makes use of some physics terminology but mathematically is reducible to solving a quadratic inequality. In addition, there are also fairly easy logarithmic and exponential equations, geometry problems (plane and three-dimensional), and a trigonometry problem. Also, there are two problems that require use of the derivative. One involves analyzing a graph, while the second one is given below.

Find the maximum value of the function

$$y = 2\cos x + \sqrt{3}x - \frac{\sqrt{3}\pi}{3} \text{ on the interval } \left[0, \frac{\pi}{2}\right].$$

It should be noted that 7 out of 12 problems in group B can be solved by a student who has never taken senior-level classes—these problems are based on the material studied in grades 1–9.

Six problems from group C require explanations and justifications and are more challenging. Two of them are devoted to geometry—one to plane and another to three-dimensional geometry. One problem is devoted to trigonometry, another to exponential or logarithmic functions. Problem C5 is devoted to solving an equation

Table 1. Relations between the raw and final scores

Raw score	0	1	2	3	4	10	11	12	27	28
Final score	0	6	12	18	24	49	52	56	94	96
% of those who received the score	0.4	1.1	1.9	2.8	4.0	9.0	9.0	8.3	0.1	0.1

with parameters, and finally, C6 belongs to the theory of numbers. Let's consider problem C1:

Solve the equation $\dfrac{6\cos^2 x - \cos x - 2}{\sqrt{-\sin x}} = 0$.

This problem is the easiest in group C. Its solution, however, involves a) solving the quadratic equation in terms of $\cos x$, b) solving basic trigonometry equations, and c) selecting among all obtained solutions only those values of x for which $\sin x$ is negative. It is important to mention that although the course for the senior-level grades (10–11) includes both solving quadratic equations in terms of $\cos x$ and checking whether the value of a given trigonometric function is negative or positive at a given x, there are basically no problems in the textbooks that combine both of these tasks.

As another example, let's consider the substantially more difficult problem C5:

Find all values of the parameter a, for each of which the system of equations $\begin{cases} a(x^4+1) = y + 2 - |x| \\ x^2 + y^2 = 4 \end{cases}$ has a single solution.

To solve this problem, one can note that if any pair (x, y) is a solution of the system, then the pair $(-x, y)$ is a solution too. This implies that the system can have a single solution if and only if this solution is a pair of the form $(0, y)$ for some y. Now, substituting 0 for x, one obtains the system $\begin{cases} a = y + 2 \\ y^2 = 4 \end{cases}$. It is clear from here that the only possible values for a for which this system has a single solution are 0 and 4.

This is not the end of the solution, however. Now it is necessary to see whether the system indeed has a single solution, given that a equals 4 or 0. If $a = 0$, then the system looks as follows: $\begin{cases} y = |x| - 2 \\ x^2 + y^2 = 4 \end{cases}$. Obviously, this system does not have a single solution. If $a = 4$, then the system looks as follows: $\begin{cases} y = 4x^4 + |x| + 2 \\ x^2 + y^2 = 4 \end{cases}$. It is clear from the first equation that $y \geq 2$; however, the second equation implies that $y \leq 2$. Therefore, this system has a solution only if $y = 2$. The equality $y = 2$ in turn implies that $x = 0$. In other words, the system has a single solution, $x = 0, y = 2$. The final answer to the problem is $a = 4$.

Data on the Results of the Examination

Each problem on the examination is assessed using the raw scores, so that the maximum possible score for all problems is 30 points. Specifically, each of the problems B1-B12 is worth 1 point, each of the problems C1-C2 is worth 2 points, each of the problems C3-C4 is worth 3 points, and each of the problems C5-C6 is worth 4 points. Then, these raw points are converted into the final scores, and the maximum possible score here equals 100 points.

Table 1 describes the relation between several raw and final scores in 2011 (only a few scores are represented); also, the percentage of students who received each of these scores is given (again, the data is taken from the website www.ege.edu.ru, which provides information up to mid-June 2011; students who were unable to take the examination at that time for serious reasons took it later, but their number was small). The USE in mathematics was taken by 762,431 students in 2011.

In particular, this table shows the percent of those who failed to reach the *minimal score*. The minimal score is defined as the score that is necessary to achieve in order to pass the examination which, in turn, is needed for receiving a secondary school certificate. In 2011 this minimal score was equal to 24 final points, and 6.2% of the students who took the examination failed to receive it.

The maximum possible score of 100 was received by less than 0.1% of the participants—by 214 students. Table 2 demonstrates in which regions the numbers of those who received 100 points are the largest.

Some schools display their students' results on the USE on their own websites. Figure 1 represents the results of the St. Petersburg school # 30 (which is considered to be the best in the city, see http://school30.spb.ru/).

Results such as those represented in the diagram, however, are fairly rare. In particular, 31.57% of those who took the examination in Russia in 2011 did not even attempt to solve any task from group C.

Critique of the USE

The USE has been strongly criticized since the very moment when the idea of this examination appeared (Bashmakov, 2010; Karp & Zvavich, 2011). In particular, critics have pointed out that the examination fails to stop corruption or provide equal opportunities for quality higher education to all students. The role of the examination as an "equalizer" of opportunities has indeed proven to be fundamentally exaggerated—as mentioned above, difficulty with coming to the universities to take examinations there is not what typically prevents students from pursuing an education in Moscow or St. Petersburg.

Critics have pointed to numerous instances in which the procedure for taking the examination has been violated. Additionally, statistics concerning the examination itself

Table 2. Number of those who received 100 points in a few regions

Region	Number of those who received 100 points
Moscow	38
Perm region	20
Moscow region (without Moscow)	16
St. Petersburg	15
Chelyabinsk region	15
Nizhny Novgorod region	13
Tomsk region	8
Orenburg region	7
Khanty–Mansi Autonomous District	7

(even very limited ones, as in this paper) suggest that this procedure has hardly been fair everywhere. For example, it is not easy to explain the large numbers of those who receive 100 points in relatively small regions, particularly given that these obviously outstanding students fail to display their gift in any other way. These "unexpected" results are even more questionable when it comes to the numbers of those who fail the examination. Indeed, some regions have dramatically improved their performance over the previous year and had no failing students at all.

Moreover, organizers are not necessarily doing the very best job, even when it concerns the relatively simple technical details. For example, the problems in group B are assessed for correctness using computers, and there are many errors here.

It is important to describe not only organizational, but also methodological issues that are criticized. Indeed, Russian textbooks for "regular" schools (recall that there are also schools with an advanced course of study) do not contain problems similar or even close to problem C5, which was discussed above. With all due respect to the beauty of this problem, one can only conclude that it is natural and logical that many students do not even attempt to solve it. By adding to those who did not attempt to solve any problem from group C those who did attempt but failed to solve any problem from this group, we see that for many students this examination is limited to the problems in group B (as is clear from Table 1, 36.5% of participants received scores of 56 or less, which corresponds to raw scores of 12 or less, which is what can be received for solving the problems in group B). Some educators disagree with offering problems in group C to all students.

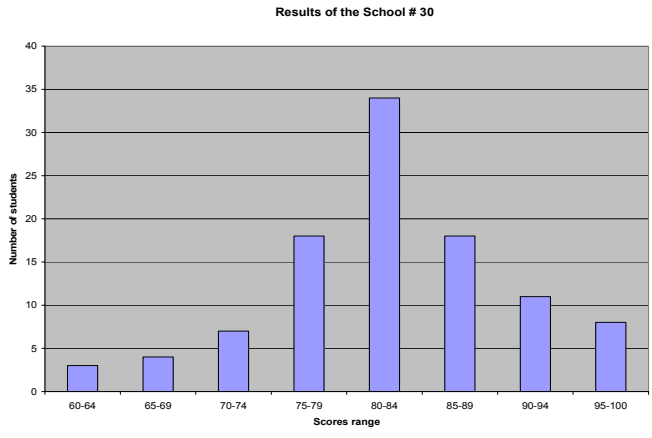

Figure 1.

Probably even more educators find it awkward that in order to pass the examination successfully (that is, to obtain the minimal passing score) it is not necessary to learn anything at all in senior-level grades. Indeed, the minimal score of 24 corresponds to 4 problems from group B, and, therefore, students can get this score by solving problems that are not based on material covered in senior-level classes. It looks like there is no difference, as far as the examination is concerned, between those who worked very hard in senior-level classes in "regular" schools and those who did nothing: neither can solve problems from group C, but both can solve some problems from group B.

On the Influence of the New System of Assessment

The results of the USE are currently considered to be a major indicator of the success of the work of schools. A characteristic example is the award of quite substantial grants to the "best Moscow schools" in 2011. The decision on which is the best was based on the results of Academic Olympiads and the USE (Nasyrov, 2011). The results of the USE prove to be important for all schools (not only for the winners of this competition). For one thing, these results are used in the process of promoting teachers. It is not a surprise, therefore, that preparation for the USE takes up a substantial portion of school time.

The content of the USE is viewed as stable and predictable. Problem B1, for example, is supposed to be a real-world problem involving percentages. Even the problems from group C are supposed to be similar to some extent to what is given in the version for display on the examination website. Thus, C4 is supposed to be a problem in plane geometry that has two solutions. These

expectations are supported by the publication of books on preparing for the examination, which have such characteristic titles as *USE 2011. Mathematics. Problem B4. Plane geometry: angles and areas. Workbook* or *USE 2011. Mathematics. Problem B6. Plane geometry: areas. Workbook* (Smirnov, 2011 a, b). One can assume that, in some schools, this kind of preparation limits teaching to practicing solving only specific types of the problems, which narrows down the teaching of mathematics and sometimes even makes it senseless. Contrary to this, however, one can assume that, at the same time, in some schools the USE has encouraged teachers and students to get acquainted with new challenging problems, and has, therefore, enriched mathematics education there.

Conclusion

The idea of a national examination in mathematics is becoming more and more popular in the United States, while high-stakes exams have been implemented already. The Russian USE gives an example of the difficulties that the organizers of a uniform national examination can face. This examination is created to be the same for all students, but although it is clear that all students can be mandated to take it, it is less clear whether it is worth mandating this. In Russia, the preference obviously has been given to the "strong" students: everybody has been mandated to take the same examination as those who are most motivated and prepared. In this situation, the minimal passing score has been set very low to prevent mass failures. The small number of those who fail, compared with the number of those who fail, say, the New York State Regents, seems surprising only until one notes that in order to pass the USE it is sufficient to solve 4 problems from the "easy part."

The opposite of the Russian approach, which would victimize "strong" students rather than "regular" ones, does not seem any better. These "strong" students can not be limited to solving something that is very basic for them and be deprived of substantial challenges.

The practice of using USE results for the identification of the best schools can be questioned. It is important to keep in mind, though, that similar ideas have been offered outside of Russia as well. Cheating on examinations in response to improper pressure to improve grades also does not seem to be an exclusively Russian prerogative.

On the other hand, the USE has introduced a number of rich and beautiful mathematical problems. Acquaintance with them seems to be useful for mathematics educators outside of Russia.

References

Bashmakov, M. (2010). Challenges and issues in post-Soviet mathematics education. In A. Karp & B. Vogeli (Eds.), *Russian mathematics education: History and world significance,* (141–186). London-New Jersey-Singapore: World Scientific.

Karp, A. (2011). Schools with an advanced course in mathematics and schools with an advanced course in the humanities In A. Karp**,** & B. Vogeli (Eds.), *Russian mathematics education: Programs and practices* (pp. 265–318). London-New Jersey-Singapore: World Scientific.

Karp, A. (2010). Reforms and counter-reforms: Schools between 1917 and the 1950s. In A. Karp & B. Vogeli (Eds.), *Russian mathematics education: History and world significance,* (43–85). London-New Jersey-Singapore: World Scientific.

Karp, A. (2007). Exams in algebra in Russia: Toward a history of high-stakes testing. *International Journal for the History of Mathematics Education, 2*(1), 39–57.

Karp, A. (2003). Mathematics examinations: Russian experiments. *Mathematics Teacher, 96*(5), 336–342.

Karp, A. & Zvavich, L. (2011). Assessment in mathematics in Russian schools. In A. Karp & B. Vogeli (Eds.), *Russian mathematics education: Programs and practices* (pp. 319–374). London-New Jersey-Singapore: World Scientific.

Nasyrov, E. (2011). Moskva razdala shkol'nye granty [Moscow Has Distributed School Grants]. *Moskovskie novosti,* November 8. 2011.

Nekrasov, V., Guschin, D., Zhigulev, L. (2007). *Gotovimsya k EGE -2007. Matematika. Uchebno-spravochnoe posobie.* [Preparing for USE-2007. Mathematics. Manual and Resource book]. St.Petersburg: Prosveschenie.

Smirnov, V.A. (2011a). EGE 2011. Matematika. Zadacha B4. Planimetriya: ugly i ploschadi. [Mathematics. Problem B4. Plane geometry: angles and areas]. Workbook. Moscow: MTSNMO.

Smirnov, V.A. (2011b). EGE 2011. Matematika. Zadacha B6. Planimetriya: ploschadi. [USE 2011. Mathematics. Problem B6. Plane geometry: areas]. Workbook. Moscow: MTSNMO.