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## Playing With Mathematics: How Play Supports Learning and the Common Core State Standards

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**ABSTRACT** International rankings show children in the United States perform well below average in mathematics. There are also large mathematics achievement gaps between children of lower- and higher-socioeconomic status. As today's teachers face these challenges, they are also faced with the pressures of sweeping educational reforms that arrived with the adoption of No Child Left Behind and continue into the Common Core State Standards era. These strict standards and the implications of low-performance can easily push teachers and parents towards the belief that direct instruction is the only way to help children learn effectively. In this article, we review evidence from the literature about playful learning as an alternative and powerful pedagogical approach. We apply the principles of playful learning to specific state standards for mathematics and illustrate promising ways to improve mathematics learning in the classroom.

**KEYWORDS** *Common Core State Standards, games, guided play, playful learning*

It is not uncommon to hear parents proclaim, "I send my child to school to learn, not to play." The nation's test scores are low—performing only 27th in mathematics—below countries like Japan, Germany, and Slovenia (Organisation for Economic Co-operation and Development [OECD], 2012). If children in the U.S. are to succeed in the global economy of tomorrow—if we are ultimately to raise our international standing, we need to prioritize stronger outcomes from our children's earliest educational experiences. Play is seen as the antithesis of this strategic plan. Ironically, however, it might be the elixir that can boost mathematical motivation and learning in ways that will increase those scores.

*Playtime* is often perceived as separate and mutually exclusive from *instructional time*. Instructional time in U.S. schools is often seen as limited to direct instruction, where children are told what they are supposed to learn in an effort to boost academic test scores. But despite the increased emphasis on structured activities in school as well as the elimination or limitation of recess (Elkind, 2008), we are not seeing increases in academic perform-

ance. According to the 2012 Programme for International Student Assessment (PISA) test results, the U.S. not only ranked below average in Mathematics (27th), but only about average in both Reading (17th) and Science (20th) as well. This pattern has remained steady (OECD, 2012) even though this latest testing cycle included children who were educated entirely under the No Child Left Behind (NCLB) initiative. And yet, despite the good intentions of NCLB, we have not seen a performance increase relative to other countries, and the achievement gap remains (Reardon, 2012; National Center for Education Statistics, 2013). *What we are doing is not working.*

With mathematics as the content area in which the United States performs the weakest relative to other areas tested on the PISA, there is a pressing need for utilizing effective mathematics instructional approaches. One attractive option is that of *playful learning* (Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009)—a broad pedagogical approach that challenges the mutually exclusive interpretation of playtime and instructional time that has led us astray. Playful learning addresses learning goals

through a variety of child-directed play methods, including *free play* and *guided play*. Guided play in particular has been shown to be more effective than either free play or direct instruction in some circumstances. For example, Fisher, Hirsh-Pasek, Newcombe, and Golinkoff (2013) compared children’s ability to learn about the features of shapes via three different types of instruction: (1) *guided play*, in which an adult partner followed each child’s lead during play with shapes and asked questions helping children to discover key information; (2) *didactic instruction*, in which an instructor delivered the same content while the child simply watched and listened; and (3) *free play*, in which the child could interact with shape materials without adult involvement. Children experiencing guided play learned more about shapes than those participating in didactic instruction or free play. When children were asked to extend their knowledge of shape properties to new, atypical shapes, children who learned through guided play exceeded those who learned through didactic instruction by over 30% and by those in the free play condition by about 55%.

These findings do not deny that children likely learn important information through free play, such as how to negotiate or share knowledge with peers. Indeed, some theorists propose that free play is sufficient for learning and leads to happier, more independent students (Gray, 2013). The advantages of both free play and guided play are captured under the umbrella term of *playful learning*. While many might suggest that play should stop with recess to make more time for academic instruction, the concept of playful learning suggests that incorporating play into the classroom can increase learning. When it comes to mathematics, the Common Core State Standards for Mathematics (CCSSM) provides a framework for structuring deep and meaningful mathematics learning at all grade levels (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). By offering some background from previous studies and giving real-world examples, we demonstrate that playful learning is a powerful instructional approach for meeting the requirements of the CCSSM.

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### Mathematics, Play, and the Common Core State Standards

Why might playful learning be effective for mathematics? Play helps set the stage, or creates a *mise en place*, for learning (Weisberg, Hirsh-Pasek, Golinkoff, & McCandliss, 2014). *Mise en place* may tap into the neural mechanism of *proactive control*—meaning that play helps put

children in a *prepared* mindset for learning. Optimal conditions for learning are created by inspiring children’s active exploration rather than having children adopt a passive role. Setting a playful and positive tone may be especially important for the learning of mathematics (Fisher, Hirsh-Pasek, & Golinkoff, 2012), as children vary widely in their attitudes towards the subject. The concept of *mise en place* suggests that playful learning, with its scaffolding via appropriate materials and guidance, may help create a positive stance toward learning. That stance may, in turn, result in significantly increased learning.

In the remainder of the article, we demonstrate an evidence-based approach for using playful learning to support CCSSM objectives. We first review an official CCSSM Objective and then present empirical evidence that suggests playful learning can serve as a potential mechanism to support that Objective. Given the breadth and depth of the objectives across early childhood grades, we cannot review evidence for each and every objective; instead, we offer these as case examples but urge educators to think about the ways playful learning can be used for any objective.

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### Case Examples: Using Playful Learning to Support CCSSM Objectives Across Grade Levels

#### Case 1: Know Number Names and the Count Sequence (Kindergarten)

**CCSS.MATH.CONTENT.K.CC.A.2**

Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

We first turn to the Kindergarten standards since early mathematics knowledge lays the foundation for children’s later mathematics performance. In Kindergarten, children are tasked with learning how to begin counting at any point in the count sequence. With four- and five-year old children, Ramani and Siegler (2008) found that playing a linear number board game—the *Great Race Game*—for four 15- to 20-minute sessions within a 2-week period increased low-income children’s mathematical knowledge, and these performance benefits lasted at least 9 weeks. Game play is one method by which playful learning approaches can be integrated into mathematics curricula. When games are merged with mathematics content, their playful, active, and engaging components may increase children’s motivation to learn.

So, how might teachers incorporate this kind of

evidence-based, playful learning in a classroom setting? One can imagine a Kindergarten teacher creating a game modeled after Ramani and Siegler's game, consisting of a ten-frame of 2 rows and 5 columns, with boxes numbered 1-10, to use as a game board. Imagine that the spinner for this game features the numbers +1, +2, -1, and -2 on a circle divided into four parts. A small group of three children plays the game as the other groups rotate through different learning activity stations. Each player takes a turn spinning and moving the appropriate number of squares. The teacher models how the children should count during the game by saying, "Suppose I was on number 3 on the board and I spun +2 on the spinner. I would take my piece and move 2 spaces on the board, saying, 'I'm on 3. I'm going to move 4, 5.'" While learning skills like turn-taking, the children would also get a chance to practice the count sequence and associate the names of numbers with their written representations. While children may think they are simply engaging in game play, this type of playful learning can simultaneously help them to learn the concept that counting can start from a different value than one.

**Case 2: Reason with Shapes and Their Attributes (Grade 1)**

**CCSS.MATH.CONTENT.1.G.A.3**

Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Fisher et al. (2013) found that playful learning was effective for helping children learn about shapes. In first grade, instruction focuses on reasoning about shapes and their attributes, including the concept of equal shares. How can playful learning help children learn this concept? Let's take another peek into a hypothetical classroom.

To help introduce equal shares, the teacher has created laminated pictures of pizza slices of varying sizes (i.e., halves, quarters, and eighths) and gives them to small groups of children. Whole pizzas can be made up of either two, four, six, or eight pieces. First, the children are given some time to just play with the pizzas. Then, the teacher asks each group to figure out how to give each person an equal share of the pizza.

While the children play around with dividing the pizzas, the teacher walks around to each group and helps

them decide if they have shared equally. While visiting each group, the teacher also introduces the terms *halves*, *fourths*, and *quarters* to help the children describe how they have divided the pizzas. The students are then asked to determine how many halves and quarters make a whole and how a half can be broken into smaller shares. Giving the children time to play with the objects before diving into instruction helps establish the *mise en place*, or mindset for learning. Children are given the opportunity to explore the materials—stacking the pieces, seeing that two quarters can be put together to make up a half, or two halves can be placed together to form a whole. Then, when instruction happens, children are not only familiar with the actual materials but may have already begun to see the mathematical concepts taught in the later lesson. By using pizza slices rather than basic triangles and circles, the teacher is showing them that shapes exist in the world and are meaningful in their everyday lives. Simply put, pizza mathematics matters. This use of playful learning will not only help increase engagement but also help children to see this lesson at home. As they learn that the mathematics they are doing in school connects with the real world, extending this knowledge to novel situations will be even easier.

**Case 3: Represent and Solve Problems Involving Multiplication and Division (Grade 3)**

**CCSS.MATH.CONTENT.3.OA.A.2**

Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

By third grade, children are asked to interpret whole-number quotients of whole numbers. Traditionally, children learn this concept through both partitive division (determining how many items go into a given number of groups) and measurement division (determining how many groups are needed when you know the number of items in each group). Habgood and Ainsworth (2011) designed a game that tapped into children's motivation to play computer games. They compared children's learning and time-on-task from two different versions of a game that involved division: *Zombie Division*. In one version, division was intrinsically motivated in that completion of a division problem was integral to progression through the game. In the other version of the game, division was extrinsically motivated in that the same division problems were given as a transition between levels of game play and were not inherently tied to game pro-

gression. Children from ages 7 to 11 years old who played the version that featured intrinsic, integrated division content, learned more division concepts and spent more time playing the game than children who played the game with division content that was extrinsic to game play.

Consideration of the details of the content-integrated version of *Zombie Division* illustrates possible reasons for why the game worked so well. This adventure game featured sword fighting, where children used different types of attacks to divide opponents in order to partition the dividends shown on their chests into whole numbers. For example, children could try to divide a zombie by 2 using a sword swipe, by 3 using a charge with a shield, by 5 using a gauntlet punch, or by 10 using a sword swipe *and* a gauntlet punch. When divided using a proper attack (e.g., using a sword swipe to divide a zombie displaying “10” into 2), the ghost of a defeated skeleton appeared and then split into equal-size portions showing the quotient. Using an improper attack led to the zombie skeleton defeating the child, requiring the child to restart the level. Children were tasked with different quotients and different weapons allowing them to use both partitive and measurement division based on the problem presented in each level. Teachers should keep in mind that this is simply an example of a game. Whether the gameplay is through digital games or classroom exercises that engage children in problem-solving through play, the key to leveraging playful learning is to provide children with the opportunity to play their way to increased understanding.

**Case 4: Apply and Extend Previous Understandings of Numbers to the System of Rational Numbers (Grade 6)**

**CCSS.MATH.CONTENT.6.NS.C.6**

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Riconscente (2013) explored the effectiveness of the *Motion Math* digital game to further develop fifth graders’ fraction knowledge. The game required children to connect visual fraction models, percentages, decimals, or fractions to a number line. *Motion Math* was played on an iPad with children tilting the device to direct a star to the correct place on a number line at the bottom of the screen. Each star displayed either a fraction, percentage, decimal, or pie shape. Incorrect answers resulted in instructional hints, starting with an arrow

pointing either left or right, then pictorially breaking the number line into the appropriate number of segments, and eventually showing labels on the marks. *Motion Math* had three levels, and each level had 24 increasingly difficult sublevels. After playing for 20 minutes per day over 5 days, children’s fractions test scores improved an average of over 15%. Perhaps more interestingly, children’s belief in their ability to understand fractions improved an average of 10%; their liking of fractions (e.g., “Fractions are fun”) also improved an average of 10%. Playful learning creates a positive *mise en place*, which is consistent with this increase in children’s learning, self-efficacy, and enjoyment of fractions. When children are motivated, they are more likely to enjoy learning and see themselves as capable learners.

In the absence of access to digital platforms, one could imagine a classroom where a 6th grade teacher presents small groups with number line “challenges.” In this game, children are given an opportunity to play with numbers that relate to the number of people in attendance at the concert of the latest pop star or the number of people in line at the local amusement park. Now, when they engage with the number line, they are doing so within a playful context instead of through more rote methods. This is not to say that playful learning is the only way to learn; instead, we argue that playful learning is one tool of many that can help children to not only learn but to enjoy learning as well.

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### **Making a Final Case for Playful Learning**

Playful learning is an instructional approach that harnesses the power of a positive *mise en place* to foster learning and engagement across domains. With younger children, play has created similar opportunities for fostering language learning (Hadley, Dickinson, Hirsh-Pasek, Golinkoff, & Nesbitt, 2015; Han, Moore, Vukelich, & Buell, 2010; Toub et al., 2016) as well as scientific curiosity (Schulz & Bonawitz, 2007). Here, we explore the ways in which playful learning instruction techniques can support the acquisition of CCSSM Objectives across the elementary years. We recommend the integration of playful learning into the K-6 curriculum and likely for older grades, too. Learning is not incompatible with enjoyment and is more likely to “stick” when children are engaged and involved in the process. While playful learning is not the only mechanism that leads to learning, the evidence shows that it can, in some cases, lead to stronger learning than other techniques. The tools for playful learning already exist in today’s classrooms,

making this technique readily available—from board games, to object play or digital games. Play and learning does not have to be an either-or proposition; learning new concepts in mathematics and beyond can be fun. This approach has potential long-term consequences. Creating a zeal for mathematics from the earliest grades may well inspire children to continue to take mathematics courses and pursue STEM careers.

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