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Mathematics Assessment at the Postsecondary Level: Three Alternative Forms of Assessment

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ABSTRACT Postsecondary mathematics classrooms continue to become more diverse, which implores educators to continue to incorporate more diverse modes of assessment to support all learners. However, researchers have found that mathematics assessment at the postsecondary level has remained mostly stagnant, with traditional, closed-book exams dominating the field. In this paper, we present three alternative forms of mathematics assessments for teachers at the postsecondary level. We incorporate researched-based tools for implementation and grading online, oral, and project-based assessments in mathematics. Potential limitations of each type of assessment are also addressed.

KEYWORDS *mathematics assessment, online assessment, oral assessment, project-based assessment, postsecondary mathematics*

Mathematics assessment in the postsecondary classroom gathers information about student content knowledge and their mathematics learning, provides students with feedback on their learning, and helps teachers reflect and improve their teaching practices (Suurtamm et al., 2016). Strategies for mathematics assessments have remained somewhat stagnant at the postsecondary level, with traditional, closed-book exams dominating the field (Iannone & Simpson, 2011, 2015). With an increasingly diverse student body entering postsecondary mathematics courses, teachers have an obligation to accommodate a variety of academic and professional needs. Incorporating alternative forms of assessment into a mathematics course can help teachers create a well-rounded evaluation of students' knowledge and skills. This paper includes resources and strategies for three types of alternative mathematics assessments: (1) online, (2) oral, and (3) project-based. Each section will define these alternative assessments in the context of a postsecondary mathematics classroom, provide specific research-based resources for implementation and grading, and address concerns about potential limitations.

Online Assessments

Assessments must measure student comprehension of learning objectives, provide students an opportunity to self-assess, and cultivate a feedback dialogue between teachers and students (Robles & Braathen, 2002). Online mathematics assessments fulfill these requirements and most aspects of traditional paper-and-pencil assessments. Advances in technology allow for multiple choice, true/false, matching, fill in the blank, free response, and many other question types to be uploaded into a digital format and instantly graded through the use of algorithms (Herbet et al., 2019; Pelkola et al., 2018). Further, many online assessment resources provide immediate feedback to student responses (Joglar et al., 2010), and some have the capability to generate personalized questions based on students' progress (Herbert et al., 2019). Online platforms for course management, learning, and teaching, known as *learning management programs*, are also equipped for teachers to build their online assessments and then import students' data into their grade books. Most online assessments can support mathematics editing

codes such as LaTeX and allow the teacher to insert interactive multimedia files from sources like GeoGebra and Desmos (Joglar et al., 2010). Several online assessment platforms allow teachers to share the resources they have created, thereby constructing digital banks of mathematics assessment questions (Gleason, 2012; Joglar et al., 2010; Pelkola et al., 2018).

Online Assessment Resources

Many resources are available for instructors to create online mathematics assessments. Bolster Academy, a standalone program, specializes in open-ended questions for students taking advanced mathematics courses at the postsecondary level with automated feedback and shared question banks (2020). Maple T.A., or Möbius, specializes in Science, Technology, Engineering, and Mathematics (STEM) courses, offers a variety of question types, automated feedback, and shared questions banks (Maplesoft, 2020). Maple T.A. is compatible with most learning management programs and also works as a standalone program. Both Bolster Academy and Maple T.A. are online programs that the teacher, students, or institution must pay for a subscription; however, a number of open-source, freely distributed programs that teachers can contribute to and modify to suit their needs are available. Lumen OHM, also known as MyOpenMath, provides teachers with question banks in which they can share and use questions designed for specific mathematics content areas (IMathAS, 2020). MyOpenMath also automatically grades multiple choice, numerical, and graphing solutions, providing students with instant feedback (IMathAS, 2020). WebWork, supported by the Mathematical Association of America and the National Science Foundation, focuses on formative assessment in the form of homework with automated feedback and shared question banks (The Mathematical Association of America, 2020). Teachers using Google Classroom may also want to take advantage of Google Forms. While not math specific, Google Forms allows teachers to manually enter feedback and automatically generate a statistical report of students' responses. We created a more detailed description of these online assessment resources and their capabilities that can be found in Appendix A.

Teachers can also access online resources available on their school's learning management programs to create assessments. Some of the most commonly used learning management programs such as Blackboard, Brightspace, Canvas, Moodle, and Sakai have a variety of question types that provide both automated and man-

ually entered feedback. Each of these systems generates statistical reports of students' performance incorporated into the system's grade book. It is important to note that these programs are not explicitly designed for mathematics assessments; instead, they are designed to be comprehensive enough to permit use in a wide range of disciplines. Advanced mathematics teachers and instructors may feel limited by the available built-in math editors, which often do not accommodate such things as scientific notation, graphing, or diagrams. Teachers could alternatively ask students to upload files or pictures of their work to be manually graded.

Implementing and Grading Online Assessments

Initially, implementing an online mathematics assessment will create some challenges for instructors, as we will address in the next section. Teachers must become familiar with the program's format and design while building a database of questions (Joglar et al., 2010). However, paid programs like Maple T.A. or Bolster Academy and open-source programs like MyOpenMath and WebWork have mathematics question banks which teachers can access for formatting examples or for incorporating directly into their assessments. Furthermore, online mathematics programs like GeoGebra and Desmos have open-source, pre-built models that can be embedded into online assessment resources, thereby alleviating the technological strain on teachers in building their own models. We recommend that teachers search GeoGebra and Desmos for specific diagrams, graphs, or models, and then copy the embedding code and insert it into the assessment system within their learning management program.

As noted above, most of the online assessment programs are able to incorporate most mathematics questions, excluding *Free Response/Essay* and *File Upload*, to be automatically graded and provide instant student feedback. For example, in the learning management program Canvas, teachers can select the point value of each question and customize the type of feedback to include simple correct/incorrect hints, or further directions. Blackboard, Brightspace, Canvas, Moodle, and Sakai have built-in grade books which will automatically fill in students' data and create statistical reports for teacher evaluation. Most online assessment programs also collect student data; Lumen OHM, for instance, will calculate students' scores for assessments and course averages that can be downloaded into an Excel spreadsheet.

Potential Limitations to Online Assessments

Accessibility

There are limitations to the use of online mathematics assessments as an alternative to traditional paper-and-pencil assessment. Student and teacher accessibility to computers and reliable Internet will pose the most formidable of challenges (Greenhow, 2015; Herbert et al., 2019), with the potential to unintentionally discriminate against those who do not have the resources to access the online assessments. As the COVID-19 Pandemic has forced schools around the world to move classes online, this issue has been exacerbated, and teachers have found that many students lack the resources necessary to participate (Reilly, 2020). We recommend that teachers gather information about students' accessibility privately before implementing any mandatory online mathematics assessment. Teachers, alternatively, may choose to implement mandatory assessments in school computer labs to ensure accessibility for all students.

Technological Skills

Initially, the technological skills of both the teachers and students may pose a potential limitation. Teachers and students need time to become comfortable using these online mathematics assessment tools and familiarize themselves with the selected programs before using the assessment in a high-stakes environment (Herbert et al., 2019). Depending on the audience, it may be useful to provide in-service training to help teachers become comfortable with online mathematics assessment programs (Joglar, 2010). To facilitate effective student use of programs, Greenhow (2015) suggests mock online assessments that allow students multiple attempts or an initial assessment in a classroom setting with the teacher present to address questions and issues in real time.

Academic Dishonesty

Teachers' apprehension over cheating and academic dishonesty is often heightened when considering online mathematics assessment (Kennedy et al., 2000; Ladyshewsky, 2015). Online question and answer websites, such as Chegg.com, have enabled cheating, particularly on mathematics assessments inclined towards single solutions and numerical responses (Klein, 2020; Supiano, 2020). A number of ways to combat the risk of academic dishonesty for online mathematics assessments include: using the question-randomization option on assessment tools, lowering the stakes of the assessment, scheduling a time and setting a time limit for the assessment, or asking critical thinking questions instead of multiple choice or true/false (Ladyshewsky, 2015). Randomized ques-

tions that have time limits reduce students' ability to share and search for answers online (Ladyshewsky, 2015). Harmon and Lambrinos' (2008) study found fewer instances of cheating when a proctor supervised the assessment. While some schools may have the resources to hold online assessments in computer testing centers with proctors, this is not always an option, especially for students taking courses online. Online assessment integrity resources such as Honorlock, ProctorU, and Proctorio provide teachers with secure online proctoring software that monitors students virtually as they assess, provide identity verification, and professional review for signs of academic dishonesty. We recommend that teachers ask their Information Technology Department to see if this type of software is available from their online campus. It is important to note that there has been recent backlash to these integrity resources, with reports of students' feeling an invasion of privacy, increased anxiety while testing, and a plethora of technological issues (Patil & Bromwich, 2020). Teachers, alternatively, may consider having students sign an honor pledge or code, where they commit to honestly completing the assessment. Honor codes have been found to reduce cheating and support integrity on assessments of all types (Miller, 2020).

Oral Assessments

Oral assessment in mathematics is not common in American classrooms, unlike countries such as Hungary, Italy, and the Czech Republic who commonly employ this assessment method in their higher education courses (Iannone & Simpson, 2012). Lee (1988) describes learning as "more than a paper and pencil activity" (p.12), and oral assessments provide a space for students to show off their "problem solving skills rather than quick answers" (Sayre, 2014, p.30). Oral assessments help develop students' communication and logical reasoning skills (Chasteen, 2018; Iannone & Simpson, 2011; Joughin, 2010). Moreover, oral assessment in mathematics can significantly reduce, if not eradicate, cheating, and plagiarism among students (Joughin, 1998, as cited in Iannone & Simpson, 2011).

Joughin (2010) categorizes oral assessments into three types: *presentation*, *application*, and *interrogation*. In the postsecondary mathematics setting, the two most prevalent forms of assessment from Joughin's (2010) model are *presentation* and *interrogation*. *Presentations* are characterized as an "in-class presentation on a prepared topic [or a] group project report to the class" (Joughin, 2010, p. 3). *Interrogations* are described as the process where a

student is quizzed or interviewed by the instructor (Joughin, 2010); we will primarily be discussing oral assessment in mathematics as an *interrogation*.

Setting the Stage for Oral Assessment in Mathematics

An instructor wishing to implement an oral assessment in mathematics needs to consider the classroom setting, which will dictate such an endeavor's plausibility. Researchers most commonly execute and study the effectiveness of oral assessments in small classroom settings (Iannone & Simpson, 2012; Odafe, 2006; Sayer, 2014), suggesting that a smaller classroom's intimacy cultivates a more suitable environment. Furthermore, the instructor needs to thoroughly plan for the assessment, asking themselves about the types of interactions and questions needed to properly assess students' mathematical knowledge, who the audience will be, and how to structure the assessment (Joughin, 2010). To reduce ambiguity and confusion for students, it is recommended to discuss the oral assessment format and expectations beforehand and provide written directions (Iannone & Simpson, 2012; Joughin, 2010). Practitioners suggest implementing practice oral quizzes and providing detailed one-on-one feedback to help familiarize students with the assessment process (Dumbaugh, 2020; Iannone & Simpson, 2012). Moreover, a comprehensive rubric of how students will be graded should be available prior to being assessed (Odafe, 2006), with the assessment's intentions clearly stated as a high or low stakes test (Iannone & Simpson, 2012).

Implementation of Oral Assessments

In mathematics, oral assessments are generally administered in two ways: group oral assessment with three to four students (Odafe, 2006) or individualized oral assessment (Boedigheimer et al., 2015; Iannone & Simpson, 2012). These types of assessments are recommended to be held outside of regular class time and should not exceed more than 60 minutes (Chasteen, 2018; Iannone & Simpson, 2012; Odafe, 2006; Sayre, 2014). Instructors may also choose to employ teaching or course assistants to increase the efficiency of administering oral assessment; however, this is only recommended for use in practice or in a low-stakes environment (Chasteen, 2018). The assessment should feature "harder, more interesting problems than...a written exam" (Sayre, 2014, p. 32) and should be both thought-provoking and not invite one-word numerical answers.

Group Oral Assessments

In a group oral assessment setting, it is important to have students accustomed to collaborating on mathematics problems in groups (Chasteen, 2018; Odafe, 2006). We recommend that teachers create groups of three to four students. In Odafe's (2006) example, students were assessed with the same group they were assigned to during class time and were provided with space, such as a whiteboard, to express mathematical ideas in written format. Some researchers recommend that scripted questions be pulled randomly from a collection created by the instructor in advance (Chasteen, 2018; Odafe, 2006). Teachers should work to create a dialogue with students as they are being assessed, asking such questions such as "Why did you do that?", "Can you explain an alternate way of solving the problem?" and allowing group members to assist each other when necessary (Lee, 1988; Odafe, 2006). The amount of time allotted for each group will vary depending on the length and difficulty of the questions. For example, Odafe's (2006) study of group oral assessments in a College Algebra course allotted approximately 30 minutes for each group, allowing students to complete two to three problems. Chasteen's (2018) Calculus courses allotted for up to an hour for each group.

Individual Oral Assessments

Current research indicates that 10-30-minute interview sessions are sufficient to assess individual students' understanding of mathematics topics (Boedigheimer et al., 2015; Dumbaugh, 2020; Iannone & Simpson, 2012). In Iannone and Simpson's (2012) study, students were tested on two questions: one in theoretical and one in applied mathematics. The student was allowed to choose the first question from either category and respond. The tutor would then randomly select a question from the other category for the student to respond to (Iannone & Simpson, 2012). When posing theoretical mathematics questions, research suggests that keywords such as prove, explain, and draw can be used to elicit responses that demonstrate students' reasoning and understanding (Chasteen, 2018; Iannone & Simpson, 2012). Applied mathematics questions in an oral assessment should ask students to implement algorithms to solve problems in front of the instructor as they talk through their thought process (Iannone & Simpson, 2012). Sayre (2014) suggests that students should not perform tedious calculations of a problem; instead, the instructor should guide the student to focus on the content and reasoning behind

the problem. Odafe (2006) further recommends that instructors incorporate ample wait time for student responses, and, at the conclusion of an oral assessment, address misconceptions or mistakes made.

Grading Oral Assessments

Grading oral mathematics assessments requires several components to facilitate a fair and equitable process. Firstly, a clear rubric that rates students on their solution(s), the communicated ideas and application while problem solving orally, and the clarity of their explanations (Boedigheimer et al., 2015; Iannone & Simpson, 2012;). Students can also be rated on how far they can carry the question through, with or without varying levels of assistance (Odafe, 2006; Sayre, 2014). It is recommended that instructors, either video or audio record the sessions for review or in the case of an appeal (Iannone & Simpson, 2015; Joughin, 2010). Teachers are advised to administer grades only after notes, comments, and recordings have been reviewed (Odafe, 2006).

Potential Limitations to Oral Assessments

Assessor Bias

One of the major concerns for oral assessment in mathematics is fairness and the mitigation of assessor bias (Iannone & Simpson, 2012; Joughin, 2010; Sayre, 2014). In addition to clearly communicated expectations, Sayre (2014) recommends postponing grading until all students have completed the oral examination to address grading fairness. While assessor bias, intentional or unintentional, may be somewhat unavoidable, the use of video or audio recording allows for “ways of subsequently monitoring the process and moderating for the marks” (Iannone & Simpson, 2012, p. 180). To minimize assessor bias, courses with multiple teachers may also randomly assign students to be assessed by instructors of other sections (Boedigheimer et al., 2015).

Stress and Anxiety

Another limitation found with oral assessment in mathematics is the high levels of stress or anxiety students can experience while testing (Iannone & Simpson, 2012; Joughin, 2010). Iannone (2020) found that student nervousness arises from two factors: (1) interacting with the instructor as they are taking their assessment, and (2) the unpredictability of the questions posed. To mitigate the first factor, Iannone (2020) found that consistent dialogue with students in the classroom made students more comfortable with the instructor. For the second factor, Odafe (2006) provides students with a pool of ten questions from which the oral exam will be composed

one week prior to the examination. Alternatively, other research has found that oral assessments may benefit students who find themselves suffering from math anxiety associated with written examinations (Heath, 1994). Some students, such as those with dyslexia or vision impairments, may find expressing their thinking and understanding of mathematics concepts orally less stressful (Joughin, 2010).

Time Consumption

Time consumption for the administration of oral examinations can often seem daunting to practitioners (Joughin, 2010; Odafe, 2006). Large class sizes can create a barrier due to the time constraints for both teachers and students (Boedigheimer et al., 2015; Joughin, 2010), particularly if scheduling assessment outside of class time. Sayer (2014) recommends that instructors consider the class size before choosing oral assessments as an alternative, noting that she will not use oral exams in classes much larger than 25 students.

Student Needs

Lastly, it is important to acknowledge that oral assessment in mathematics may not be suitable for all learners. Some students may have hearing or speech impairments that would make oral examinations discriminatory (Joughin, 2010); others may not be experienced at explaining their reasoning or thinking on-the-spot (Odafe, 2006). Also, international students may not be proficient in the language that oral assessment is conducted, once again making the assessment unintentionally discriminatory. The instructor needs to consider these limitations and make adjustments accordingly.

Project-Based Assessments

Project-based learning (PjBL), to demonstrate real-life applications, made its debut in the early 1900s in the United States (Barron et al., 1998). PjBL “is a comprehensive approach to classroom teaching and learning that is designed to engage students in the investigation of authentic problems” (Blumenfeld et al., 1991, p. 379). While project-based assessments (PjBA) are generally used in classrooms that teach with PjBL, they are versatile enough to be a part of any mathematics course. Helle et al. (2006) describe PjBA and PjBL as having five essential features: (1) the problem drives the investigation, (2) students construct a concrete product, (3) students are in control of their learning process, (4) the problems and solutions are contextual and challenging, and (5) students can represent their solutions in multiple formats.

The use of PjBA has been praised as a type of authentic assessment, that serves as a method of evaluation while continuing student learning through providing opportunities for “meaningful experiences ... [and] ... high-level thinking” (Fauziah, 2018, p. 1). Given the investigative nature of PjBA (Blumenfeld et al., 1991), they are excellent platforms for students to apply their mathematical content knowledge while exercising critical thinking skills in both mathematics and project design (Helle et al., 2006). PjBA in mathematics encourages students to engage in problem-solving, perform experiments, analyze data, or create presentations (Blumenfeld et al., 1991; Russell & Rowlett, 2019). PjBL ensures that students overcome misconceptions that could easily be overlooked in traditional learning environments (Helle et al., 2006). PjBA can also be a tool that integrates the fields of STEM, helping students to recognize and apply the relationships between disciplines (Han et al., 2016). Furthermore, traditional academic skills students acquire in typical undergraduate mathematics programs are not always employable. Researchers (Hibberd, 2005; Knight & Yorke; 2004, as cited in Russell & Rowlett, 2019) claim PjBL and PjBA can build on desirable professional skills (Hibberd, 2005), such as how to collaborate in a professional group setting while improving planning and organizational skills (Russell & Rowlett, 2019). Similar to oral assessments in mathematics, PjBA will require careful planning and clear communication of expectations to students.

Types of Project-Based Assessments

The versatility of PjBA means it can take on many different forms, such as portfolio projects, academic papers, or presentations. Portfolios consist of a collection of high-quality student work throughout a course that highlights their mathematical explorations and abilities (Knoerr & McDonald, 1999). Portfolios provide students with an opportunity “to take an active role in their own assessment and progress toward completing course objectives” as well as to “present a full assessment of learning” (Burks, 2010, p. 455-457). In academic mathematical papers, students research a topic applicable to the content of the mathematics course and write about it in their own words (Keith, 1988). Crannell (1999) promotes academic papers as a way: “(1) to improve students’ mathematical exposition; (2) to introduce new mathematics; (3) to strengthen understanding of previously encountered mathematics; and (4) to provide feedback from the student to the instructor” (p. 113). Lastly, presentations can be live or video recordings of students’ work on a mathematical problem, explanation of a mathematics

theory or concept, or application of mathematics to conduct an experiment.

Implementation of Project-Based Assessments

When implementing PjBA in a mathematics course, there needs to be a transitional phase that gives students opportunities to learn about the processes of PjBL and engage in initial low-stakes assessments (Blum, 1999; Slough & Milam, 2013). Instructional strategies such as scaffolding each step in a PjBA, modeling with exemplars of different parts of a PjBA, and encouraging perseverance will help students to transition to this new style of assessment (Barron et al., 1998; Slough & Milam, 2013). Teachers should assume an advisory or facilitator role rather than an authoritarian role (Adderley et al., 1975, as cited in Helle et al., 2006). Moreover, Slough and Milam (2013) recommend that during the PjBA process, teachers ensure: (1) content is made accessible to students, (2) the thinking process is visible, “which includes visual elements to help the learner and using learner constructed visual elements to assess learning” (p.16), (3) students are encouraged to learn from each other, and (4) the PjBA is focused on “autonomy and lifelong learning” (p. 16). Students will need to become accustomed to working together, communicating mathematical ideas, giving and receiving feedback, and understanding how to create a product in a timely and organized fashion. As PjBA ordinarily occurs within a group setting, instructors are encouraged to have students select roles, draft the goals and ground rules, or even establish cooperation agreements (Capraro & Corlu, 2013; Morgan & Slough, 2013) (see *Personal Accountability*). As with the case with oral assessments, students should be familiar with how they will be assessed at the beginning of a PjBA, provided with rubrics, prompts, and checklists.

Grading Project-Based Assessments

Rubrics and Checklists

In PjBA, “rubrics are an essential component ... that serve different purposes for those who are involved in the assessment process both at the stage of the rubric’s development and its utilization during the evaluation” (Capraro & Corlu, 2013, p.115). To promote deeper understanding and involvement of the assessment process, researchers (Capraro & Corlu, 2013; GAIMME, 2016) encourage teachers to allow students to be a part of the rubric design process and then use these rubrics as tools for self- and peer-assessment. Capraro and colleagues (2013) include rubric categories such as authenticity, academic rigor, exploration and independent research, use of technology, and application and demonstration of

learning. Checklists can also serve as a scaffolding tool for students and teachers, providing guidance on formatting and important components for completing PjBA (GAIMME, 2016). Checklists should be catered to the type of PjBA and include such items as how to structure a speech or advice on time management (Doree, 2017).

Personal Accountability

In group PjBA, students will need to be assessed both individually and as a group. Capraro and Corlu (2013) suggest that teachers use peer assessments, student reflections, group contracts, or an additional individual assessment to help increase individual accountability and fairness. Peer assessments either direct students to use the predetermined rubric(s) to assess each other or employ a Likert survey of teammates' contributions. In PjBA, students prefer to "demonstrate what they know instead of being caught at what they don't know" (Kenschaft, 1999, p.133). Teachers can provide space to demonstrate this knowledge through student reflections in the form of a survey or essay to include an account of personal responsibility and contribution to the project, in addition to a self-assessment using an agreed-upon rubric. Group contracts can include items related to behavior and social interactions in the group, responsibilities for each member, and consequences for negative actions (Capraro & Corlu, 2013). The combination of these documents, with the rubric(s) or checklist(s), can then be used to create a weighted grading system that will evaluate the whole process of the PjBA.

Potential Limitations to Project-Based Assessment

Time and Class Size Constraints

PjBA provides a versatile platform for mathematics assessment that builds on students' ability to problem solve and encourages student-centered learning throughout the process; there are a number of potential limitations. Most notable is the constraint of time; PjBA is a process that will take more than one class period to complete and often requires students to collaborate outside of class (Morgan et al., 2013). In addition to providing time for students to work on the project during class, more time will be required should teachers include presentations of projects as part of the assessment. Teachers should also consider the time they will need to grade PjBAs. It may require a more in-depth analysis of items submitted by the students; however, the use of comprehensive rubrics and clear expectations can simplify grading.

As with oral assessments, large class sizes pose a limitation to PjBA. It may be challenging for the teacher to guide and monitor students throughout the whole process. In such cases, it is recommended that the teacher use group-based PjBA and seek out "peer facilitators" that can help to supervise students (Özel, 2013).

Plagiarism

Unlike online and oral assessments in mathematics, PjBA is more susceptible to plagiarism (Johnson, 1983). Students may unintentionally plagiarize using ideas or content from the Internet without proper citation, or intentionally, by looking up solutions to problems. Unintentional plagiarism can primarily be mitigated by using explicit guidance on when and how to cite sources. Intentional plagiarism can be reduced by using original problems posed by the teacher or selected by the students.

Lack of Prior Knowledge and Skills

Lastly, PjBA may pose initial challenges to students, who may not have the prior knowledge or skills necessary to implement a PjBA all on their own (Capraro & Jones, 2013). Skills will differ from student to student, and teachers will need to consider this varying level of experience prior to engaging in a high-stakes assessment (Capraro & Corlu, 2013). Researchers suggest the use of scaffolding to include coaching students through a project or modeling the expected outcomes of the PjBA (Slough & Milman, 2013). The use of instructional strategies mentioned in the implementation section, along with time and experience, will temper these challenges with PjBL and PjBA.

Conclusion

The use of alternative forms of assessment in postsecondary mathematics classrooms provides diverse student populations multiple opportunities to showcase their strengths and abilities. Online mathematics assessments increase accessibility to both formative and summative forms of assessment while providing reusable resources for teachers that are both academically challenging and efficient at data collection and analysis. Oral assessments provide an authentic portrayal of students' understanding and knowledge of mathematical content while building communication and problem-solving skills. PjBA facilitates high-level thinking of real-world problems that develop students' professional skills.

We found it important to mention that during the COVID-19 Pandemic, teachers may modify these forms of alternative mathematics assessments to suit the needs of students in online or hybrid classrooms. We recommend that teachers continue to follow the above research for implementing, grading, and addressing potential limitations of alternative mathematics assessments. Teachers may also want to consider increasing the use of low-stakes, formative, online assessments to gauge student understanding during distance learning using the resources noted in Appendix A. Oral mathematics assessments can be administered using a video-conferencing medium, where teachers provide a digital whiteboard for students. Most PjBAs can be submitted digitally, and teachers can use video-conferencing mediums to conduct live presentations. Teachers may also want to consider asking students to create pre-recorded presentations to submit as part of their PjBA; these can then be shared through a discussion board on a learning management program or played during class.

Although this is not an exhaustive list of alternative forms of assessment, this paper was designed with the intention that postsecondary teachers and instructors will implement these or other alternative forms of assessment in their classrooms. We hope that these resources and ideas will help teachers become more flexible and innovative in their assessment strategies in a variety of postsecondary classroom settings. Teachers are encouraged to use their professional experience and judgment to decide how to most effectively use these assessment tools to elicit authentic assessment of student learning.

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Appendix A

Table A

Online Mathematics Assessment Resources

Resource	Bolster Academy, SOWISO	Google Forms	Lumen OHM MyOpenMath	Maple T.A., Möbius	WebWork MAA
Compatibility					
Standalone	x	x	x	x	x
Blackboard		x	x	x	
Brightspace		x	x	x	
Canvas		x	x	x	
Moodle		x	x	x	x
Sakai		x		x	
Other	LTI Integration		LTI Integration		
Questions Types					
Calculated	x		x		
Equation/ Expression	x		x	x	x
Fill in the blank	x			x	x
Free response/ Essay		x	x	x	x
Graph sketching			x	x	
Matching			x	x	
Multiple choice		x	x	x	
Multi-part			x		
Numerical	x		x	x	
Assessment Type					
Math Specific	x		x	x	x
Formative	x	x	x	x	x
Summative	x	x		x	
Feedback					
Instant	x		x	x	x
Computer generated	x		x	x	x
Manually entered		x	x		
Other					
Generates statistical report	x	x	x	x	x
Question Bank	x		x	x	x
Accessibility	Paid	Open-Source	Open-Source	Paid	Open-Source