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TABLE OF CONTENTS

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Two Modes of Game-Based Learning for Middle School Mathematics

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ABSTRACT Game-based learning has received more focus as a way to engage students and increase interest in mathematics. Game-based learning should be integrated with effective practices for teaching mathematics, though there is a need for further research on effective implementation models. The purpose of this article is to describe two modes of game-based learning that are aligned with National Council of Teachers of Mathematics effective practices for teaching mathematics including multiple representations and meaningful discourse. Examples will be discussed from the implementation of game-based learning in a Saturday Science, Technology, Engineering and Mathematics (STEM) program for middle school students offered at a large research university.

KEYWORDS *game-based learning, integrated STEM education, middle school*

Middle school is a crucial time for building students' interest in mathematics. In the elementary grades teachers often make use of multiple representations of mathematical concepts using manipulatives and technology, but this occurs less in the middle school grades (Van de Walle et al., 2018). Student interest in mathematics is an important consideration as too often middle school students perceive mathematics to be dull, irrelevant, or too difficult (Grootenboer & Marshman, 2016). In addition, students' interest in mathematics generally declines during the middle school years (Franzel et al., 2010). Making connections to students' interests may be one way to increase interest in mathematics. For instance, students can play sports or video games for hours with few breaks and the time can go by quickly because they are so engaged. A question thus arises: Why can learning in school not foster this kind of engagement?

Game-based learning has been suggested as one way to increase student engagement with mathematics and develop mathematical understanding (Foster & Shah, 2015; Stohlmann, 2019a; Wang et al., 2018). Game-based learning can support mathematical understanding through the incorporation of multiple representations including language, symbolic, pictorial, real-world, and concrete representations. Game-based learning also supports active student-centered learning, including learning through exploration and failure (Devlin, 2011). I adopt Oldfield's definition of *game* (1991): a game is a task that involves a challenge against an opponent, is governed by a set of rules, has a distinct finishing point, and is connected to specific mathematical objectives. When students play games they persevere in problem solving, try new approaches, and continue to develop their strategies when encountering setbacks or failures (Stohlmann et al., 2018). These behaviors are all characteristics of a growth mindset: the belief that one's skills or knowledge can be cultivated through effort (Dweck, 2006), which can lead to increased effort, engagement, and mathematical understanding.

In my prior work I investigated the impact of game-based learning with a class of middle school students that were part of a four-week Saturday Science, Technology, Engineering, and Mathematics (STEM) program. A pre- and post-survey showed that students had statistically significant improvements in aligning their mindsets with a growth mindset. They also improved in the quality of their solutions from the games; by the last week students described more complete strategies, communicated their solutions more clearly, and demonstrated more developed mathematical understanding. Additionally, student engagement was high throughout the program (Stohlmann et al., 2018).

Drawing on this work and my additional implementation of game-based learning with middle school students (Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a; Stohlmann & Kim, 2020), the purpose of this article is to describe two effective modes of game-based learning that middle school teachers can implement: context game-based learning and integrated steM game-based learning. Context game-based learning involves integrating games to pose interesting mathematical questions. In this approach the games are used as a motivating context for the related mathematical task. Integrated steM game-based learning, meanwhile, is the integration of STEM subjects with an explicit focus on mathematics through open-ended technology-based games. In this approach the mathematics is integrated into the gameplay in a substantial way beyond traditional practice problems. Here, students use mathematical thinking and ideas as they play the game. Both modes of game-based learning are aligned with National Council of Teachers of Mathematics (NCTM) effective mathematics teaching practices (NCTM, 2014) including multiple representations and meaningful discourse.

In describing the two modes, I draw on examples of game-based learning that I implemented in the Saturday STEM program. One of the goals of the program was for students to be engaged while completing interesting mathematical tasks. If students' interest and engagement in mathematics can be increased, then it will be more likely that more students will do well in mathematics and possibly be interested in STEM fields (Gandhi-Lee et al., 2015; Lesseig et al., 2016; National Science Board, 2018; Stohlmann, 2019b).

Game-based Learning

Game-based learning has drawn international interest as it has the potential to improve students' motivation and performance in mathematics (Byun & Joung, 2018; Foster & Shah, 2015; Galarza, 2019; Wang et al., 2018). A meta-analysis of studies done with K-16 students found that when compared to nongame comparison groups, technology game-based learning yielded on average a 0.33 standard deviation improvement in learning outcomes across a variety of content areas (Clark et al., 2016). It is possible that the improvement in learning outcomes could be increased with further classroom research on game-based learning that is integrated with best practices for teaching.

The increased prevalence of tablets and computers in schools has created a need to ensure that these technological resources are being wisely used to support students' mathematical understanding (Takeuchi & Vaala, 2014). In a national survey of K-8 teachers in the U.S., 74% reported integrating digital games in instruction, though 80% reported that they would like it to be easier to identify games aligned with standards and objectives (Takeuchi & Vaala, 2014).

Supporting all teachers with game-based learning implementation is important. Denham (2019) conducted case study research with three middle school teachers to investigate how the teachers began to integrate game-based learning. The teachers reported feeling a lack of confidence in how to facilitate the game-based learning effectively. They also noted the importance of thoroughly analyzing the game beforehand to identify how to make the mathematical connections explicit; identify where students may struggle; and note any prerequisite skills students might need to play the game. These findings highlight the importance of having quality examples of game-based learning implementation that can help teachers be confident and successful in implementing game-based learning. Further, the teachers noticed that student engagement was high during the game-based lessons and that the students who usually struggled in mathematics class benefited most from the game-based lessons (Denham, 2019). Their observations highlight the importance of supporting teachers with effective models for game-based learning through teacher-led workshops and/or professional development. This may lead to better game-based learning implementation as many teachers learn about game-based learning from other teachers (Takeuchi & Vaala, 2014). There is a need, then, for guidance on "how, when, for whom, and under what conditions to integrate digital games into formal education" (Van Eck, 2015, p.13).

Implementation and Structure of Game-Based Learning

Guidance and recommendations for effective implementation of game-based learning are important because game-based learning is often not implemented with best practices for teaching mathematics in mind (Byun & Joung, 2018). In 2018, Byun and Joung conducted a meta-analysis of research studies examining the effect of game-based learning on K-12 students' mathematics achievement between 2000 to 2014. Of the 71 authors in the studies reviewed for the meta-analysis, only five of these authors had a background in mathematics education. The effects of not including mathematics education expertise in the majority of the research studies were seen in how the games were structured. Most of the games used in the studies involved drill and practice (Byun & Joung, 2018). In drill and practice-type games, students only receive feedback if answers are correct or incorrect and do not receive support for conceptual understanding. These types of games also emphasize that mathematics is about speed and memorization of facts instead of building conceptual understanding (Bay-Williams & Kling, 2015). Game-based learning for mathematics should move beyond games where drill and practice is the focus so that mathematics is not learned as memorization without understanding. This prior research supports the need for further work in game-based learning in the mathematics education research community so that game-based learning is implemented with effective mathematics teaching practices that support conceptual understanding (NCTM, 2014).

In implementing game-based learning, it is important for teachers not to view games as stand-alone instruction (Galarza, 2019). Instead, teachers should provide students with the opportunity to discuss and reflect on the mathematics within the games. This can involve whole-class discussion and reflection prompts to further develop the mathematical knowledge presented in the games. Foster and Shah (2015) detail one model for this. They propose a four-phase Play Curricular-activity Reflection and Discussion (PCaRD) pedagogical model for game-based learning implementation. In the first phase, play, students are given adequate time to play the game without interruption. During this phase, students are encouraged to discuss the game with other students. The teacher's role is to monitor students while they are working to gather student ideas to share during whole class discussion. The next phase, curricular-activity, is when students complete activities to connect their experience in the game to desired mathematical learning objectives. In the reflection phase, students individually think about and reflect on the connections between the mathematics and the game-play. During this phase the teacher continues to gauge student understanding through listening to or looking at students' ideas and providing assistance as needed. The teacher leads a whole-class discussion in the final phase to address any misconceptions and highlight important student insights.

In my teaching and research with game-based learning, I found that the discussion phase is better as phase three and the reflection phase as phase four. Having the whole class discussion before individual reflection allows for students to better develop their understanding, clear up any misconceptions, and make the mathematics more explicit. It has been noted that students can have difficulty with reflection on game-based learning (Denham, 2019). Having the whole-class discussion first enables students to make mathematical connections and have the opportunity to determine what they know and what questions they still may have.

The final reflection phase can then be implemented using formative assessment strategies to guide future instruction. Formative assessments used at the end of class often include exit slips or journal prompts. This helps students process what they have learned. For example, students can be asked to write three things they learned that are connected to the mathematical objectives, two things they did well, and one question they still have. Reflection through formative assessments can help students metacognitively monitor their understanding and provide the teacher with information to inform further instruction. Teachers should then use students' responses to identify learning needs or make modifications for future lessons (Black & Wiliam, 1998).

In addition to the implementation of game-based learning, learning how the games are structured is vital. From my research I have developed four general principles for how game-based learning should be structured. First, the mathematical objectives should be connected to grade-level mathematics standards. This is important for any mathematical task or activity so that students learn the required mathematical content (Stohlmann et al., 2011). Second, multiple representations should be integrated. Multiple representations help to develop conceptual understanding through students' use of realistic, language, symbolic, pictorial, and concrete representations (Lesh & Doerr, 2003). There should be support for conceptual understanding of the mathematics in the games and not just problems posed symbolically (Stohlmann, 2019a). Third, discussions and activities make explicit connections to the mathematics in the games (Foster & Shah, 2015; Galarza, 2019). Fourth, students should receive feedback from the game or other students during gameplay. Feedback is an important benefit of technology integration (Schenke et al., 2014; Stohlmann, 2019b) and student-to-student feedback can benefit students' mathematical understanding (Keeley & Tobey, 2011).

Game-based learning implementation is most suc-

Table 1
NCTM Eight Effective Teaching Practices

Effective Teaching Practice	Description
Establish mathematics goals for learning	Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations	Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse	Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions	Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding	Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics	Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking	Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

cessful when it is aligned with effective mathematics teaching practices (Kiili et al., 2015). In this way, game-based learning is not a new teaching technique to be learned, but aligns well with existing teaching practices. Table 1 outlines NCTM's (2014) effective teaching practices that connect well with my principles for game-based learning.

The eight practices align with how game-based learning should be implemented. Clear mathematical goals should be set for game-based learning. Game-based learning should promote reasoning and problem solving as students explore and try new ideas. To be the most effective, game-based learning should incorporate multiple representations to support conceptual understanding. Discourse is included as students discuss their ideas with classmates and in whole class discussion. Teachers should plan purposeful questions to ensure students make connections that are aligned with the mathematical objectives for the lesson. Conceptual understanding can be developed through learning done in relevant contexts supported by multiple representations. Productive struggle is seen as students stay engaged with the work and persevere in problem solving. As teachers monitor students working, they elicit student thinking and use this as the basis for discussion. In this article I focus on four of the effective teaching practices that align especially well with game-based learning: (1) establish mathematical goals for learning, (2) use and connect mathematical representations, (3) facilitate meaningful mathematical discourse, and (4) pose purposeful questions.

Two Modes of Game-Based Learning

I now describe two ways to implement game-based learning that I implemented with middle school students (Stohlmann et al., 2018; Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a): context game-based learning and integrated steM game-based learning. I will then discuss how the four previously mentioned effective teaching practices were integrated into each mode. The students were participants in a Saturday STEM program focused on game-based learning. The students who consented to participate in the study were sixth to eighth grade students, most of whom were in sixth and seventh grade. The implementation was structured as a teaching experiment (English, 2003).

Context Game-based Learning

In context game-based learning, games are used as an interesting context to pose related mathematical problems. Context game-based learning does not need to be technology-based. When students play the games, they are not doing mathematics. The games are played for students to collect data or to become familiar with the game so that mathematics questions can be posed. Students do mathematics after the game when solving these mathematics problems. Games can be taken from popular culture to increase student engagement.

Water Bottle Flipping Activity

The first example I will consider is a water bottle flipping activity. The mathematical objectives for this activity are for students to analyze and solve proportional and linear equations through tables and equations. In this activity students play a game to see how many times in a minute they can flip a partially filled water bottle and get it to land straight up (Stohlmann, 2020b). Students are directed to do five one-minute trials in which they record how many "lands" of the water bottle they can make. The world record for this is 47 lands in one minute. The implementation of this activity was completed in the following sequence: (1) The teacher explains the game to the students and an overview of the accompanying worksheet students will complete (Figure 1). (2) Students play the game. Then, for the mathematical part, students calculate their average number of lands per minute based on the data they collected. Students then fill in a table using this average and answer follow-up question (Figure 1). (3) Students complete the questions on the worksheet. (4) The teacher facilitates a whole-class discussion on students' ideas from the worksheet. (5) Using equations from students, the teacher facilitates additional questions on interpreting graphs in the context of the game. The questions in this game-based learning implementation are planned so that students work with multiple representations—realistic context, table, graph, language, and equations—aligned with the lesson goals.

Class Discussion. Students were able to answer the questions in Figure 1 and some students created their own questions to see how long it would take them to do different numbers of lands, such as 1,000. After students completed the questions in Figure 1, a *Desmos* graphing calculator was used to display one of the students' equations for number of lands: $y = 7x$. The students were asked what the slope of the graph meant. They were able to share that the slope was the number of lands per minute. Another student had an equation of $y = 5.5x$ and so I asked if it was possible to get five and a half lands in a minute. A student replied, "that you cannot have half a land." I followed up with asking, "How did he arrive at 5.5 for the slope?" A student shared that it was from five minutes of flipping bottles. Then I asked, "What was done with the number of lands to get 5.5?" Through further discussion students came to see that in the context of how the data was collected, average number of lands per minute was more precise for the description of the slope.

A couple points on the line $y = 7x$ were then highlighted and students were asked what the points represented. For (4, 28) a student responded that "4 is the number of minutes and 28 is the number of lands." I then asked students how they could use the graph to

Figure 1

Water Bottle Flipping Activity Questions

Fill in the table below using your landing rate for 1 minute. Is the relation between time and lands linear?

Time (minutes)	Lands
0	
1	
2	
3	
4	

Write an equation for number of lands (y) made based on time in minutes (n)

$y =$ ____________________

- Use your equation to determine how many lands would you make if you tossed for 10 minutes?
- Use your equation to determine how long it would take you to make 100 lands?
- If you were given a 10 lands head start, how long would it take you for your total lands to be 145?

solve the questions from Figure 1. After a few minutes of thinking, a student shared, “You can go on the line where the number of minutes is 10 and then see what the number is for the number of lands.” Another student shared, “If you know the number of lands, then you start with that and then see what it matches on the line for the number of minutes.” Students were able to see that a graphical approach can be used in addition to a symbolic approach for solving equations.

Analysis of Learning. The water bottle flipping game implementation connected well with effective teaching practices. There were mathematical goals for the lesson that aligned with questions asked related to the game. Through monitoring students while they were working and the whole class discussion it was evident that students were able to make connections between multiple representations through graphs, tables, equations, and the context of the game. For example, students demonstrated the ability to connect multiple representations when they noted the possibility of extending the graph to predict future outcomes. Meaningful discourse about mathematics occurred as students discussed and shared their ideas in the context of the game. Highlighting this is the discussion on the meaning of slope in the students' equations and what the points on the lines represented. Purposeful questions were posed to have students solve problems in the context of the game and interpret the graph in the context of the game. The questions in Figure 1 and the whole class discussion questions were planned so that students would use graphs, tables, and equations. Students found the planned questions engaging and also the questions asked in the whole class discussion. Doing mathematics in a related problem to the context of water bottle flipping engaged students through movement and healthy competition. The mathematical tasks posed in an interesting relevant context also engaged students in the mathematical work and enabled the mathematics to be discussed in the relevant context of the problem. This engagement was seen as students' conversations were focused on the game and the associated mathematical work with no off-topic discussions.

Integrated steM Game-based Learning

The second mode of game-based learning, integrated steM game-based learning, involves open-ended technology-based games. The integration of the STEM disciplines has the potential to bring together overlapping concepts and mathematical and science practices in meaningful ways. This integration is becoming more important because many jobs rely on STEM knowledge (Fayer et al., 2017). However, there is no widely agreed-upon idea for how this best translates into K-12 education. Schools, districts, and researchers can have different interpretations of STEM education that lead to different implementation models (Johnson, 2013). Further research is needed to investigate effective models for integrated STEM education (Lesseig et al., 2016).

It has been noted that mathematics is often not emphasized in the integration of STEM subjects (English, 2017; Gravemeijer et al., 2017). In response to this, I have proposed that mathematics teachers and researchers focus on integrated steM. Integrated steM is the integration of STEM subjects with an explicit focus on mathematics (Stohlmann, 2018b). It is an effort to combine mathematics with at least one of the three disciplines of science, technology, and engineering, into a class, unit, or lesson that is based on connections between the subjects and that has open-ended problems (Stohlmann, 2018b).

My definition of integrated steM education through game-based learning has several important features that help ensure quality implementation. First, the technology integration should allow for significant task redesign or the creation of new tasks that would not be possible without the technology (Puentedura, 2006). Second, the tasks used should be worthwhile tasks. Hiebert et al. (1997) define worthwhile tasks as tasks without prescribed rules, methods, or perceptions of a specific "correct" solution method. Third, the tasks should be aligned with grade-level standards. Fourth, the tasks should enable students to work with multiple representations. Fifth, the technology should provide students feedback. Finally, the tasks should be open-ended so as to allow for discussion and multiple solutions (Stohlmann, 2019a). When structured well, technology-based mathematics games can engage students in mathematics and help develop their conceptual understanding (Stohlmann, 2019a).

The DiRT Dash Game

The DiRT Dash game from NCTM's Calculation Nation website (NCTM, 2020) is one example of game-based learning through integrated steM education. The mathematical objectives for this game are for the students to use proportional reasoning to solve real-world problems. The implementation of this activity was completed in the following sequence: (1) The teacher explained the directions for the game and did an example game for the whole class. (2) Students played the game for approximately 20 minutes. (3) Students discussed in small groups how to do well in the game. (4) Students provided a written response for how to do well in the game.

(5) Students completed follow-up questions to explore if a straight line is always the best path. (6) The teacher facilitated a whole-class discussion on how to do well in the game and if a straight line is always the best path to take. (7) Further questions were posed using the DiRT dash game end of race information.

Focusing on the relationship between Distance, Rate, and Time, the object of the online game is to drive a virtual car on a variety of terrains to beat one's opponent to the finish line. The race terrain is a patchwork of rectangles that each represent one of three terrains: pavement, dirt, or water (Figure 2). Before the beginning of each race there is the option to select a vehicle. Each vehicle has different speeds for the three terrains. Selecting the right vehicle for each game terrain increases the chances of winning.

Class Discussion. After playing this game several times students were asked to explain how to do well in the game. This question was posed to see if students were considering the relationship between the rate of a vehicle for the different terrains and planning the best path to take. This mathematical reasoning is essential for doing well in the game. Students discussed in small groups how to win the game and then provided a written response. The following are examples of three student explanations from when I implemented this game with middle school students.

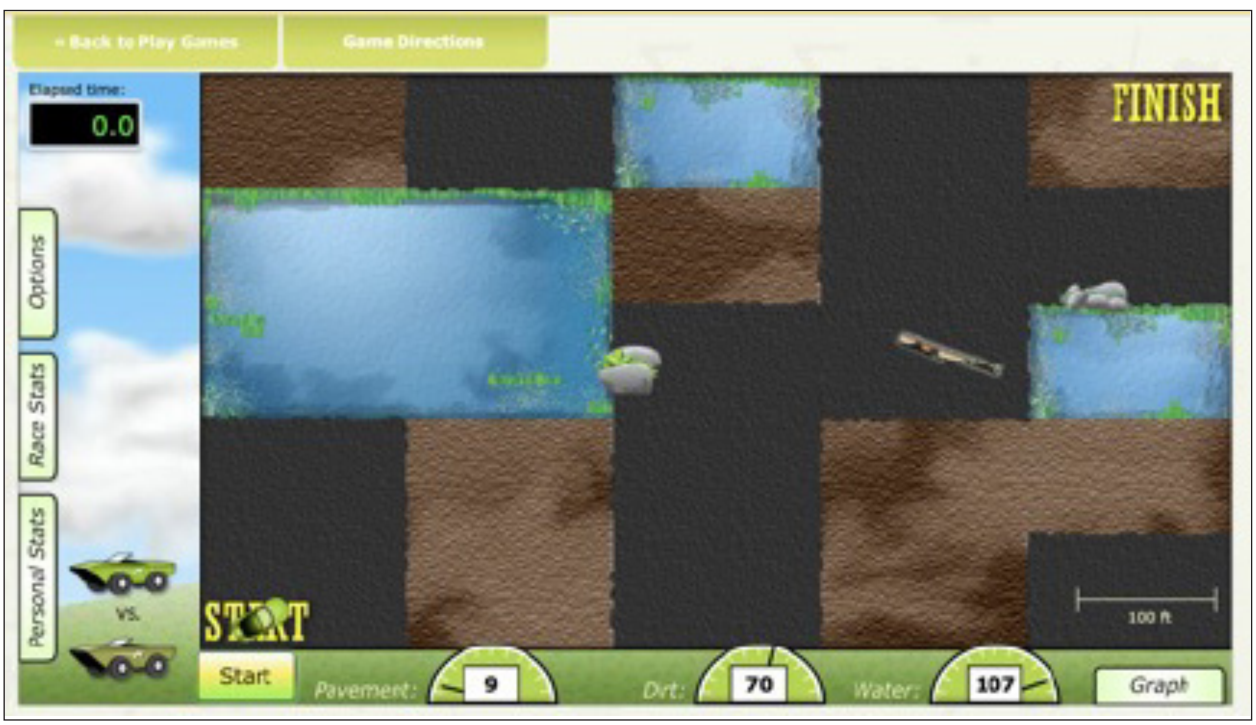
"To win the dirt game you would have to pick a car that would match the map. Also you would have to find the fastest way to get to the finish line."

"1. Just cut them off in the start of the race, then just make sure the other car is behind you the whole time.
2. Choose the car that has the most advantages on the area.
3. Try to find the fastest way to the finishing area."

"The best way to win the Dirt Dash game on Calculation Nation is to measure the amount of each element on your path to the finish, and then prioritize certain elements, and choose the car with the high speeds for the element(s) you prioritized."

Though the written examples above do not incorporate specific rate calculations, students used this ratio and rate reasoning in analyzing the terrain of each map including the length of each type of terrain, the best path to take, and the fastest car for this path. Students looked

Figure 2
Example DiRT Dash Game Terrain (NCTM, 2020)



at the terrain and determined the best car to pick, keeping in mind the length of each type of terrain and the available cars speed. This involved determining the car that had the most advantages on the area (terrain type) and then finding the fastest way to the finishing area, keeping in mind the terrain type and the car's speed for each terrain type. Students can realize that a straight line is not always the best path to take in the game in order to win. If there is a vehicle with high speed for a different path in the game, this can lead to a first place finish. Students can use the scale provided in the game to determine the distance that would need to be traveled on the different terrains for different paths to the finish line. As they do this they consider the speed for the different vehicle options. Students did not do the exact calculations but used this mathematical reasoning in the game.

Extension. In order to incorporate further mathematics work with exact calculations by exploring the path taken across different terrains, NCTM has an *Illuminations* activity connected to the DiRT Dash game (Slowbe, 2020). In the DiRT Dash game students did not calculate the exact time it should take them to get to the finish line before playing the game. In this *Illuminations* activity students calculate the exact time for different paths using distance and rate. The *Illuminations* website provides standards-based resources and materials that can be used to implement NCTM's effective teaching practices. The DiRT dash game *Illuminations* activity poses several scenarios to students on the quickest way to get from one place to another by moving through the terrain of sand and/or water. For example, in Figure 3 students are asked to calculate the time it would take to get to their brother when traveling in a straight line towards him. Students can then determine the quickest route to take.

Further, questions can also be posed from the DiRT Dash gameplay. At the end of each race the distance traveled and the time for each terrain are given. A graph is also provided that shows each vehicle's distance traveled and the terrain for the path that was taken (Figure 4). Students can calculate the rate for each terrain and compare this with the ideal rate for each vehicle. The ideal rate may not always be achieved if students have trouble steering or hit an obstacle. Sample graphs can be given as well for students to interpret the graphs in the context of the game for what occurred in the race. For example, I used Figure 4 to ask students questions related to average speed and sections of the graph where there are horizontal lines. Students were asked which vehicle had the greatest average speed between 3 seconds and 12 seconds. Through discussion, students came to realize that they needed to look at the distance traveled for each vehicle in relation to time. I also asked students to explain what was happening when a horizontal line was on the graph. A common misconception was that the vehicle is moving at a constant speed. Since the x-axis is time and the y-axis, distance traveled, students should explain that the vehicle is not moving when there is a horizontal line.

Analysis of Learning. The DiRT Dash game implementation connected well with effective teaching practices. There were mathematical goals for the lesson, for students to use ratio and rate reasoning, that aligned with the mathematics students used during the game and questions asked related to the game. Students were given the opportunity to make connections between multiple representations through discussions about the game with other students, the real-world context of the game, and the symbolic mathematics. Meaningful discourse occurred as students discussed and shared their ideas in the whole-class discussion and as I monitored their small group discussion. Students considered the relationship between distance and the rate of speed of

Figure 3

Example DiRT Dash Game Illuminations Activity Question (Slowbe, 2020)

Your brother notices a starfish on the ocean floor and calls for you to see it. You want to get there as fast as possible, but you are starting in the sand. Refer to the picture below.

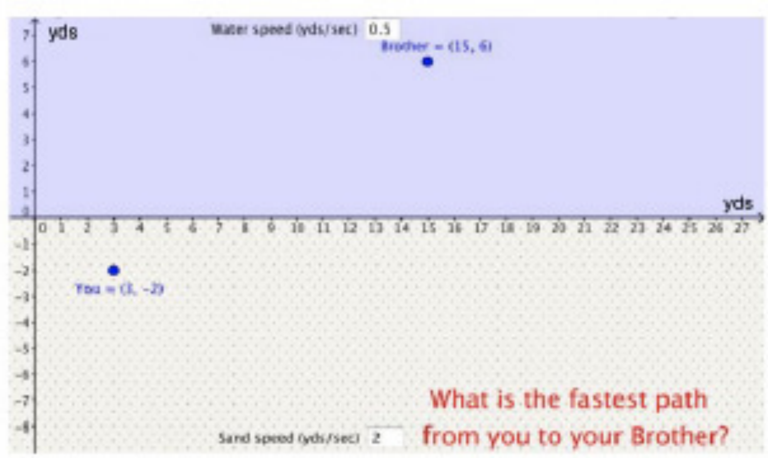
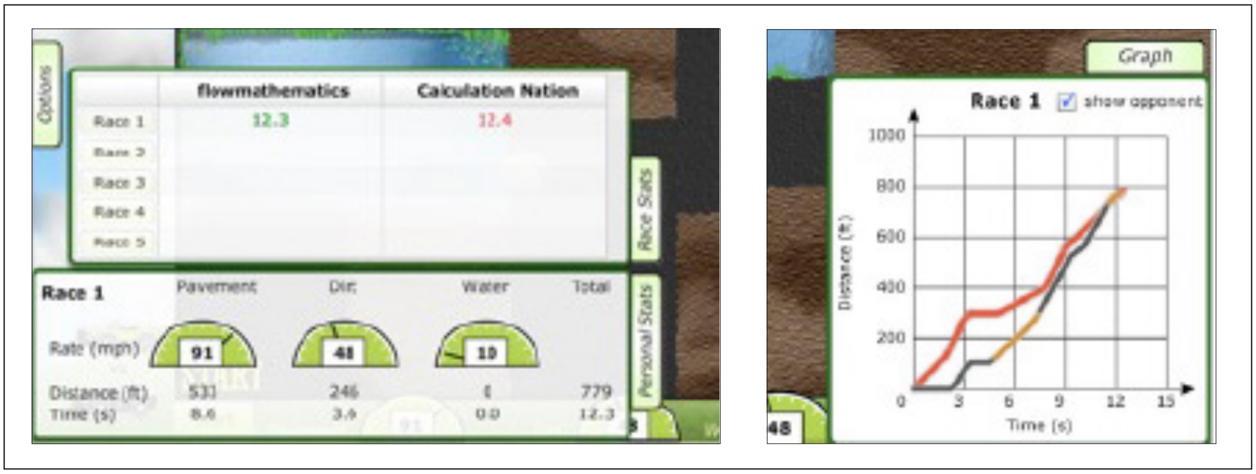


Figure 4

Example DiRT Dash Game End of Race Information (NCTM, 2020)



the vehicles to get to the finish line in the quickest time. For example, a class discussion involved students solving $d = rt$ for t and then discussing how to minimize the time. In the context of the game students discussed generally wanting the distance to be smaller and the rate larger for $t = \frac{d}{r}$. Purposeful questions were posed to have students solve problems in the context of the game and interpret the graph in the context of the game. For example, the questions posed in the *Illuminations* activity had students determine the best path to take to arrive in the quickest time. Students were also asked to describe an end of race graph for someone else's race to see if they could explain what happened in the race. The students were engaged with the game and were able to describe strategies for doing well in the game through the written responses and whole class discussion.

Additional Game-based Learning Activities

A few more examples of types of games that fit within the two modes of game-based learning are robotics and escape rooms. Both of these types of games have engaged students and enabled students to demonstrate mathematical understanding through multiple representations (Stohlmann, 2020a; Stohlmann & Kim, 2020). Students can use programming and robotics, a type of integrated steM game-based learning, to investigate the speed, time, and distance traveled of a robotic ball and then use this knowledge to play several games. For example, this can be done through programming a robotic ball with different speeds to travel in a straight line for three seconds and measuring the distance traveled. Students can then record this information in a table and work to come up with a method to predict the distance traveled given a speed. Along with this task, various games can be incorporated with the robotic balls in which students can use their knowledge from the previous task. Mini golf holes can be drawn on paper or setup with blocks and students can program the balls to end up in the hole. Along with proportional reasoning and measurement, students also apply knowledge of angles to get their ball to go on a path to the hole. Bowling pins can be setup as another game to be incorporated with the programming. A track can also be setup and students can program the balls to race each other around the track (Stohlmann & Kim, 2020). In addition, Dunbar & Rich (2020) describe how a robotic ball may enable students to use proportional thinking to program a ball to go through a maze. This is similar to the idea of mini golf holes as students must program the ball to move in a route from the start to the end of the maze.

Escape rooms are another type of context game-based learning that engages students and encourages mathematical communication. Escape rooms are a "game during which teams solve multiple puzzles using clues, hints, and strategy to determine how to escape from a locked room" (Stohlmann, 2020a, p. 383). Mathematical escape rooms can be designed with a unifying theme, a brief backstory, structures to help students persevere in problem solving, the inclusion of hints if needed, and a compelling twist (Stohlmann, 2020a). I have developed and classroom-tested escape rooms based on the topics of lines (Stohlmann, 2020a) and ratios and proportions (Stohlmann & Kim, 2020). Students displayed high levels of engagement while completing the escape rooms. Moreover, they demonstrated mathematical understanding through modeling the content using symbolic, verbal, and pictorial representations (Stohlmann, 2020a; Stohlmann & Kim, 2020). As in the other examples of game-based learning, design and implementation of escape rooms should align with the NCTM (2014) effective teaching practices to ensure its success.

Conclusion

Difficulty with mathematics can prevent students from degree completion and from pursuing certain careers (Lee, 2012). When students see mathematics as interesting and relevant, this can help students do well in the subject (Kim et al., 2015). The ability to persevere in problem solving is a vital part of mathematics, and game-based learning helps students to do this more effectively (Stohlmann et al., 2018). When structured well, game-based learning has the potential to engage students and support the development of mathematical understanding. Context game-based learning and integrated steM game-based learning are two promising modes of game-based learning that deserve further research work. Yet, many teachers report challenges in implementing game-based learning effectively (Denham, 2019) and there is a need for more guidance on how to integrate games to help develop conceptual mathematical understanding (Van Eck, 2015). Moreover, teachers have reported that they would like it to be easier to identify games aligned with standards and objectives (Takeuchi & Vaala, 2014).

In this article I described two modes of game-based learning that I found to be effective with middle school mathematics students: context game-based learning and integrated steM game-based learning (Stohlmann, 2019a; Stohlmann & Kim, 2020). Both modes incorporate multiple representations, student feedback, and an explicit focus on mathematics. The main difference between the two modes is that integrated steM game-based learning includes open-ended tasks and always includes technology integration.

Both modes enable teachers to implement effective teaching practices including posing purposeful questions, having students learn through multiple representations, generating meaningful discourse, and having clear mathematical goals (NCTM, 2014). These effective teaching practices are aligned with my general principles for game-based learning implementation. Clear mathematical goals help teachers focus on grade-level mathematics standards. Students make use of multiple representations in demonstrating their understanding. Mathematical discussion occurs in the context of the game and questions are posed to make the mathematics explicit. Students also receive feedback from the technology or from other students. Both modes of game-based learning allow for student-centered learning and students to socially construct their knowledge. In the examples that I described in this article and my prior work with game-based learning (Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a; Stohlmann & Kim, 2020) students have been engaged with the games and the mathematical work that went along with the games. This is important as students' interest in mathematics generally declines in the middle grades (Franzel et al., 2010). Future research with game-based learning can focus on supporting teachers for effective implementation of game-based learning, further development of games that are well-structured and support conceptual understanding, and investigating the impact of game-based learning on low achieving mathematics students. Game-based learning is a promising method that deserves further focus.

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