Journal of Mathematics Education at Teachers College

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A Century of Leadership in Mathematics and its Teaching

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Place, Poverty, and Algebra: A Statewide Comparative Spatial Analysis of Variable Relationships

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Place matters in moderating variable relationships between algebra performance and educational variables because there are differences on the socioeconomic (SES) poverty-affluence continuum that shape local contexts. This article examines relationships between variables for school district demographic composition, teaching and financial contexts, student behavior variables, and Algebra I performance as measured by a statewide test aggregated at the district level. The purpose is to investigate how these relationships vary across 471 districts within a state using spatial mapping. Local R2's from geographically weighted regressions (GWR) are mapped using a geographic information system (GIS) to demonstrate variation in relationships across districts. Results show the importance of allowing relationships to "vary" between local contexts and that using a global measure of variable relationships based on aggregated data fails to capture important local variation. This analysis suggests that policy focused on addressing the influence of poverty on algebra performance should be targeted based on region specific models.

Keywords: algebra, poverty, spatial mapping

> Not only have the rich and poor been pulling apart economically through transformation of the income distribution: since 1970 they have also been separating spatially through a resurgence of class separation…Whether one looks south, north, east, or west, or at whites, blacks, Hispanics, or Asians, America became a more class-segregated society during the 1970s and 1980s. (Massey, 2009, pp. 18–19)

The demographer Douglas Massey describes the spatial transformation of the United States as an age of extremes, where concentrated affluence and poverty and their related effects represent important problem spaces for social science research and policymakers. Poverty has been a long-standing focus of social science. However, the challenges associated with poverty continue for educators as more than one in five children in the United States (15.75 million) lived in poverty in 2010. According to the American Community Survey [ACS] (Macartney, 2011), more than 1.1 million children were added to the poverty population between the 2009 ACS and the 2010 ACS. In addition, the number and percentage of children in poverty increased in 27 states between 2009 and 2010 survey administrations. No states experienced a decrease in the percentage of children in poverty.

Massey (2009) argued that the disadvantages of one's class position in society are compounded and reinforced by the systemic process of geographic concentration. Trends related to childhood poverty when coupled with concentration effects are important to matters of child development and learning (Gagnon & Mattingly, 2012; Wilson, 2009). Darling-Hammond (2010) posited that a major building block of inequality in the United States education system is the high level of poverty and lack of social supports for low-income children's health and welfare, including early learning opportunities. One of the challenges in the literature examining the inter-relationship of education and SES status is that simple explanations as to how poverty combines with other factors or how it impacts in different locations is scarce (New South Wales Department of Education and Training, 2005). There is evidence to support the view that concentration of disadvantage rather than disadvantage per se is the critical mechanism influencing educational outcomes and other psychosocial factors (Galster, 2012; Sampson, 2012). Examination of concentrated poverty-related disadvantage has not been widespread in education research. While many studies have examined the influence of SES as measured by family income, parental occupation (s), and education attainment of primary caregiver (typically the mother) there have been few efforts to determine how poverty is differentially impactful by geospatial location (Lubienski & Crane, 2010).

Understanding poverty effects has been an important part of the mathematics education research literature for several decades (Tate, 1997; Campbell & Silver, 1999; Secada, 1992). The dominant methodological approach has been to compare mathematics assessment outcomes using SES categories. This is a common practice in state mathematics assessment programs. The other dominant methodological strategy is to use a nationally representative

sample of students surveyed in different time periods. In many of these mathematics education studies, SES is a control variable. In some studies the research design is organized to determine the influence of SES on mathematics achievement. However, none of the methodological approaches attempt to determine if poverty influences mathematics achievement differently by location. This limitation is not acceptable today as there are methods and tools that have the capability to inform equity policy in mathematics education with greater specificity about poverty effects in context.

One area of mathematics education that provides a uniquely important problem space to explore the role of place is algebra. Algebra has been a central focus of the equity movement in mathematics education (Ham & Walker, 1999; Anderson & Tate, 2008). The conceptual ideas associated with the content in algebra appear throughout the elementary and secondary curriculum (NRC, 1998). Moreover, algebra and algebraic thinking are drivers for more advanced mathematics, college attainment, and many career opportunities. A large majority of state graduation requirements have mandated a course in algebra for all secondary students (Perkins, Kliener, Roey, & Brown, 2004). This requirement targets an opportunity gap experienced by minority and low SES students for many decades in mathematics education (Tate, 2005). Like most curriculum mandates, requiring algebra has both benefits and costs associated with it (Lillard & DeCicca, 2001). The equity literature in mathematics education has not attempted to estimate whether poverty influences algebra achievement differently by location within a state. Unfortunately, this void in the literature limits insight into the benefits and costs associated with mandating algebra for all students as part of minimum requirements for high school graduation. Addressing this void in the literature is consistent with the National Council of Teachers of Mathematics (2008) call to create a culture of equity including continued assessment of community to ensure that students have access to the resources with the greatest potential to promote learning. This study contributes new evidence in support of a culture of equity as it addresses a particular void in the mathematics education and equity literature.

In order to address the lack of attention to the role of location, this research investigates how the relationships between Algebra I end-of-course (EOC) performance and education variables associated with neighborhood SES differ across school districts in various local contexts. Traditionally, educational and social science research has sought to discover variable relationships in samples that represent "true" relationships in the overall population. However, due to the complexity and variety of many local contexts in terms of educational, social, cultural, political, financial, infrastructure, and housing factors (among others), it is at best, extremely difficult to adequately specify a model for variable relationships that is applicable across all SES contexts. Researchers now recognize that "place matters" and it is important to account for location in examining variable relationships.

In an attempt to represent some of the factors that reflect the SES poverty-affluence continuum across school districts, the present article examines the relationships between variables for school district demographic composition, teaching context, financial context, and student behavior/academic performance variables aggregated at the district level. The study investigates how these relationships vary across districts within a state using spatial mapping. The results of geographically weighted regression (GWR) are mapped using a geographic information system (GIS) to demonstrate variation in relationships across districts. The results show the importance of allowing variable relationships to "vary" between local contexts and that using a global measure of variable relationships based on aggregated spatial data fails to capture important local variation.

Importance of Local Context

Factors related to school district demographic composition, teacher context, financial context, and student behavior in predicting educational outcomes have been widely studied (Hogrebe & Tate, 2010; Gregory, Skiba, & Noguera, 2010; Wayne & Youngs, 2003; Wood, Lawrenz, Huffman, & Schultz, 2006). Research findings have not always arrived at the same conclusions and that inconsistency can be attributed to a number of factors such as studies using different grade levels, types of schools, measurement/assessment procedures and instruments – to name only a few sources of variation. However, one source of difference typically not accounted for is variation due to location and spatial proximity-distance. It may be that variable relationships found in one local context are not the same in a much different local context, even though the research design and controls are constant. The present study is designed to show how variable relationships can differ across the local contexts of school districts within an entire state.

The concept of a local context arises from the fact that within a specific location variable values tend to be more similar which produces a clustering effect. Most researchers are now aware of the need to account for correlations in data that originate from the same group or cluster. Data that come from the same group tend to be correlated in that the factors unique to a specific group (or local context) influence all group members in a similar way. When data points within groups are dependent, then statistical methods that assume independent observations underestimate standard errors and increase Type I error rates. In cases where observations are nested within groups, multi-level models (MLM) are appropriate to account for correlation among group

members (O'Connell & McCoach, 2008; Raudenbush & Bryk, 2002; Snijders & Bosker, 2012).

But what about situations where the grouping or clustering is based on spatial proximity such as adjacent schools or districts? Most likely, students within a district tend to be similar. If the geographic unit is a school district, then at what point do students in one district become different from students in another district? If similarity is defined by a district boundary like a street, then it is unlikely that students on one side of the boundary are much different than those on the other side. However, they may be different from students in a district twenty miles away. The point is that similarity based on spatial proximity is better represented as a continuum which does not start and stop at artificial lines such as district boundaries. So in the case of spatial clustering, multi-level modeling is not appropriate because it focuses on within group correlation and ignores similarity between adjacent clusters (Chaix, Merlo, & Chauvin, 2005; Fotheringham, 2009; Fotheringham, Brunsdon, & Charlton, 2002).

The present study uses geographically weighted regression (GWR) to account for the spatial clustering of districts. (GWR is described in the method section.) Like MLM, GWR allows relationships to vary across groups (in this case, districts), but also takes into account the underlying spatial continuum that MLM ignores. Mapping the GWR results with GIS shows that variable relationships are not constant across districts and the underlying spatial continuum.

Data Source and Variables

The data set for this study started with 565 school districts in the state of Missouri in 2009–2010. The focus was on Algebra I performance at the high school level since performance at this level is critical for success in college and beyond, as well as in light of current policies such as mandating algebra for all students. Since some of the districts do not have a high school or did not have any students take the Algebra I end-of-course exam, the actual number of districts used in the analyses was 471. The variables in this analysis were chosen because they are prominent in the school composition literature as factors that influence student achievement in successful and struggling schools (Hogrebe, Kyei-Blankson, & Zou, 2008; Hogrebe & Tate, 2010; Newton, 2010) (see Figure 1). The aim of the analysis is to determine if, and how, these variable relationships are a function of place. All variables were aggregated to the district level since the study examined the large geographical area of the state. In addition, the area of a district was assumed to approximate a reasonably homogeneous local context with the understanding that the underlying processes operate on a spatial continuum and are not restrained by man-made district boundaries.

Variables

Outcome

- *Algebra I MAP Index Score*. End-of-course scaled score, which was derived from an end-of-course exam, given to all Algebra I course participants. The exam was developed by the Missouri Department of Elementary and Secondary Education (DESE) under its Missouri Assessment Program (MAP).

District Composition

- *Free- or reduced-price lunch percentage (FRL).* Percentage of the district enrollment receiving free- or reduced-price lunches.
- *Minority student percentage.* Percentage of the district enrollment consisting of the total number of students in the following minority groups (Minority pct.): African-American, Hispanic, Asian, and American Indian.

Student Behaviors

- *Discipline incident rate.* Number of incidences reported divided by total enrollment in the district. An incident was reported when a student was removed from the traditional classroom setting for ten or more consecutive days. Multiple short sessions (cumulative removals adding up to 10 days) were not included.
- *Attendance rate.* Average daily attendance rate for the district.

Teaching Context

- *Students-to-classroom teacher ratio.* The ratio of students in the district to regular classroom teachers, excluding special education, remedial reading, Title I, and vocational teachers.
- High school teacher average salary. The average regular term salary of teachers in the district. Fringe benefits were not included.
- *Teacher with master's degree percentage*. Percentage of teachers in a district with master's degrees in any field.

Fiscal Context

- *Expenditure per average daily attendance.* District expenditures divided by average daily attendance.
- *Local revenue percentage.* Percentage of district revenue from local sources.

Figure 1.

The overall research question is: Do the relationships between the outcome variable of Algebra I performance (aggregated at the district level) and district composition variables, teaching and fiscal context variables, and student behavior variables differ across districts throughout the state?

Method

In order to determine if the relationship between district Algebra I performance and selected district composition, context, and student behavior variables differed across districts, variable relationships were defined and tested using geographically weighted regression (GWR) (Fotheringham et al., 2002). The GWR analysis was conducted using the geographically weighted regression procedure in the spatial statistics toolbox of ArcMap (ESRI, 2009). Subsequently, the local regression R^2 values and statistically significant beta coefficients from the GWR were given spatial perspective by mapping them with ArcMap. ArcMap is a geographic information system (GIS) software that integrates spatial data (e.g., geo-referenced coordinates such as latitude and longitude) and non-spatial data to produce geographic maps as well as providing procedures to analyze spatial relationships. GIS is based on the philosophy that location is important because variables and their relationships can vary by place and the space between them.

More specifically, GWR is designed to account for spatial dependence in clustered data and the fact that variable relationships may differ by location. GWR reflects geographically clustered data as a continuous spatial process whose variation can be represented on a map. This variation in relationships by location is referred to as "nonstationarity" and GWR is able to incorporate these local spatial relationships in the analysis approach (Fotheringham, 2009; Fotheringham et al., 2002). However, unlike a multilevel model, the coefficients in GWR are *not* assumed to be random, but instead are a direct function of their spatial location as determined by the geographic weights (Fotheringham et al., 2002, p. 52).

Using a spatial kerning process, GWR weights data points according to their proximity to a specific location (see Figure 2). Data points are not weighted equally across observations and thus vary by location. Data points closer to the specific location are weighted more heavily than more distant points. Through this process of differential weighting by location, GWR calculates an optimum number of "nearest neighbors" which are used to derive each local regression model.

GWR modifies the standard regression equation to include a geographic weight (u_i, v_i) which represents the coordinates of the *i*th point in space. The weights represent the proximity of each data point to the location of *i* such that points closer have more weight in the parameter estimation for location *i* (Fotheringham et al., 2002, p. 52). Incorporating geographic weights (u_i, v_i), the standard regression equation for GWR can be rewritten as follows:

$$y_i = \beta_0(u_i,v_i) + \beta_1(u_i,v_i)x_{i1} + \beta_k(u_i,v_i)x_{ik} + \varepsilon_i$$

GWR computes a local regression equation for each district based on the data from the district and the group of its nearest neighbors (Fotheringham, 2009; Fotheringham et al., 2002). In this study, data points are district polygons and the spatial kerning process employed is adaptive in that the size of the kernel changes as a function of the density or number of districts in an area (see Figure 3). Using the Akaike Information Criterion (AIC), the adaptive spatial kerning process determines the optimal number of nearest neighbors for each district, which results in the best fitting local regression equation.

In the present study, the relationship between each predictor variable and Algebra I performance was calculated separately using GWR. R^2 values and statistically significant beta coefficients were mapped with GIS to show district variation in local regression values across the state. In order to demonstrate the improvement in model fit using GWR, ordinary least squares results were also reported. In addition, two other models were included: one that used both FRL percent and minority percent, and one that added the interaction of FRL percent and minority percent to this model.

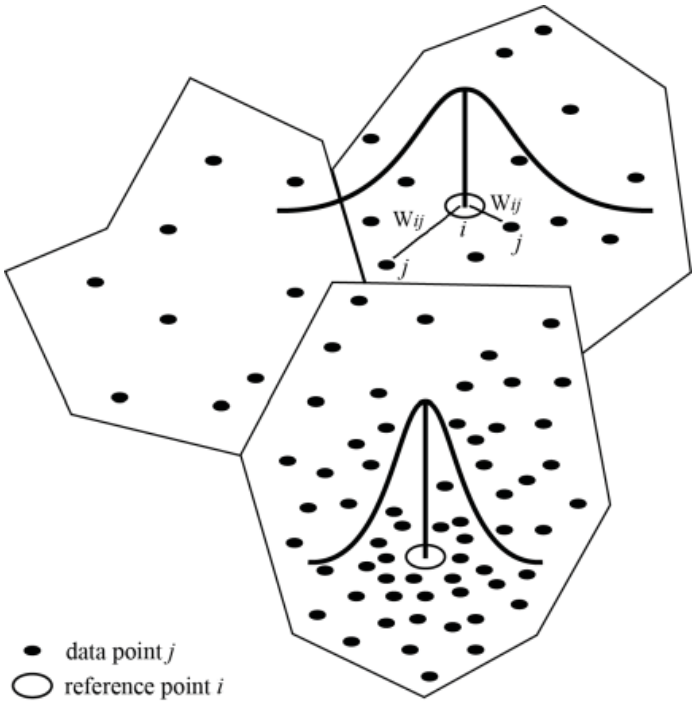

Figure 2. Adaptive spatial kerning for single data points

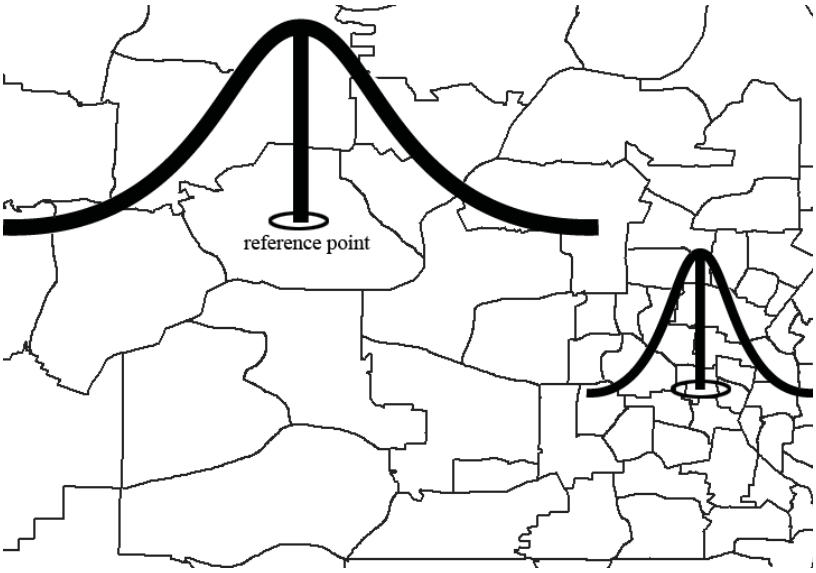

Figure 3. Adaptive spatial kerning for district polygons

Table 1. GWR and OLS Results of Individual Variables Predicting End-of-course Algebra I Scores

	GWR Results				OLS Results		
Category	[a]R^2	Adjusted R^2	[b]AIC	[c]Neighbors	[d]R^2	Adjusted R^2	AIC
District Composition							
FRL pct.	.210	.175	4504.9	141	.094	.092	4879.7
Minority pct.	.272	.172	4530.7	48	.068	.066	4888.6
FRL + Minority pct.	.231	.196	4494.2	190	.132	.129	4855.0
FRL pct. + Minority pct. + Interaction	.219	.181	4836.4	218	.144	.139	4850.2
Student Behavior							
Discipline rate	.221	.132	4545.3	64	.046	.044	4900.4
Attendance rate	.017	.007	4583.9	393	.017	.015	4915.3
Teaching Context							
Student/teacher ratio	.068	.043	4561.6	228	.013	.011	4917.6
Teacher salary	.108	.066	4563.9	132	.010	.008	4918.7
Master's degree	.263	.131	4561.8	44	.019	.017	4914.1
Fiscal Context							
Expenditures per ave. daily attendance	.019	.012	4581.1	471	.031	.029	4908.3
Local revenue pct.	.118	.079	4556.8	142	.032	.030	4907.7

[a]R^2 for all districts computed from observed and GWR predicted values at each location that gives a measure of model fit.

[b]Lower Akaike Information Criterion (AIC) values reflect better fitting models.

[c]Optimum number of "nearest neighbors" which were used to derive the GWR local regression model for each district.

[d]R^2 for ordinary least squares solution using all districts.

Results

The results of the GWR analyses were mapped using GIS to demonstrate how the relationship between Algebra I performance (aggregated for district) and the district variables of composition, teacher/fiscal context, and student behavior differed across districts throughout the state of Missouri. Table 1 lists the overall relationship between the variables for all districts as well as the adjusted R^2 values (also see Figures 4 and 5). The overall R^2 value for each variable is conceptually the squared correlation between the observed district Algebra I scores and the predicted scores based on the local models. This overall R^2 value provides an indication of model fit along with the Akaike Information Criterion (AIC). Variables with lower AIC values are better fitting models and tend to reflect the higher R^2 values. The overall R^2 values ranged from a high of 0.27 for the relationship between district minority percent and Algebra I scores to a low of 0.02 for the two attendance related variables. Since the global R^2 values for the attendance rate and expenditure per average daily attendance were near zero, the local R^2 values were not investigated.

For the purpose of further assessing the model fit using GWR, the OLS results were also reported in Table 1. It can be seen from the higher AIC values and the lower R^2 values that the OLS approach was masking the different local patterns in variable relationships across districts throughout the state. For example, the OLS R^2 for the relationship between Algebra I and minority percent was .068, while the GWR R^2 was .272. Similarly, the OLS R^2 for teachers with master's degrees was .019, but with GWR R^2 was .263.

The approach of reporting only the global OLS R^2 values is limited in that it fails to show the substantial heterogeneity in variable relationships across districts within the state. GWR computes local R^2 values for each district based upon its group of "nearest neighbor" districts. Figures 6 through 12 map the *local* R^2 values for the relationships that had the higher overall GWR R^2 values. Mapping local R^2 values for each district clearly shows that variable relationships can be quite different across the state. The relationships for Algebra I scores with FRL percent, minority percent, discipline incident rate, master's degree percent, and local revenue percent tended to be stronger (higher local R^2 values) around the larger urban metropolitan areas (i.e., St. Louis and Kansas City). The relationship of each of these variables with Algebra I scores cannot be adequately described with a single statewide OLS R^2 value. In similar fashion, the relationships for student/teacher ratio and teacher average salary with Algebra I scores varied across districts with the highest local R^2 values concentrated in the southern part of the state for student/teacher ratio, and specifically in the southwestern corner for teacher salary. GIS mapping of the GWR local R^2 values shows that variable relationships differ by local context and can be portrayed as representing an underlying spatial continuum.

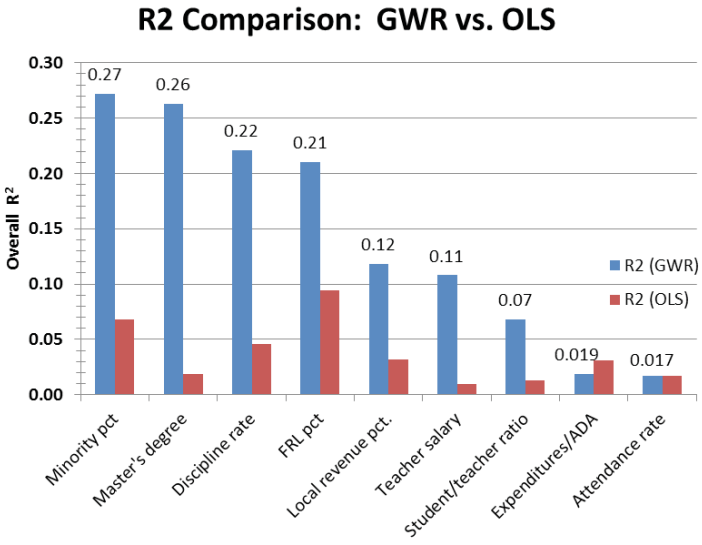

Figure 4. Data from Table 1, R^2 comparison between geographically weighted regression and ordinary least squares for relationship between each variable and Algebra I scores. (Left bar in pairs represents GWR.)

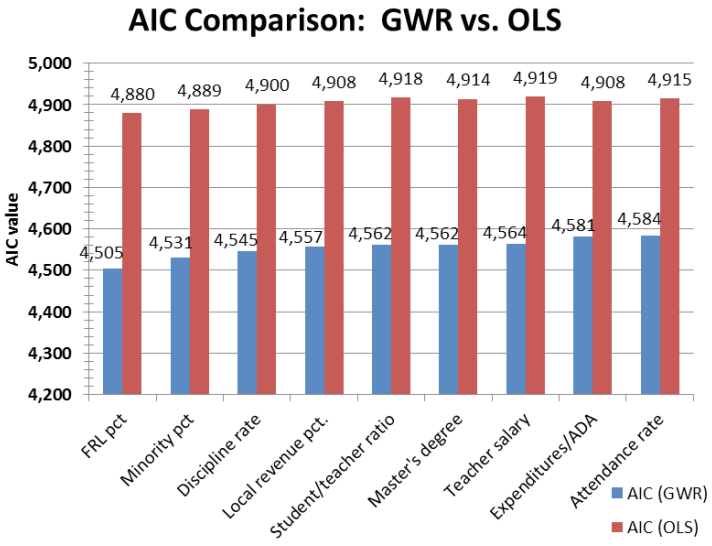

Figure 5. Data from Table 1, Akaike information criterion comparison between geographically weighted regression and ordinary least squares for relationship between each variable and Algebra I scores. (Left bar in pairs represents GWR.)

Two additional models were run that included more than one predictor variable in order to see if adding variables improved the model fit. Since both FRL and

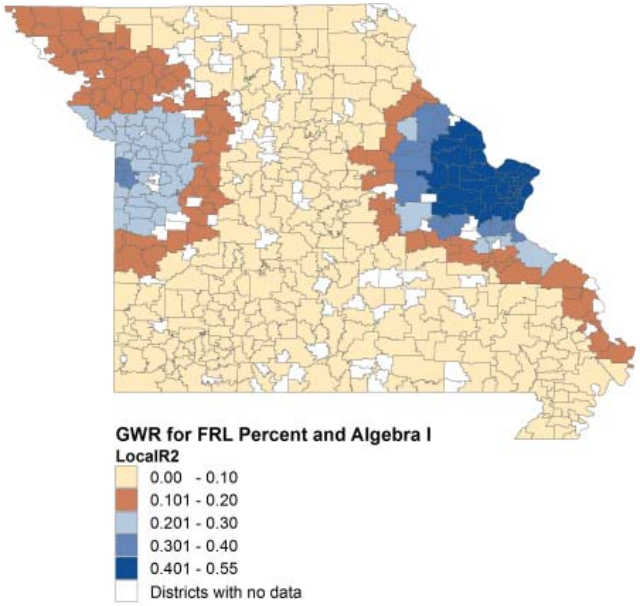

Figure 6. Local R^2 values for the relationship between free-reduced priced lunch and algebra I scores in Missouri school districts.

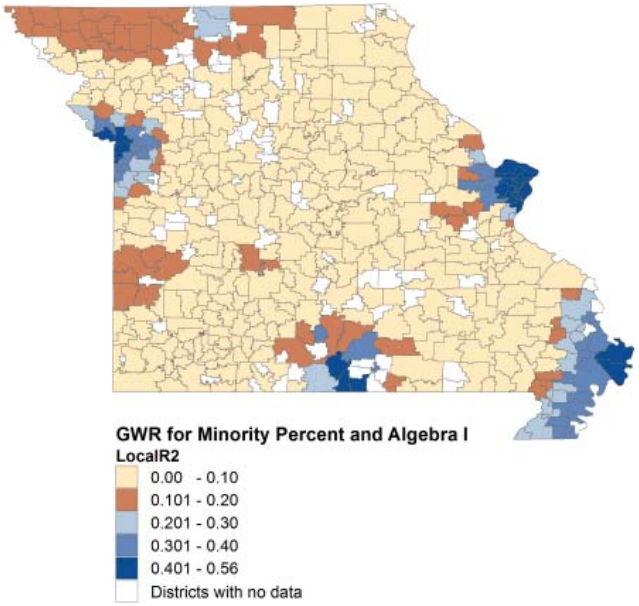

Figure 7. Local R^2 values for the relationship between minority percent and algebra I scores in Missouri school districts.

minority percent variables produced high local R^2 values when run separately, they were introduced into the same model. Also, another model added their interaction as well. The results reported in Table 1 show that entering both FRL and minority in the same equation produced a slightly lower AIC value for better model fit, but adding the interaction yielded a poorer fitting model.

Figures 6 through 12 show the variation in local R^2 values throughout the state, but how do we know which local regression models are statistically significant? One method is to plot the *t*-values of the beta coefficients for the predictor variable from each local regression equation. However, evaluating 471 district *t*-values at $p < .05$ would certainly produce significant *t*-values due to chance. In order to control for the high family-wise Type I error rate from 471 tests, we used the Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002) instead of the overly conservative Bonferroni technique. Thissen et. al. describe a practical implementation of the B-H procedure for controlling the false positive rate in multiple comparisons. Using the B-H technique to control the false positive rate for the 471 *t*-values, Figures 13 through 19 show the clusters that demonstrate where local variable relationships are statistically significant.

Discussion

This section discusses in more detail the maps in Figures 6 through 19. The clustering patterns of local R^2 values were compared to the clustering patterns that remained when only beta coefficients that were statistically significant were considered. Only the beta coefficients that were statistically significant after using the B-H technique to control the false positive rate in the 471 multiple comparisons were used in Figures 13 through 19, so the discussion below refers to those beta coefficients that remained significant after the correction.

Free-reduced price lunch percent. The local R^2 values for FRL percent and Algebra I scores ranged from near zero to .55 with the highest R^2 values clustered around the two major urban metropolitan areas of St. Louis and Kansas City (see Figure 6). Figure 13 shows that the significant beta coefficients covered the same metro areas and actually expanded to include much of the eastern and western regions of the state. Evidently, even some of the beta coefficients for FRL in equations with local R^2 values less than .10 were statistically significant. The relationship between FRL and Algebra I scores as depicted by the beta coefficients suggest that districts with a higher percentage of FRL students tend to have lower average Algebra I scores.

Minority percent. There were several distinct cluster patterns of local R^2 values for minority percent and Algebra I scores with values ranging from near zero to .56 (see Figure 7). Two distinct clusters of higher local R^2 values emerged again around the two large urban areas of St. Louis and Kansas City. In addition, there were clusters of higher local R^2 values in the southeast Missouri boot heel and along the south-central border. A long cluster of local R^2 values ranging from .10 to .30 was located along the northwestern state border. Figure 14 shows that the significant beta coefficients covered the same large metro areas, the boot heel

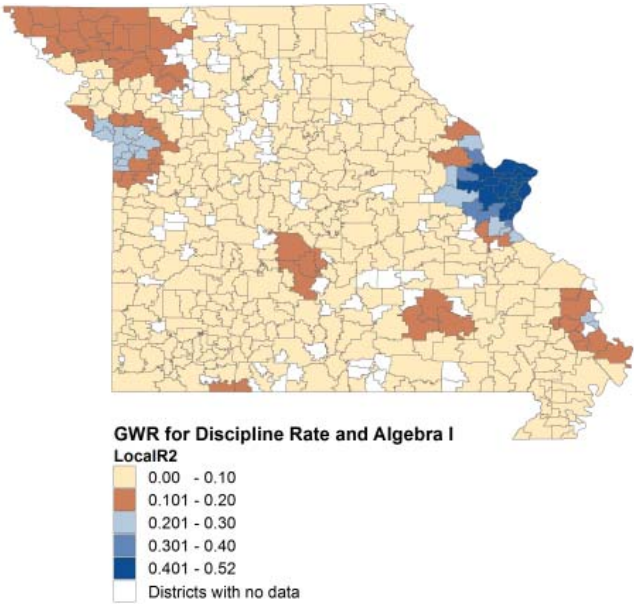

Figure 8. Local R^2 values for the relationship between discipline incident rate and algebra I scores in Missouri school districts.

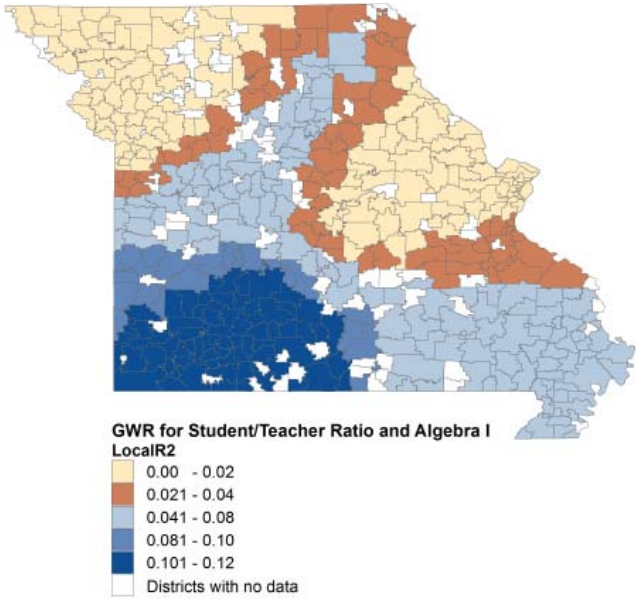

Figure 9. Local R^2 values for the relationship between student/teacher ratio and algebra I scores in Missouri school districts.

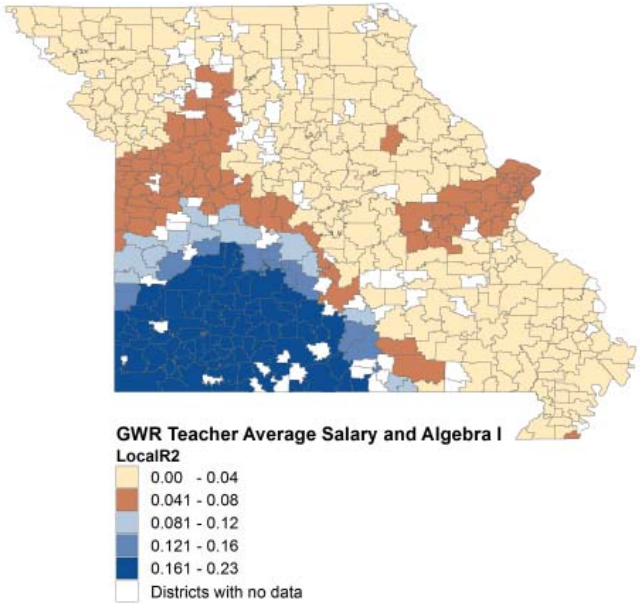

Figure 10. Local R^2 values for the relationship between average teacher salary and algebra I scores in Missouri school districts.

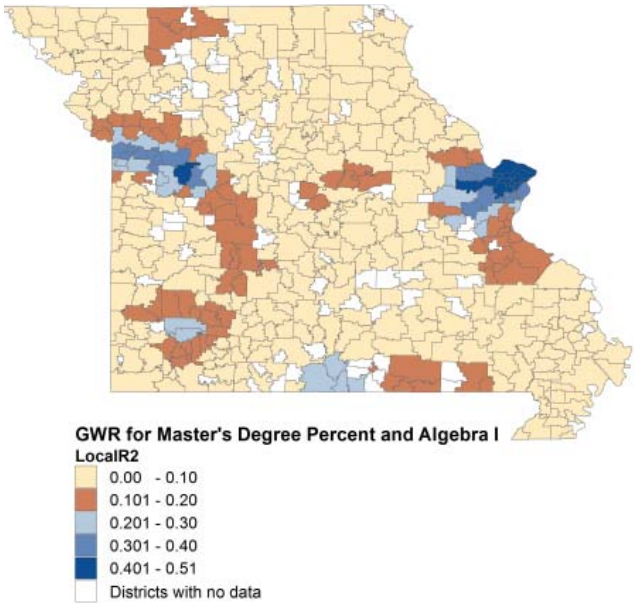

Figure 11. Local R^2 values for the relationship between teacher master's degree percent and algebra I scores in Missouri school districts.

region, and the northern border region. The negative sign for these coefficients indicated that lower Algebra I scores were associated with higher minority percent. However, the central southern border cluster and the southern western border cluster both had positive beta coefficient signs that associated higher minority percent with higher Algebra I scores. This is an example of how variable relationships can differ as a function of local contexts. It would be important for future study to determine why the positive relationship between minority percent and Algebra I scores exists in these clusters. One hypothesis may be that since the percentage of minority students is low in these regions, the larger enrollment districts tend to better accommodate the few minority students. The larger districts may have more

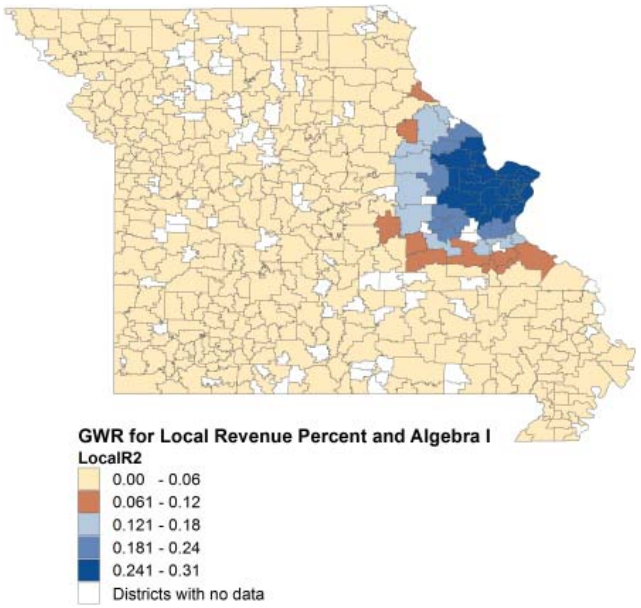

Figure 12. Local R^2 values for the relationship between local district revenue percent and algebra I scores in Missouri school districts.

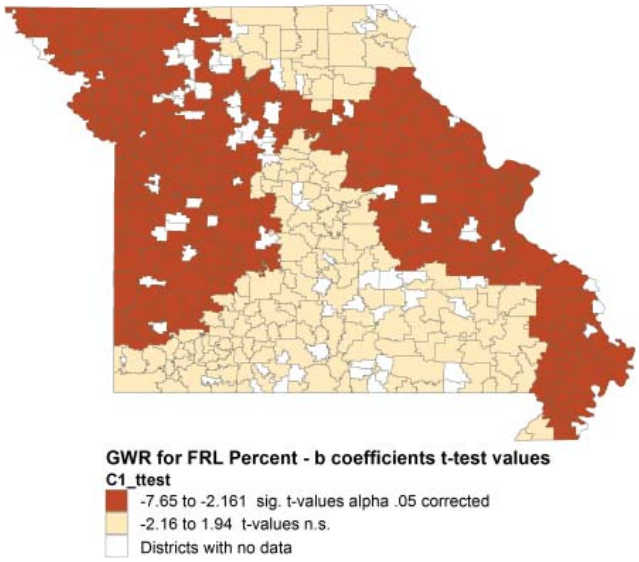

Figure 13. Statistically significant beta coefficients for the relationship between free-reduced priced lunch percent and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

resources to support students learning and their ability to score higher in Algebra I than smaller enrollment districts that have almost no minority students. The main point is that GWR identified clusters where the local context influences variable relationships in the opposite direction.

Discipline incidence rate. For the discipline incident rate and Algebra I scores, the local R^2 values ranged from near zero to .52 (see Figure 8). Clusters of higher local R^2 values were found around St. Louis and Kansas City, the northwest corner of the state, and several small clusters across the state's southern region. The negative relationship between the discipline incident rate and Algebra I scores can be seen for all the clusters except the one in the northwest corner. In the northwest corner, the local context shows positive relationship between discipline incident rates and Algebra I scores. In this instance, like in the discussion of minority percent above, it may be that the positive relationship is a function of an overall low discipline incident rate in the area. If the few discipline incidents occur in the larger enrollment districts that tend to have more resources and higher Algebra I scores than the smaller districts, then the relationship would be positive. Once again, more study is needed to determine what is unique about the context in this region that produced the unexpected positive relationship between the discipline incident rate and higher Algebra I scores.

Student/teacher ratio. The local R^2 values for the student/teacher ratio and Algebra I scores were lower and ranged from near zero to .12 (see Figure 9). The higher R^2 values for student/teacher ratio were concentrated most heavily in the southern part of the state with a narrow band running to the north through the center of the state (see Figure 16). This result contrasts with the findings for FRL percent, minority percent, and discipline incident rate in which the higher local R^2 values clustered around the two major urban metropolitan areas of St. Louis and Kansas City. Although the local R^2 values did not suggest a strong relationship, the positive direction of the beta coefficients indicated that higher student/teacher ratios tended to be associated with higher Algebra I scores in these areas. Average student/teacher ratios for districts in these areas did not exceed 22/1, which was similar to the ratios around the larger urban metro areas. The weak but significant positive relationship suggests that in the more rural areas of the state where some districts have low Algebra I enrollment, they also tend to have student/teacher ratios that are lower than the 22/1 along with lower Algebra I scores.

Teacher Average Salary. For teacher average salary and Algebra I scores, the local R^2 values were moderately low and ranged from near zero to .23 (see Figure 10). The higher local R^2 values formed a definite cluster in the southwestern corner of the state. The statistically significant beta coefficients were all in the southwestern cluster (see Figure 17). Evidently, only in the southwestern corner of the state were higher average teacher salaries associated with higher Algebra I scores.

Percentage of Teachers with Master's Degrees. There was a wide range of local R^2 values for the percentage of teachers with master's degrees and Algebra I scores that extended from near zero to .51 (see Figure 11). When

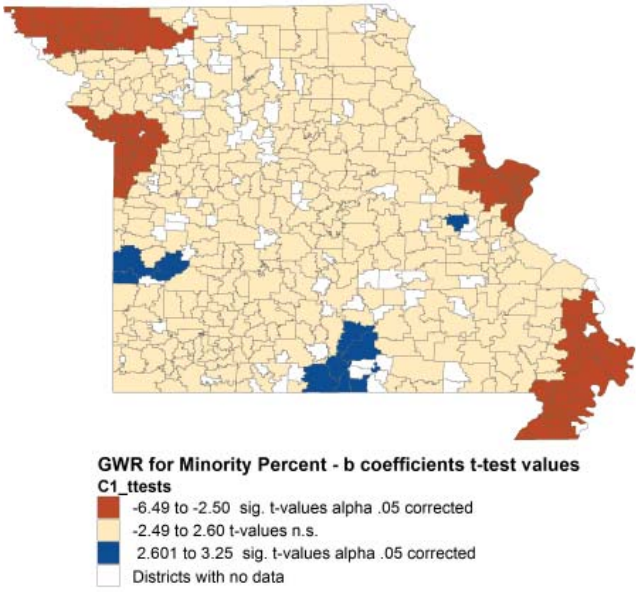

Figure 14. Statistically significant beta coefficients for the relationship between minority percent and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

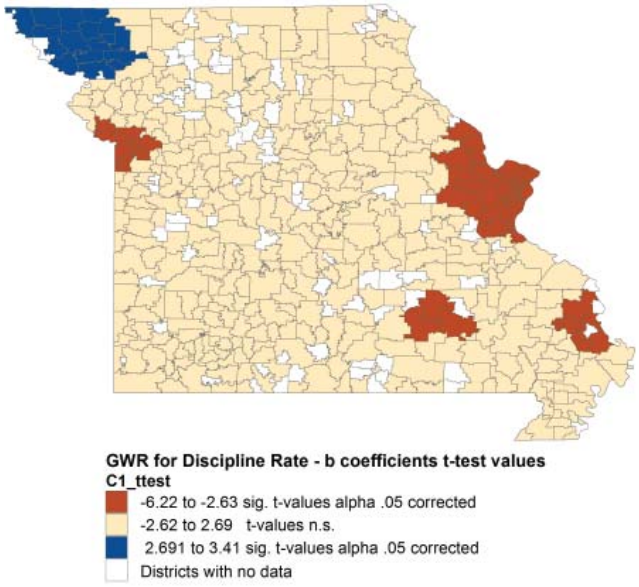

Figure 15. Statistically significant beta coefficients for the relationship between discipline incident rate and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

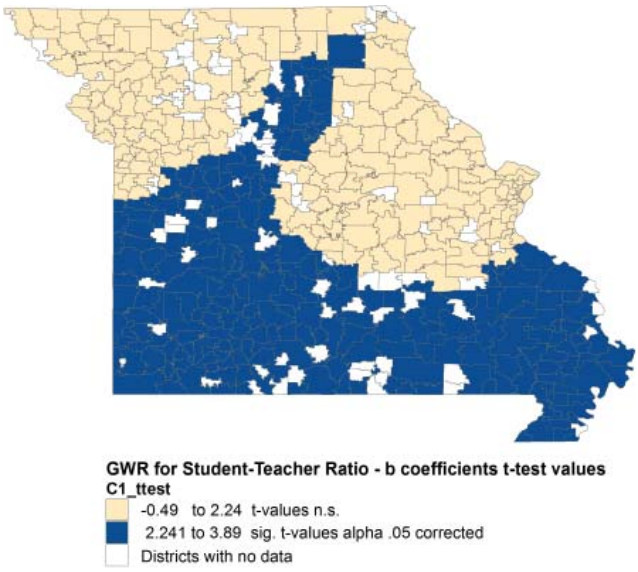

Figure 16. Statistically significant beta coefficients for the relationship between student/teacher ratio and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

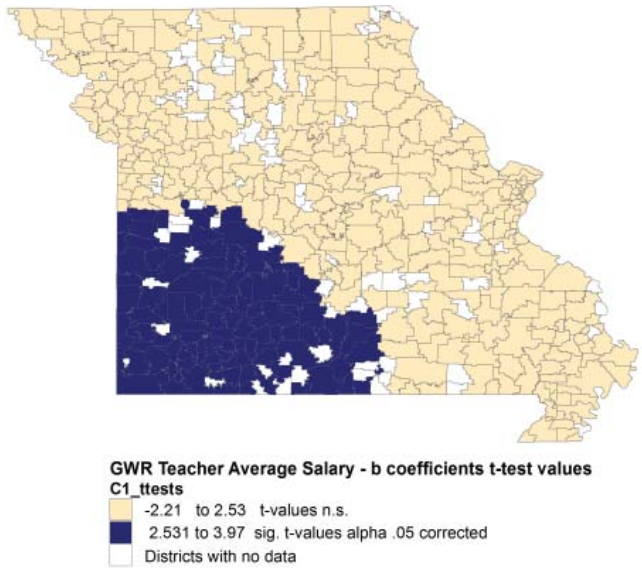

Figure 17. Statistically significant beta coefficients for the relationship between teacher average salary and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

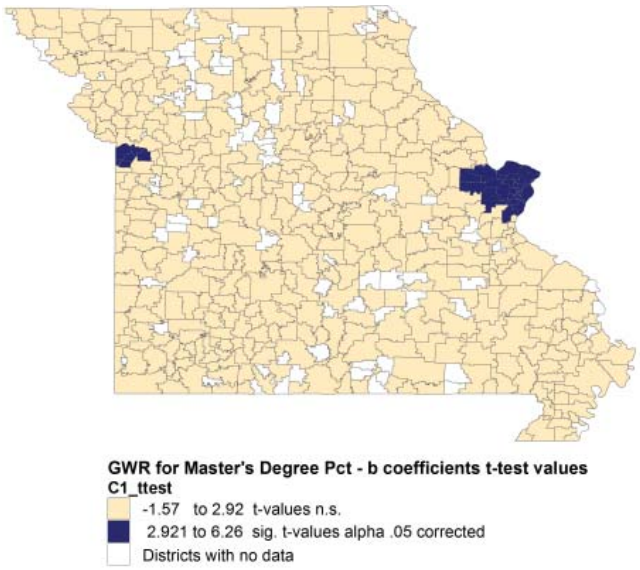

Figure 18. Statistically significant beta coefficients for the relationship between percentage of teachers with master's degrees and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

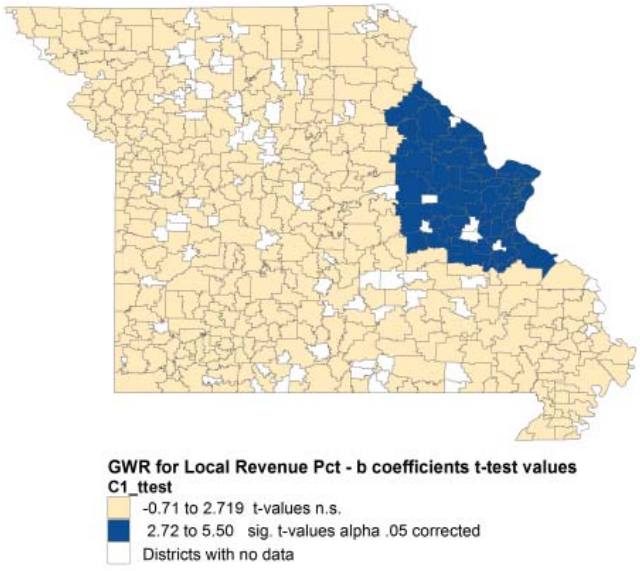

Figure 19. Statistically significant beta coefficients for the relationship between local revenue percent and algebra I scores in Missouri school districts. Multiple t-tests for beta coefficients corrected for false positive rate by Benjamini-Hochberg (B-H) procedure (Thissen, Steinberg, & Kuang, 2002)

statistically significant beta coefficients were mapped in Figure 18, the two clusters of high local R^2 values above .40 were in the St. Louis and Kansas City metro areas. Evidently, in these higher density urban areas, districts that have more teachers with master's degrees were associated with higher Algebra I scores. It is interesting to see how this relationship was concentrated in the larger urban areas. In light of debates about appropriate credentialing for mathematics teachers, this represents an area for further study.

Local Revenue Percent. The local R^2 values for percentage of revenue derived from local sources and Algebra I scores ranged from near zero to .31 (see Figure 12). The statistically significant beta coefficients shown in Figure 19 revealed that the only cluster of high local R^2 values was in the St. Louis area. It is interesting that only in the St. Louis metro region was a higher percentage of district revenue from local sources associated with higher Algebra I scores. This relationship represents an opportunity for additional study.

Conclusion

The results of this study demonstrate how variable relationships with Algebra I EOC scores differ based upon where school districts are located within the state of Missouri. It was striking to see how the strength of relationships varied and concentrated in clusters across regions. The cluster patterns differed as a function of variables and location. For example, there was a strong relationship between percentage of teachers with master's degrees and Algebra I scores only in the St. Louis and Kansas City areas, while a significant relationship with teacher average salary existed only in the southwest corner of the state. The use of geographically weighted regression and GIS produced maps that displayed where these clusters of stronger variable relationships existed.

These findings suggest that research on the equity movement in mathematics education, as well as discussion of mandatory algebra for all students must take local context into account. The analysis suggests that urban cities in Missouri are important targets for mathematics education interventions. While some variables may not appear to be strongly related to Algebra I performance when examined globally across a state, significant relationships existed regionally. In the case of FRL as a proxy for SES, the results show significant relationships to Algebra I scores along the eastern and western borders of the state, but non-significant relationships throughout the central regions (Figure 13).

When variable relationships are not seen as "stationary" but allowed to vary across location and geographic context, the inadequacy of sole reliance on a global measure becomes transparent. A single weather forecast for an entire state is usually inadequate for local areas since weather can vary dramatically across a region and depends upon local conditions. In similar fashion, variable relationships depend on local context and may not be the same everywhere. The bottom line is that "place matters" in moderating variable relationships between Algebra I and education variables

associated with SES and the poverty-affluence continuum. Location is important because there are differences in regions related to educational, social, cultural, and political influences, as well as differences in financial resources, employment, infrastructure, and housing factors. The results of this study demonstrate the need to take local context into account when analyzing variable relationships that may be moderated by geographic location and clustering. Using ordinary least squares for data that spans a continuous geographic space such as school districts across a state will most likely mask important relationships that exist within regions as a function of local contexts. We strongly encourage researchers to account for geographic clustering and spatial processes when examining variable relationships. This practice will greatly inform the knowledge base guiding the equity agenda in mathematics education.

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