Journal of Mathematics Education at Teachers College

Fall – Winter 2012

A Century of Leadership in Mathematics and its Teaching

TABLE OF CONTENTS

Preface

Articles

TABLE OF CONTENTS, continued

EQUITY NOTES FROM THE FIELD

Other

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A Conversation With Uri Treisman

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Dr. Uri Treisman, professor of mathematics and public affairs at The University of Texas at Austin and the director of the Charles A. Dana Center, has deep and active roots in mathematics and mathematics education. Dr. Treisman is well known for his early work at the University of California at Berkeley, where he developed the Calculus Workshop Model in response to differential outcomes in Calculus among African American, white, and Asian students. His continuing work in mathematics education, particularly his impact on equity in the eld and his current work focusing on mathematics teaching and learning in community colleges was re ected in his remarks on September 29, 2012, during the annual Program in Mathematics Colloquium Series. Dr. Treisman had initially planned to give a talk on the technical aspects of scaling educational innovations, but seeing that the audience was composed largely of young teachers and teachers-to-be, he instead gave an impromptu talk on teaching mathematics. This is an edited version of those remarks. —Ed.

On Teaching Mathematics

So because this is the famous Teachers College, I'll start with a confession: Teaching is a bitch. It's incredibly difficult, and one of the central problems that we have to solve is helping people understand that teaching is difficult. It's something you get better at over your whole life, and you only get better at it if you think of it in terms of the problems it presents in knowing your students, the content, and the ways in which we can introduce beautiful ideas to students in settings in which they can learn. I've been working at trying to get better at teaching for close to 50 years now, ever since I had a tutoring business in high school.

So picture my class of 120 calculus students and I'm about to get at the punch line of the fundamental theorem of calculus. I have set up the trap, right? I introduced it in a way in which they're about to have an enormous surprise. Just as I get ready to deliver the example that's going to let them see the magical connection, the student seated right there let out this gigantic yawn. Of course I thanked him for his honesty, and I asked him, "Is this interesting to you?" He said, "Well, Professor T," (he called me Professor T) "you care so much about us." I said, "That wasn't the question." (laughs) He said, "Well, to tell you the truth, you're funny, but this is sort of boring." I said, "Look, there are things that are going to be boring because math, like music, requires you to be skilled at it. No musician in his or her right mind fails to understand the importance of practice in order to play something. Some of that could be boring, but not the fundamental theorem of calculus!" (laughs)

There is no universe in which I can imagine the fundamental theorem of calculus is boring. I know most of my students—I memorize their names and learn their backgrounds. I have a system that probably some of you also have where every one of my classes elects three students who are the class reps. If you don't do this, you should think about something like it. They survey the students, and they come to me every two weeks when we meet with a list of things, two things in particular, that I can do to help them learn better. In return, I give them two recommendations that I think will make them learn better, be better students, and make better use of the class. Then in the alternate weeks, we evaluate each other publicly. This is a very powerful technique.

So right after this occurred in class, the three representatives came to me and said, "Professor T, we hurt your feelings, didn't we?" You have to be honest with your students, right? I said, "The truth is it did hurt my feelings. Not that you hurt my feelings, but I was disappointed in myself because obviously I didn't think hard enough how to set this up." One student said, "What can we do to make it up to you?" So I said, "Look, just once in my life, when I introduce the punch line, when I get you to discover the punch line of a beautiful theorem, I want a student to be so overwhelmed that he or she uncontrollably gets up and goes, 'Holy shit! That can't be true!' "

Five weeks later, we are about to discover some other theorem. That day a respected colleague came in with a young faculty member that he's mentoring and they sat in the back of the room. Of course, because I'm being observed, I immediately reverted to all the things that I know are going to work, right? I became much less experimental. When I delivered the punch line of the theorem, someone gave a signal. All 120 students got up and said, "Holy shit! That can't be true!" And to this day, my colleague—a good friend of mine—has never mentioned it. It was so out of the ordinary it was like the gorilla in the room.

When you actually study teaching, it's incredibly hard to believe that anyone can do it well. It's almost amazing when you look at the microscopic level of teaching. For all

the years I've been teaching, I've videotaped and audiotaped classes to listen to what's going on. When you look at a fine level, it's almost impossible to believe that teaching can achieve its intended purpose. You start noticing crummy things, like every time you say, "You got it?" No one ever says "no." There are all these things that happen in your classroom that have the exact opposite meaning of what you literally said.

For those of you who want to teach, it's a source of extraordinary pleasure. I've now had more than 200 low-income, African American and Latino students who have gotten Ph.D.s or M.D.s or who have advanced degrees in engineering. As teachers, we're in this privileged position where we can open up possible universes for our students to explore and seek membership in. We can introduce them to worlds that they couldn't even imagine, and we can help them do things that are good uses of their life. So we have tremendous power as teachers, but when you look at the fine details of it, it's almost impossible to imagine how we can affect how people think about things. It's very, very hard work. Indeed, a lifetime of work.

In the beginning, when you're learning teaching, as about a third of you in the audience are, there are all these skills you have to learn that have been well studied. You don't give directions when you're moving around. You learn very quickly you stand still when you're asking students to do something—all the stuff that good teachers learn and that have been studied and analyzed. However, once you get past the 50 or 60 basics, it's going to be a lifetime of practice. It's like becoming a musician—it's something you will get better at your whole lives. One tragedy is that the data show that when you look at value-added measures (which certainly are highly imperfect) on average, teachers don't get better at helping students learn past their sixth year of teaching because there are very few structures and supports for teachers to keep getting better. There are tragically very few incentives for it. The people who do get better are driven internally. It's like art in that way or music or religion. So I hope that you already think about this as a lifelong quest. If you do that, it's going to make it so much more interesting to you.

On Equity and Democracy: Algebra For All, Algebra Forever, and Mathematics For Upward Mobility

One thing I agreed to talk about is what math educators actually do when working at scale on real problems that have societal implications, deal with equity, deal with social justice, and affect how people live their lives. The problem I'm working on now is the problem of remedial education. Twenty years ago, I was one of a band of maybe 100 mathematicians and math educators who argued for "algebra for all." At that time, they were teaching math courses in poor schools, things called *general math* or *nancial math*—things of that sort. We used to call general math "our friends the numbers." (laughs) The irony about financial math was that if people were in there, the only thing you knew for sure was they would never have any money to spend. So we used to teach these courses that had no mathematical integrity, which was effectively a betrayal of our role as teachers.

Unfortunately, our movement of "algebra for all," as my friend David Foster has asserted, has turned into "algebra forever" for millions of students. We're now in a situation where people who want to become nurses, EMTs, firemen, or interior designers are taking algebra courses for the fourth or fifth time. In California two years ago, tens of thousands of students were taking a developmental course in community college, mostly math, for the fifth or greater number of times. This is bad. For those of you who are religious, this is Old Testament bad. I'm talking about rivers of blood, locusts, frogs. (laughs) This is extraordinarily bad, and it's extremely bad for mathematics because people associate our beautiful and powerful discipline with essentially being a barrier to improving their lives. So this is something we have to work on as a field.

It's why math education is so important—it plays an important role in controlling mobility in society. So many math teachers, when they tell me they're interested in equity, want to make better algebra courses. There'll be arguments about factoring versus completing the square or why we need this topic or that topic. However, if you are concerned about the social implications of our field, it's important that you actually think about the big picture.

The big picture is that America is a wonderful country, but we have one or two character flaws. One flaw is we prefer nostalgia to history. We like to think that democracy is always going to be here because the founding fathers had a brilliant idea and envisioned a country built on ideas rather than blood. This is a beautiful democratic principle. Well, the fact is that democracies are extremely fragile things—the Weimar Republic in Germany, for example. A social contract on which democracy depends is the notion that people can be comfortable with inequality as long as they believe that through education and hard work, they or their children can have better lives. American democracy is based on the potential of upward mobility.

Well, guess what? We now have lower upward social and economic mobility than most European countries. That's really bad. Why is this bad? Because we in mathematics are one of the vehicles for actually increasing social and economic mobility. It used to be the military for poor kids, but that's closed off as an option for many for health reasons or entry tests. Small businesses are in a terrible state right now in terms of job creation. Government jobs also used to be a major vehicle for upward mobility. The other avenue is education, but math education is the primary barrier for people, particularly adults coming back to school, who want a better life. In too many cases, they need to take college

algebra in order to be an interior designer—that's crazy and we need to reflect on it carefully.

Most education improvement takes place at a very small scale. In this stage of my career, however, I am interested in working on problems at a larger scale. I am now working on the redesign of pathways to upward mobility that are connected to programs of study. These are the remedial courses in community colleges and the remedial courses now appearing in the eleventh and twelfth grade in high schools. It's a big issue. We estimate that two-thirds of high school students will be deemed *not college ready* on the new Common Core exams; that means it has all kinds of implications for their ability to go to college as well as their perception that they can go to college. There's enormous engineering work to be done, reengineering work on the math that people take beyond the Common Core. What we offer students who we deem not college ready by instruments not yet designed is this infrastructure of remedial courses that form the bulk of the enrollment in American community colleges. In fact, only about 6 percent of community college math enrollment today is in calculus and beyond. Almost all of the enrollment is in courses that contain topics that were taught as part of the middle school and high school curriculum. So this is a major, major problem.

On the Meaning of Education and Creating Effective Learning Environments: "A Joyful Conspiracy"

In my first career, which was as a landscape designer, I was not the typical student. I was much older than the other students. I went to the horticulture program in community college, and when I got to UCLA, I was 26. I didn't know what I wanted to study, and my grandfather said, "Study what you're worst at. That's the only way you'll learn something." So I said "I'm good in math, I'll study that, and what am I worst at?" Studio art. So I double-majored in studio art and math because I wanted to know what it was like to constantly confront the struggle to learn. I became a landscape designer because I actually got pretty good at drawing and I liked it.

One thing that's really disturbing about education is that most students who finish our programs are positive that they're not artistic or philosophical or scientific, and they're only somewhat convinced that they might be good at certain things. They're positive they're bad at a whole list of things, and only a little bit are sure that they're good at others. This is a horrible outcome of educational systems because as soon as the people start believing that they're not mathematical or scientific or artistic, it begins to erode their personal freedom. It has devastating impact on one's sense of power in the world. For the teachers in the room, think about this: Are you allowing students to believe that they're not good at something? What are the ways we as teachers respectfully challenge our students to refuse to accept self-limiting characterizations? It's a very important task for teachers.

For those who are going to be teachers, remember that the first big hurdle is learning the craft. We know a lot about the 50 or 60 things that good teachers do that are necessary but not sufficient to produce great learning. You have to perfect that craft. You're not going to be a good guitar player unless you practice the scales. However, if you think you're going to be able to improve education in your classroom without affecting education in your school, you're completely wrong. Tony Bryk,[1] in one of the greatest pieces of education research in the century measured teachers' relational trust in schools—the way teachers respect each other and their administrators. He showed that in schools with no relational trust, the likelihood that professional development or interventions led to sustained meaningful increases in student learning was low. In schools that actually produced meaningful improvement, the kind that actually affects the life chances of children, we see a culture of respect, collaboration, and transparency because that was a culture in which teachers were willing to take risks. If you're a new teacher and you're going to require homework done in a certain way and none of the other teachers do, you have an almost impossible hill to climb. However, when all the teachers in the school share the same norms and culture, students are more likely to cooperate.

So this question of innovation as a set of practices, sort of the technocratic view of improvement, is essentially wrong. The really hard question is, "What are the catalysts in these institutions that turn those practices into a place where children can learn and adults can learn and encounter what learning entails, which is productive struggle?" This is the hard work of design in these environments. To give you another example, Adriana Kezar,[2] professor at University of Southern California, studied about 600 proposals to the federal government to improve education. It's a wonderful study. She found that virtually all of them focused on one level of the system: We'll make teaching better, we'll make governance better, we'll have better leadership training for these leaders. It turns out that the only way you create schools where children and adults can learn is to have coordination at different levels of the system. It's what I call the "joyful conspiracy." Actually it's my wife's term. Unless there's the articulation of improvement at different places in the institution, it's going to be almost impossible for your classroom to improve, other than through external events that are hard to sustain.

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[1] Anthony Bryk, president of the Carnegie Foundation for the Advancement of Teaching, professor at Stanford University Graduate School of Business, and the author of numerous books and articles about hierarchical linear modeling, school reform, and relationships and trust within schools.

[2] Adriana Kezar is a professor of higher education at the University of Southern California who examines change and leadership within institutions.

So for you who are becoming teachers, the first step in your life as a teacher is learning the craft—others in the room know this from deep experience. The hardest part of learning your craft is switching from technique to learning how to listen and manage student thinking. Surfacing student thinking is *incredibly* difficult to do, and learning when you can follow student thinking without the rest of the class falling apart is not for amateurs. With only two years in the classroom, you don't yet have the skill to surface student thinking and work with it. After three to five years, you begin to get it. One of my students did her dissertation on teaching dilemmas in the third year of teaching. She found that there were no dilemmas. Teachers just had predicaments. (laughs) You know, trying to follow and manage a discussion where students were creating ideas on the spot. This is really, really hard to do. Unless you spend hundreds of hours watching yourself and your peers do it, you're not going to get it. This is beautiful; this is the beginning of becoming a mid-career teacher.

The next thing is most of the improvement efforts in our schools are driven by data on failure, but there's very little knowledge about what's actually working other than anecdote. Lee Shulman,[3] one of my heroes, said, "The plural of anecdote is not data." It's a beautiful line. In very few places where people are working to improve, do they actually know the faces of the students they've really affected positively? What we're seeing when we look at improvement is that when system improvement comes in, the first things that get killed off are all the quirky interesting positives that produce the successes. Almost all the data are organized around the failures of the system, and none is organized around the successes of the system.

On Public Education

Public education for generations was perceived as a beacon of hope. Now, as David Cohen has remarked, it's perceived as a symbol of government failure. Do you understand how dangerous that is? Because you are the future generation of mathematics educators, that's one of the principal problems you need to take on as a profession, not just as an individual, but as a group. How do we make sure that math education is organized so that people understand the power of the discipline to create extraordinary opportunities for advancement? That schools actually can work? People will argue about teacher evaluation, morale will sink, and public trust in education will drop. People will begin to define higher education as a private good, not as a mixture of public and private goods. This is dangerous for those who care about equity because, in the end, people with a lot of money go to great places and get educated. It's the people who come from where many of you may come from or I come from—who depend on education working to enhance our life possibilities—we're the ones at risk in this.

On "Getting the Problem Right"

The last general comment, and then I'll describe a little of the work itself, is that you've got to get the problem right, and there's a lot of energy spent on designing solutions to problems and very little energy in crystallizing what the problem is you're actually trying to solve. When I started working in the first part of my career, I was trying to solve the problem of how to help students who I believed were poorly prepared to succeed. We surveyed the faculty at Berkeley and got roughly 400 responses.[4] Why are African American students doing so much worse than white students in this institution? The faculty said, "Look, the students are motivated. They're very good, but they're not as motivated as other groups." People believed that it was the families of these students, so people had a deficit-based approach. They actually had the problem wrong. All our studies of students the week before they start college show they're incredibly motivated, right? They were the students who survived the schools and actually came to these institutions that were elite, and people were trying to remedy families, the punitive weaknesses of these students, rather than take what I found in the end to be the right problem—how to determine their strengths and practical ways and build on them so that they could achieve their hopes. The nature of the problems you're thinking and working on is incredibly important for the decisions you make and the work you do toward it.

Getting the problem wrong is a big issue, and that brings me to developmental education. Everywhere in every state, people are working to improve developmental education and redesign it. However, many states are restricting access to students who have been poorly prepared. For example, there are 470,000 students in California on waiting lists to get into community colleges, and in some cases they're biasing admissions toward students they believe will succeed. You want an equity problem to work on? This is the problem. When you look at the problem of developmental education failure, people say, "Yeah, if you start in arithmetic, the probability that you're going to finish Dev Ed is pretty much zero." In one community college, I went to interview all the successes and I asked for all the students who started in arithmetic who finished calculus in three years. Laura Delgado [pseudonym] was her name!

One thing that people have noticed is that when you look at data in states such as California and Texas, more than 50 percent of African American and Latino students get

[3] Lee Shulman is President Emeritus of the Carnegie Foundation for the Advancement of Teaching and the Charles E. Ducommun Professor of Education Emeritus at Stanford University.

[4] See Treisman, U. (1992). Studying students studying calculus: A look at the lives of minority students in college. *College Mathematics Journal, 23* (5), 362–372.

WDs or Fs in regular college courses. About 42 percent of white students are also getting WDs and Fs in all the regular college courses. So the framing of this as a *remediation* problem will lead to the building of a six-lane highway into a swamp, right? The problem of remediation came up, and it's actually the wrong problem. In working on this problem, we must learn from Einstein. He has this beautiful quote that when you work on a hard problem, make everything as simple as possible but not simpler.

The work that the Dana Center is doing now in Texas at scale, with all 50 community college districts, is to replace remedial courses with a system that enables students placed into developmental mathematics to complete a credit-bearing, transferable mathematics course on an accelerated timeline while simultaneously building skills for long-term success in college and life. It is a systemic approach to improving student success and completion through the implementation of processes, strategies, and structures built around three mathematics pathways and a supporting student success course.

In this work, there are about five dimensions that we have to deal with. The first is the content. The content of traditional courses is shaped more by the weight of history than by the actual nature of mathematical practice today, by the needs of majors and careers today, and certainly by workplace needs. What do most people need? Bureau of Labor Statistics studies show about 20 percent of adult workers use algebra or anything past it. I think the most common use of algebra in society may be helping kids with their algebra homework. (laughs) It's not that algebra isn't extremely valuable and important, but it's not the only branch of mathematics. Many people need to learn statistics—they need to deal with the inexorable uncertainty in the world around them. They need to understand basic mathematics at a much deeper level in the spirit of modeling, solving real problems, and learning to seek and find mathematics in the middle of real-world phenomena. However, for historical purposes in many states, people are taking college algebra so they're [doing things like] factoring trinomials creatively, and it should tear at your mathematical hearts. It's not right.

So the first step is changing the content. Remember I said a high bar for change. This requires the professional associations that we're working with to develop sanctioned standards for the outcomes in these courses so they're worthy of students and our discipline. What we cannot do is betray our profession and teach mathematical content that has no integrity. This is critical work. Step 1 is to determine the appropriate content.

Step 2 is the structure. The normative pattern of remedial education is four courses. That means students have to pass a course, choose to enroll, pass, choose to enroll, and so on, and then retire when they finish the whole sequence. Two-thirds of all students who take remedial programs in the United States pass every course they take. The biggest attrition cause is not failure. It's that students with complex lives must complete a long sequence of courses. So we need to think of different structures that are highly accelerated. Most students are struggling with what college means, and they need immersive experiences in which they can figure out clearly what they're supposed to be doing and what the actual demands on them are. We know how to do that.

Step 3 is the way courses are delivered. It turns out that students in community college have very complicated lives and are a very diverse group of people, and there are no silver bullets. This Brooklyn boy has become a Texan. I do not believe in silver bullets, but I think I believe in silver buckshot. So there are three or four different delivery vehicles (for example, modularization and hybrid courses), and the challenge is to figure out which students can use them well and how to get the right student in the right pathway at the right time. For example, some faculty in states like North Carolina, Virginia, and Indiana are working incredibly creatively on developing modularized programs, organized around better content with more customization to what students actually need. They're learning that there are certain groups of students who typically do much better in these courses. First, we are seeing, for example, that veterans who have learned to learn through technology can also learn from modularized programs. The second group is adults who had a good education, but are moderately terrified and are rusty. They're very good users of these individualized self-paced programs. Third are high school students who want to accelerate and have the skills. They're good users, defined narrowly, of these programs.

The students who are bad users are those without self-regulation skills and with less familiarity of the enterprise of higher education. It's not that we don't want self-paced modularized hybrid forms, but we're responsible to figure out whom they're actually helping. They're not silver bullets.

I want to spend a bit more time on Step 4 because it's more related to the people who are going to be teachers. This is about understanding the support systems that have to be in place so that students can learn, especially students who have been led to believe that they're not good in mathematics. These are students who when you ask them, "When was the last really positive math experience you had?" can't think of one or tell you something that happened in the second grade—there are a lot of these people. You know all the people who tell you that they can't balance their checkbook. You should tell them, "It's okay. I can't read." See what they say. Socially acceptable? (laughs)

What have we learned about these students? So here's some data, and this is the work I'm doing right now in productive persistence and calibration. We're looking at hundreds of students. It turns out that about 90 percent of students, even those placed in arithmetic, are positive they're going to succeed on the first day of instruction. In fact, more than half of them say they're going for a B.A. or advanced degree. So motivation is not a problem. They're not only

highly motivated, they recognize they made a sacrifice to start this program, they see themselves as distinguished by going to school to learn, and they're sure they're going to succeed.

We've also found that many of these students have no conception of the relation between effort and achievement. If you can't connect your effort and work to your outcomes, it's highly demotivating and it accelerates the disengagement from the process. So the question is, "What are the psychological techniques that one can use that teach calibration and motivation?" What if half of the students in these courses believe that your intelligence is fixed? They actually believe that you're good at math or not good at math, and what you're doing is demonstrating your level of goodness at it. They don't realize that you're supposed to struggle; that math involves a creative, productive struggle; and that if you don't have a language for that and attributions about what that feels like, you're not going to actually persist. So learning to teach students Aha! experiences and giving them a vocabulary for describing their feelings—this is a great dissertation topic—dramatically increases the amount of energy that people put toward their goals.

Step 5 is faculty support because in community colleges, there's virtually no investment in higher education and very little investment in faculty improvement. About two-thirds of the developmental courses in community colleges are taught by adjuncts who are paid by the course (often at a very low rate) and have no office hours or benefits. When you have a large number of people who are contingent workers, they're not paid to improve your offerings. It's a difficult situation.

If you work with remedial students, the first time they have a success will embolden them to raise their aspirations. It will jar them, open up questions about who they actually are, and be a liberating experience. That's how powerful the stereotypes about math are in this country. We need tools for using this—for liberation purposes, deepening, and improving the quality of the lives of our students. Sitting there pushing buttons meaninglessly and trying to guess an answer as with some mathematics software programs is an affront to our discipline.

Reprise: On Teaching as "A Lifetime of Work"

Teaching is a lifetime of work. You're just starting. The first step is knowing your students and the basic techniques of teaching. There's a lot to know, and you need to honor the work of people who came before you. Learn from them, learn the skills. Don't make it up all yourself. The second stage has to do with learning to hear the voices of your individual students and manage them collectively—this is really hard work. The third stage, when you're really a math educator and a member of the profession, is when you start assuming responsibility for the collective failure of mathematics education. That means you're working in schools to improve them so that math can be better in every classroom, and you're not just working to improve learning and outcomes for this particular group of students. You're working to improve all students' performance. You're active in the profession and shaping society's image of mathematics, which needs work. It will only change if we as a profession are able to communicate the actual and extraordinary power of the discipline that we're privileged to teach and if we define professional norms and practice that we instantiate and advocate for in our communities. That's what you're starting on a long pathway toward. The job of the teacher—I'll end with this—and our job as mathematicians and mathematics educators is to make sure that the next generation is better than us. Your job as our students is to surpass your teachers. That's the way society works. That's your job. It's not disrespectful. The most respectful thing you can do is challenge us, learn from us, and surpass us. Thank you