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# Leadership in Mathematics Education: Roles and Responsibilities

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This article partitions leadership in mathematics education into two categories: leadership in defining and maintaining important principles in teaching mathematics, and leadership in informing the public about the importance of mathematics today and in the future. Examples of both types of leadership are given in the article. Teacher leaders in the field should be skilled in enriching instruction and linking best practice with research activities and results in the professional development of the teachers they lead.

Keywords: leadership in mathematics education, motivating students, enriching mathematics instruction

There are basically two kinds of leadership required in mathematics education; there is leadership in defining and maintaining the important principles of teaching mathematics, and there is leadership in educating the public as to the fundamental role mathematics plays in our technological society. Those of us who have had leadership roles in mathematics education have tried continuously to demonstrate the importance of mathematics as an application in thinking logically, in real life circumstances, and as a subject which enjoys a unique power and beauty in and of itself.

#### Guiding Policy and Influencing Public Opinion

A leader in mathematics needs to take on issues of public policy as they relate to mathematics education. For example, in recent years the use of student standardized-test results as a way of assessing mathematics instruction has become quite controversial. Naturally a student's performance is dependent in some measure on the teaching received in school; however, there are many other factors which affect these performances, such as support students get at home, the amount of tutoring they get outside of school, the mathematics foundation they received in previous grades, and a student's natural ability-a topic often avoided in public conversation. Teachers, whose professional position depends on their assessment, will do everything they can to influence a favorable result. This often finds teachers "teaching to the test," a practice typically frowned on by educators as one that avoids providing students with a proper understanding of the subject matter. A leader in mathematics education needs to address this issue in a public forum, such as at professional conferences, parent association meetings, and in various print media. Where possible, opinion columns in newspapers can serve a mathematics education leader well to bring a point across to the general public.

Parents play an important role in mathematics education. They need to be supportive at home and monitor a student's progress with great interest. A leader in mathematics education should use the public pulpit to drive this point home. For one thing, many research studies show that expectation on the part of an adult-teacher or parent-is a paramount motivator of a student's performance. At a recent parents association meeting, I was asked by a parent how I would expect her to help her son with his homework when she recalls none of the material he is currently learning in school from her own high school days. I indicated that she could very well play an important role in supporting her son's homework effectiveness. First, I told her to make sure that her son's homework environment is clear of any distraction. She indicated that this was clearly the case in her home. I then told her that when her son finishes his homework he is to explain to her what he has done in his homework assignment. This serves two purposes: first, it entices the student to do his assigned work in order to really understand it, knowing that he will be required to explain his work upon completion, and, second, it allows the student to reinforce his understanding of the material by verbalizing it as he explains it to his parent.

Rewarding successful teaching of mathematics is also a concern that should be addressed by leaders of mathematics education. Publicly recognizing outstanding teaching through awards is a fine motivator that these leaders can exercise. Taking positions on the salaries of mathematics teachers in public forums also may be one of the considerations for such leaders. Some school districts are now providing additional stipends to mathematics teachers—as a way to motivate outstanding teaching of this important subject.

What is taught in mathematics in K–12 schools has always been the purview of leaders in mathematics education. After the Sputnik event in 1957, the School Mathematics Study Group (SMSG) was formed and stimulated a new approach to teaching mathematics known popularly as the "new math." This highly controversial change in what and how we taught mathematics became a very significant agenda item for mathematics education leaders. In subsequent decades issues such as "return to basics," the use of computers and other technology in teaching mathematics, and the assessment of teaching mathematics are among the issues that leaders of mathematics education addressed. Most recently the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010), an attempt to standardize mathematics instruction throughout the United States, has become a topic discussed by many leaders in mathematics education. The issue of the teaching of geometry at the high school level as described in the Common Core State Standards has come under fire. Some leaders of mathematics education are opposed to the fact that these standards indicate that the course be taught through transformations rather than in the deductive Euclidean style. Mathematics education leaders must address these and other topics for the general public as well as other professionals in the field.

An issue that surfaces periodically is how mathematics should relate to other subjects being taught in the schools. Many European countries require that secondary school teachers be certified to teach more than one subject. Consequently a mathematics teacher will feel quite confident applying mathematics to the other subject in which she/ he is certified. In the United States, the application of mathematics to other fields has been rather sparse. Leaders in mathematics education need to be prepared to respond to this issue. Typically, teachers who choose applications of mathematics will select them from areas of personal interest to them and not necessarily to their students. Leaders in mathematics education should emphasize the use of real world mathematics applications appropriate to the level and interest of the students.

Perhaps one of the chief responsibilities for leaders in the field of mathematics education is to ensure that the instruction in schools is as effective as possible. Mathematics education leaders should be a resource for best practices in the schools. Those leaders who are responsible for the teaching in a school or school district should bring to the teachers in their schools the latest findings of effective teaching by involving teachers in research projects.

#### Leading Teachers to Motivate Students

One of the key elements in the teaching of mathematics is to get teachers to motivate students properly. Mathematics education leaders should be in a position not only of encouraging teachers to begin every class with a brief motivational device, but also exhibit the appropriate enthusiasm while teaching. There are also a number of motivational devices that leaders of mathematics education should encourage teachers to use each and every lesson. Some motivational devices are rather common, while others are not, and yet can be every bit as effective as the more commonly known ones. The following paragraphs explore some of these motivational devices (Posamentier & Krulik, 2012).

#### Indicate a Void in Students' Knowledge

Students usually have a natural desire to complete their knowledge of a topic. This motivational technique involves making students aware of a void in their knowledge and capitalizes on their desire to learn more. For instance, a teacher may present a few simple exercises involving familiar situations followed by exercises involving unfamiliar situations on the same topic. Alternatively, a teacher may explain (or demonstrate) how the topic to be presented will complete students' knowledge about a particular part of mathematics. The more dramatically delivered, the more effective the motivation. Often, guiding students to self discover a void in their knowledge is particularly effective.

#### Discover a Pattern

Setting up a contrived situation that leads students to "discover" a pattern can often be quite motivating, as students take pleasure in finding, and then "owning," an idea. Some mathematicians find pattern discovery a key to their research. The trick is to use the pattern to develop the concept which is to be taught in a particular lesson.

#### Present a Challenge

When students are appropriately challenged intellectually, they react with enthusiasm. Great care must be taken in selecting the challenge. The problem (if that is the type of challenge used) must not only clearly lead into the lesson, but it must also be appropriate for the students' abilities. A challenge should be short and not complex. It should not be so engrossing that it detracts from the intended lesson. This would certainly defeat the purpose for which the challenge was intended. Thus, challenges providing motivation for one class may not do so for another. Teacher judgment is most important here.

#### *Entice the Class with a "Gee-whiz" Amazing Mathematical Result*

Unexpected results often intrigue the students and stimulate their curiosity. To motivate basic belief in probability, a very effect motivation is to discuss with the class the famous "Birthday Problem" (Lesser, 1999). Its amazing (and we dare say, unbelievable) result will have the class in awe and eager to pursue further study of probability.

#### Indicate the Usefulness of a Topic

Here a practical application is introduced at the beginning of a lesson. The applications selected should be of genuine interest to the class. Once again the applications chosen should be brief and not too complicated in order that they motivate the lesson rather than detract from it. Student interest must be considered carefully when selecting an application. Usefulness is appropriate only when students have the proper background knowledge of the application.

#### Use Recreational Mathematics

Recreational mathematics consists of puzzles, games, paradoxes, and other forms of mathematics entertainment. In addition to being selected for their specific motivational gain, these devices must be brief and simple. A student should realize the "recreation" without much effort in order for this technique to effectively motivate a mathematical topic to be explored. The recreational aspect of the activity should be the impetus of the lesson, not its high point; it should be a means to an end, not the end in itself.

#### Tell a Pertinent Story

A story of a historical event or of a contrived situation can motivate students. All too often, teachers, already knowing the story they are about to tell and eager to get into the "meat" of the lesson, rush through the story. Such a hurried presentation minimizes the potential effectiveness the story may have as a motivational device. Thus, a carefully prepared style of presentation is almost as important as the content of the story itself. A teacher's enthusiasm for the story being told is of prime importance.

## Get Students Actively Involved in Justifying Mathematical Curiosities

One of the more effective techniques for motivating students is to have them justify a pertinent mathematical curiosity. The students should be familiar with the mathematical curiosity before you "challenge" them to justify it, to ensure a complete result within a reasonable time.

#### Teacher-Made or Commercially Prepared Materials

Here motivation can be achieved by presenting the class with concrete material of an unusual nature. This may include teacher-made materials, such as models of geometric shapes, mechanical gadgets that demonstrate mathematical concepts, or practical "tools" that illustrate a specific geometric principle. There are fine commercially-prepared materials available, ranging from geometric models to videos of various kinds. Materials selected should be reviewed carefully and their presentation carefully planned so as to motivate students for the lesson and not to detract attention from it.

#### **Enriching Mathematics Instruction**

An important role that leaders in mathematics education must play is to provide teachers at all grade levels with ideas and materials that they can use to enrich their instruction. The time spent on genuine enrichment—which brings with it a high level of contagious enthusiasm on the part of the teacher—should result in more motivated learning.

Consistent with all versions of Standards issued over the past few years (National Council of Teachers of Mathematics, 1989; 2000)–including the Common Core State Standards (NGA & CCSSO, 2010)– is the notion that mathematics instruction should be enriched wherever possible and feasible. First, it must be emphatically stated that mathematics enrichment is not reserved only for gifted students. Students of all ability levels should have their mathematics instruction enriched.

Enrichment of mathematics can manifest itself in at least three different ways. The easiest and least creative to implement is *acceleration*. This refers to the process of moving a stronger student along the mathematics curriculum more rapidly. Acceleration has a few drawbacks. First, the student may finish the coursework a high school has to offer and then be left with no further mathematics courses to take, while not having completed the coursework in other subjects. In this situation, the student may work privately with a teacher who volunteers to be a mentor for the remainder of the student's high school career, or the student may enroll in a course at a nearby college (if that option exists), or the student "takes a vacation" from mathematics. The last would be an inexcusable shame, for a bright student might ultimately be lost to mathematics study later on.

Another reason for enriching a bright student in ways other than to accelerate him or her is that such enrichment can stimulate the student to pursue mathematics study more seriously later on, or it may simply motivate the student to deepen his or her understanding of mathematical concepts and ideas. There are many topics to investigate outside of the regular school curriculum, depending on the individual student's interest.

Another form of enrichment may be called *expansion*. This term refers to the resourcefulness of the mathematics teacher to delve into regular curriculum topics in greater detail than normally required. This will spur the better students to investigate the topics in greater depth.

*Digression*, another form of enrichment, refers to taking class time to consider a topic that is not in the regular curriculum but relates to one that is. Moving out of the confines of the syllabus to related topics and studying them in appropriate detail can open up new areas of interest for students. One must keep in mind that younger students on average do not have as sophisticated an ability to abstract as do older and more mature students.

The time taken away from syllabus topics to do enrichment activities is not to be seen as time lost from the regular instruction. Quite the contrary! The time used for enrichment activities is actually an investment of time. Students will become more active participants in the classroom and more efficient learners when their instructors spend time to motivate and interest them in mathematics. A "turned on" audience needs less time to learn new concepts.

It is a popular myth that a mathematics class is lifeless and dull. Unfortunately, that myth is too often true-though it need not be so! We frequently find ourselves concentrating on the teaching of mathematics to reach a deadline, such as giving a test or completing a course of study. The luxury of teaching what mathematics is all about seems to be beyond our grasp. But is it really? We can easily teach where mathematics came from, who first thought of it, and who later developed and refined it. In short, we can use the *history of* the mathematics, including the lives, loves, successes, and failures of the people who created it, to breathe life into what might otherwise be considered rather dull. History enriches the instruction of mathematics and needs to be encouraged by leaders in mathematics instruction. Some sample topics of historical interest that may enhance students' perspective of mathematics are listed in Table 1.

#### Support Research & Best Practices

A leader in mathematics education must serve as a stimulus for innovative procedures and concepts being taught both locally and nationally. Of particular importance is to have teachers actively involved in appropriate investigation of the work they do. This would imply that teachers should

#### Table 1. Topics of Historical Interest to Enrich Instruction

- The History of Non-Euclidean Geometry
- How Eratosthenes Measured the Earth
- Ancient Egyptian Arithmetic
- The Four Color Map Problem
- The Birthday Problem
- The History of Computation in Other Cultures
- Albrecht Dürer's Contribution to Mathematics
- Constructing Radical Lengths from an Historical Perspective
- Napoleon's Contribution to Geometry
- The History of the Golden Ratio
- The Nine-Point Circle and its History
- The History of Zero
- Carl Friedrich Gauss's Summation of the Numbers from 1 to 100
- The Golden Rectangle and its Appearance in Historical Sites and Paintings
- The Pythagorean Theorem's History
- The Fibonacci Sequence
- Diophantine Equations
- Pascal's Triangle
- The History of Pi
- The Three Famous Problems of Antiquity

Note: For information on these topics see Posamentier (2007, 2010), Posamentier & Krulik (2008), and Posamentier & Lehmann (2004, 2009, 2011)

engage in research that enhances the field and perhaps more importantly allows them special insight to their professional practice. Leaders in mathematics education must then do all they can to popularize best practices as verified by both action research and formal research studies.

#### Professional Development

Leaders in mathematics education also need to play an active role in the pre-service and in-service preparation of mathematics teachers. After all, the future and present of the teaching of mathematics lie in the professional development. These leaders should be involved actively either by working directly with pre-service and in-service teachers or by providing innovative ideas for enhancing the teaching practice.

#### Conclusion

It is the responsibility of leaders in mathematics education to popularize the subject, to inform the public of its importance, and make sure that teachers provide all students with the competence appropriate now and in the future. Leaders in mathematics education must be the champions of the power and beauty of mathematics! Teachers who aspire to leadership not only must influence teachers, but also must sway parents and the community at large in the necessity of mathematics competence in an increasingly complex technological society. Without skilled teacher leaders future generations will be controlled by technology rather than controlling technology for the benefit of their community and the nation.

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