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Modeling as Story-Building and Storytelling: Developing the Mathematical Identities of Adolescent Girls of Color

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ABSTRACT "Algebra-as-gatekeeper" is a powerful paradigm that structures students' experiences within mathematics class, as well as their future educational trajectories. In this paper, I show how modeling—when conceived as a culturally relevant practice—can support students to make sense of the key ideas of algebra as well as develop positive mathematical identities. This requires two important facets: 1) that students see themselves reflected in the contexts that are mathematized and 2) that they are positioned to be creative, sense-making story builders and storytellers of this mathematics. Results from my semester-long design study with adolescent girls of color illustrate the potential of modeling, especially for students who often experience the greatest barriers to participation, engagement, and learning within the high school mathematics class.

KEYWORDS *modeling, algebra, students of color, culturally relevant pedagogy, storytelling, design study*

For many, algebra is not simply a required high school course but a determiner of students' educational trajectories. Unlike other secondary mathematics courses, it not only mediates access to higher-level secondary mathematics but also serves as a gatekeeper to college. In fact, it has functioned as a gatekeeper for decades (Ladson-Billings, 1998; National Research Council, 1998)—effectively creating a system such that "passing an algebra course becomes a barrier to educational and economic advancement" (Greer, 2008, p. 425). Further, "knowing algebra" can be considered a cultural hallmark of a mathematically literate person; and since literacy and citizenship are profoundly linked in our culture, algebra shapes notions of citizenry as well (Moses & Cobb, 2001).

More troubling, it is well known that students of color—specifically Black and Latinx—and low-income students are more likely to be placed (and kept) within lower-track mathematics courses such as algebra (Cortes et al., 2013). Tracking has been widely criticized for impeding the academic progress of students and exacerbating, rather than minimizing, existing inequalities (Oakes, 2005; Powell et al., 1985). In fact, tracking is considered socially unjust since low-income students and students of color are overrepresented in these "dead-end" courses (Oakes, 2005), where they experience both low-level content and diminished expectations of them as learners (Powell et al., 1985). "Algebra-as-gatekeeper" is a tracking paradigm held together by high-stakes testing. Typically, when students do not pass their high school algebra course—or its affiliated exit exam or end-of-course exam—they are labeled as "repeaters" (a reductive, deficit label) and placed in a variety of intervention models, many of which have already been shown not to be effective (Boaler & Sengupta-Irving, 2016; Cortes et al., 2013; Fong et al., 2014). "Repeaters" courses disproportionally affect students of color, and particularly those from low-income communities, who seem to exist in a sort of educational purgatory I call "endless algebra."

I wanted to understand the experiences of students caught within this paradigm in order to design alternative experiences. Over the course of a semester, I examined the effect of changing the conditions in which the participants came to experience mathematics. I chose an all-girls public high school as the research setting precisely so that I could study, and respond to, the specific issues that adolescent girls of color face within the secondary mathematics classroom related to their agency, their engagement with mathematics, and the intersection of their mathematical identifies with other identities (Joseph et al., 2019; Radovic et al., 2017). Scholars have demonstrated the ways that mathematics socialization and mathematics identity development are critical aspects to the learning and participation of students, particularly Black students (Berry, 2008; Martin, 2007). Because learning mathematics and developing a mathematical identity are inseparable, I pursued two related research questions:

1. How does modeling develop key ideas in algebra, such as functional reasoning?
2. In what ways does modeling support the development of mathematical identities for adolescent girls of color?

In this paper, I describe one important finding from this larger qualitative design study: When mathematical modeling is reconceived as a form of story-building and storytelling, a wider range of learners will a) see themselves within mathematical contexts, b) begin to consider themselves creators of new mathematics, and c) make sense of key concepts within algebra. First, I will define mathematical modeling as it is commonly understood. Then, I will briefly describe the research methodologies used. Next, I will provide evidence of this salient finding. Finally, I will offer a brief discussion of the potential and the limitations of mathematical modeling as an intervention for students like the ones in my study.

What is mathematical modeling?

The relationship between mathematics and the real world lies at the heart of mathematical modeling. That mathematics is a "human endeavor" (Jacobs, 1994), inextricably linked to cultural and social phenomena, is a well-established idea. The term mathematical modeling arose out of the field of mathematics and can be found in almost every facet of science and social science (Pollak, 2011). Often used broadly with conflicting meanings, there is some consensus about two types of central activities to modeling: translating some aspect of the real world into mathematical terms (Gravemeijer, 1997) in order to solve or analyze a situation; and a cyclic process that includes: describing the problem, establishing limiting assumptions, structuring a situation mathematically, selecting and omitting variables, predicting results, interpreting, and validating possible results (Blum, 2002).

Since modeling always originates *outside* of the mathematical domain—a situation, a context—it is often conflated with the solving of "word problems." Learning to solve "real-life" problems is often assumed to be more challenging than solving their "anesthetized counterparts in textbooks and tests" (Lesh & Doerr, 2003, p. 4). But modeling differs from solving word problems in a few important ways: it does not result in "short answers to narrowly specified questions" (Lesh & Doerr, 2003, p. 3) but, rather, a generalizable method that can be shared and reused across related situations; the process tends to be less sanitized, more iterative, and more complex; and more than one mathematically valid model typically emerges.

Methodology: Why a Design Study?

Design studies have been characterized, with varying emphasis, as iterative, process-focused, utility-oriented, and theory-driven (Cobb et al., 2003). Given these key features, design studies afford several advantages: testing theories iteratively and in-the-moment within the context; treating participants as co-constructors of knowledge; tackling everyday dilemmas in classrooms, schools, or systems; recognizing the constraints of theory as it applies to actual participants; and sharpening theory within an educational context. Of course, all scientific research must provide evidence to support its theoretical claims; with design studies, this evidence typically emerges from highly complex and dynamic situations such as classrooms, or what has been called the "crucible of practice" (Shavelson et al., 2003).

Given my desire to study learning and identity development up close and in-depth, a design study, or design experiment, provided the best opportunity to investigate while generating some emergent theories. I situated my study within a high school algebra class in a large, urban, public, all-girls school. After conducting initial classroom observations and in-depth interviews, I selected a "repeaters" course where both the teacher and students were interested in supporting the research. The students—all girls of color—were programmed for this class because they had not passed Algebra I somewhere between one and four times, including summer school.

The class had twenty students enrolled, so I narrowed my gaze in order to focus more deeply on six students who consistently attended class, agreed to be participants, and were representative of the class as a whole. I selected, designed, and modified a sequence of model-eliciting and model-interpreting activities that were designed to support the development of the concept of function while also deepening the participants' mathematical identities. That is, I hoped to engender not only a deeper conceptual understanding of function, a central object of algebra, but also to provide opportunities for the participants to re-engage and re-identify positively with mathematics. Over the semester, I collected and analyzed sets of rich data: video recordings of all class sessions, including small group work, formal interviews and informal interactions with the students, and all student work related to the modeling process.

Whose worlds are being mathematized?

Research consistently describes the ways that students do not see themselves or the "real world" within many school mathematics contexts. Boaler (2008) claims that to do well in mathematics class, children must suspend reality and accept nonsensical problems where, for example, trains travel towards each other on the same tracks, and people paint houses at identical speeds all day long. Further, she has argued that students come to understand that "if they think about the problems and use what they understand from life, then they will fail" (p. 51).

Modeling attends to notions of the "real world," requiring learners to situate mathematical ideas within a realistic or believable world, but the question remains: whose worlds are being mathematized? Often, teachers select contexts that we hope our students will relate to. Yet, the same cultural mismatch that students tend to experience with traditional word problems is not necessarily resolved when modeling. There are a few attempts within the modeling literature to attend to the cultural lives of students when designing and enacting tasks (Cirillo et al., 2016). But research on modeling, as a whole, doesn't concern itself with aspects of students' culture. For this reason, the modeling tasks used in this study were deliberately re-situated in imaginable culturally relevant contexts. This meant studying the participants as girls of color within the classroom, as well as other social spaces within the school setting. In listening, observing, and developing relationships with the participants, I came to understand their interests, passions, beliefs, and lived experiences, which became fodder for contexts to be modeled.

That the participants saw themselves in the mathematics was critical. But it was equally vital that they were treated as both story-makers and storytellers, giving them ongoing opportunities to envision, participate in, and shape the narratives. It was important that the students came to associate storytelling as mathematical in nature—not a diversion (e.g., "off-task" behavior) nor a gimmick (e.g., getting them to do some mathematics), but the vital sense-making work that modeling demanded. As I will demonstrate, this crafting of narratives—personal, cultural, individual, and shared—created conditions under which mathematical identities grew more agentic and more aligned with other salient identities.

With these two ideas in mind—seeing oneself within the contexts and seeing oneself as a storyteller of mathematics—I began to envision modeling as a form of culturally relevant pedagogy (Ladson-Billings, 1995; Gay, 2000). Specifically, I was drawn to story-making and storytelling as a culturally relevant teaching practice. Dyson and Genishi (1994) describe humans' "basic need for story," which they define as a process of "organizing our experiences into tales of important happenings" (p. 2). This idea can also be found in mathematics education research, particularly among those interested in humanizing aspects of mathematics. Su (2020), for example, explains how story functions for math learners, particularly those on the margins:

> Learn a bunch of separate mathematical facts, and it is just a heap of stones. To build a house, you have to know how the stones fit together. That's why memorizing times tables is boring: because they are a heap of stones. But looking for patterns in those tables and understanding why they happen—that's building a house. And house builders perform better in mathematics; data show that the lowest-achieving students in math are those who use memorization techniques, and the highest-achieving students are those who see math as a set of connected big ideas. (p. 38)

Here, he posits that story can serve as a unifying force—the metaphorical "glue"—to hold together and make sense of otherwise abstract, disconnected, or disparate ideas. As such, the idea of mathematics-as-story could be especially powerful for those students in lower-tracked mathematics who were subject to a curriculum of disconnected and unrelated skills and facts and not as a "set of connected big ideas."

In the sections that follow, I will illustrate how I enacted modeling experiences to invite participants to be both story-makers and storytellers, giving them continued opportunities to envision, imagine, and participate in the narratives while developing important ideas within algebra such as function.

Narratives as a form of identity

Drawing upon socio-cultural views of identity—particularly those that recognize the complexity and intersectionality of identities (Martin, 2000; Nasir & Saxe, 2003)—I will demonstrate how the girls' existing identities were shaped and developed within the context of our modeling investigations. First, I will situate our modeling work in the students' own descriptions of their experiences both in school and in math class specifically. Drawing upon data from pre-and post-interviews, I will provide a glimpse into the complex identities that existed for the students.

Mathematical identity and its relationship to story

As I designed the mathematics of the study, I was mindful of the themes that emerged from my initial interview data. For example, eleventh grader Natasha described her favorite classes and why mathematics had never been among them:

Natasha:

> English, you don't really have a right or wrong answer in that class. It's just like…you do your own, like, thoughts—you write down your own thoughts, you read. It's just like… it's not like how math is, like there's one answer and one answer only. In English, there's not really an answer.

The reductive right-wrong binary that Natasha experienced in math class was repeatedly described by the participants, where there was little room for "your own thoughts" outside of an "answer." Later, she expanded upon this distinction:

Natasha:

> Because in English, you can sort of like…project your opinion, like it's not just—you could show more of yourself. Like math class is not like—it's not really like—you're not showing much. It's just about numbers and stuff. It's not like about feelings. It's not about like conflicts. It's not about nothing. Like English is about all that.

Here, Natasha located both her feelings and opinions as *outside of* and *separate from* math class. She did not claim they were incompatible, but that for her, they were not experienced within mathematics. The idea that math class was a place where "you're not showing much" spoke directly to her sense of mathematical identity. Natasha explained why, within her English class, she was positioned to reveal more of who she is and why math class had not afforded her this same opportunity. Mathematics class, in her view, was not a place to "do your own thoughts," "project your opinion," or "show more of yourself." These aspects were salient to her own sense of self. In her fullest critique of math class, she claims that "it's not about nothing." That is, mathematics is about nothing that is relevant, important, or meaningful to her.

Tenth grader Danya expressed a different form of disconnectedness from mathematics, noting that she rarely felt successful as a math learner. She described how she currently experienced mathematics teaching and learning:

Danya:

> The way they do it here, it's all math steps, so like that's what makes it more confusing. It was like we did a whole bunch of steps, and it confuses us.

Her conception of mathematics as "all math steps" was indicative of the emphasis within testing environments on procedures (usually at the expense of reasoning, justifying, or developing conceptual ideas) and described by each of the participants in the study. It was consistent with Williams and Miner's (2012) notion that "standardized tests also come packaged with demands for more standardized curriculum. These calls are part of a broader effort to promote a narrow version of what children should learn" (p. 10). The narrowing of mathematics that Danya alluded to is in direct contrast to the expansive work of modeling. As Gann et al. (2016) explain, "When students are engaged in modeling, they are not using an algorithm or following a prescribed set of steps" (p. 102).

In describing her teachers, Danya reinforced the fact that identities are intersectional and constructed socially. She explained how the intersection of race, ethnicity, and gender shaped who she is and how she learns. Most of her teachers, particularly her elementary teachers, were "just so confusing." One middle school teacher, however, was noticeably different:

Danya:

> Then in seventh grade, that's when it was just like, "This is easy." Like, "I know this." I don't know. I felt

> like there was a connecting or something, the way they taught it. Because, like, I'm not trying to be racist, but my teachers have been like American or something, and my seventh-grade teacher, she's actually from Jamaica. So I'm also from the Caribbean, so I don't know if there was like some type of relationship…I just understood the way she taught it.

Learning from an Afro-Caribbean woman helped Danya to make sense, to see math as "easy," and to feel as if "I know this." Instead of "steps," she spoke of understanding, a noticeable shift. She suggested that what made this learning possible was "some type of relationship"—that this personal connection was critical to her understanding of mathematics. The importance of an emotional bond between teachers and their students—to motivate, to engage, to generate learning—is well-established (Waller, 1932; Lee et al., 1997). In this case, the bond between Danya and her teacher was grounded in their shared racialized and gendered identities—a vital connection between them, even if it was never articulated outright.

For Danya, this relationship with her mathematics teacher also re-oriented her towards mathematics and led her to consider her mathematical identity related to, not separate from, other intersecting identities. Research confirms that racial identities develop early (Tatum, 1997), yet Danya's middle school experiences also bolster what Noguera (2003) has noted: "Adolescence is a period when young people begin to solidify their understanding of their racial identities" (p. 20). Understanding how Danya came to see herself as an Afro-Caribbean girl has a significant bearing on the development of her mathematical identity, as these are dynamic and intersectional. Given the associations of mathematics (and for that matter, all things considered as "the norm") with whiteness, maleness, and middle-class status (Rubel, 2019), it was vital that Danya saw herself, and others like her, within the community and practice of mathematics.

Because of these early analyses, it was critical that the modeling investigations evolved from imaginable, culturally relevant stories so that each of the girls felt some personal connection and interest. Typically, this happened either by introducing an artifact (e.g., an object, an image, a video) or by orchestrating some common referent (e.g., a shared experience). It was necessary, but not always sufficient, simply to situate mathematics in contexts that might resonate culturally with the students. After all, there is a well-established tradition of "dressing up" mathematics in contexts to engage or inspire learners, often with limited success. What was equally important was to elevate the students as story-makers and storytellers as well—that is, to give them a specific role in creating the mathematics through story.

Of course, storytelling can be a remarkably intimate act, full of the possibility of connecting with others. The desire to connect with others in school and beyond has been found to be particularly important for adolescent girls (Belenky et al., 1986; Gilligan, 1990). These salient aspects of stories are bolstered by Gay (2000):

> Stories are means for individuals to project and present themselves, declare what is important and valuable, give structure to perceptions, make general facts more meaningful to specific personal lives, connect the self with others, proclaim the self as a cultural being, develop a healthy sense of self, forge new meaning and relationships, or build community. (p. 3)

The use of storytelling in this study was situated within a gendered and racialized context, consistent with what Joseph et al. (2019) described as "transformative pedagogical models" for Black girls within the high school classroom. They proposed that when students were encouraged to "personalize mathematics with examples from their own communities and histories" (p. 138) and given meaningful ways to co-create knowledge, they were more likely to be repositioned within the mathematics classroom. Aguirre et al. (2013) extended this idea, situating it within mathematics:

> Mathematical identities can be expressed in story form. These stories reflect not only what we say and believe about ourselves as mathematical learners but also how others see us in relation to mathematics. (p. 14)

Therefore, the relationship between storytelling and identity-building was critical to the study: we construct and tell stories about ourselves to reveal to others who we are and to become members of communities who share salient aspects of our identities. Within the mathematical context of this study, story and story-building were vital in several ways: allowing students to locate themselves within the contexts of the mathematics; illuminating mathematical identities and connecting to other intersecting identities; and offering new, meaningful roles to students in math class who experienced barriers to learning or participation.

Using artifacts as stories: Determining what is real

In the first modeling task, the girls were invited to study and mathematize a high school graduation program shown in Figure 1. First, I invited them to share their own stories of graduation. A wide-reaching conversation ensued, as nearly every student sharing a detailed story relating to a graduation ceremony they experienced. They described "gap and gown," "diplomas," and the ritual of family and friend photos. The stories transitioned across time and space, from fifth-grade graduations to imagined college graduations. For some, like Keisha, the ceremony was a significant feature: "In the graduation, my parents are going to cry. They take photos and more photos. My mother cries through everything." For others, like Jaila, it was the meaning of the graduation that mattered. "The only thing I care about," she explained, "is getting my diploma. I don't care about the graduation. I just need to know I'm not in high school anymore." When storytelling is connected to larger pedagogical goals, this small example of personalizing the mathematics shows how it is both appropriate and useful within the context of teaching and learning.

Once the idea of graduation was connected to personal experiences, I introduced the artifact. Tenth grader Danya noted publicly, "I have a lot of questions. Now let me see if I can find someone with my name." This inspired her classmates to look for their own names as well:

Keisha:
Oh my gosh, there was a Keisha there. Natasha, I found you.

Jaila:
There's no Jaila.

Danya:
There's no Khadijah.

Keisha:
There's a Kristin. Do you want a Kristin?

Figure 1

High School Graduation Program


San Lorenzo Valley High School
Class of 2011

Micah Stephen Adams
Xavier Josue Alvin
Connor James Antisdel
David Zachary Baker
Bradley Garrett Barnard
Joseph Barnes
James Alexander Barnes
Michael Barton
Sarah Raisha Beasley
Allison Rose Beasley
Emily Anne Bechtle
Brandon Scott Beevers
Patrick Fereva Belardi
Kaitlyn Ella Benson
Jordan Starr Thomas Bereman
Taylor Renee Foster Berritto
Ashley Michelle Ann Billington
Morgen King Biswa
Jaclyn Kate Black
Xitlali Borreson
Chere Nicole Brandon
Sydney Lauren Garcia Breili
Curtis Martin Brewer
Katherine Evelyn Brown
Annika Bruce
Amanda Lynn Bruce
David Michael Burgler
Jourdan Donovan Burk
Nicholas Ryan Burks
Joseph Thomas Burton
McLean Avery Camacho
Alexander Austin Campbell
Jason William McGregor
Taylor James Casey
Kory Daniel Chadwick
Audrey Elizabeth Chapin

Amber Anne Chase
Faye Elise Chavez
David Thomas Chesus
Zoe Darlene Chinn
Emily Elizabeth Clements
Claire Elizabeth Cloud
Steven Dennis Connelly
Conor Thomas D'Amato
Cassandra Suzanne Davis
Lauren Marie Dempewolf
Blake Bradley Dennis
Jake Madison Debreuil
Shannon Elaine Eisner
Emily Anne Engel
Samantha Marie Ferguson
Dominique Angelina Jones
Cherina Marie Freitas
Anju Friend
Nathaniel Justin Fruzza
Mariah Nichole Galmez
Robin Lavendar Garcia
Ronja Andrea Franklin
Mason Garrett
Kayle Anne Genis
Melanie Lauren Gleim
Andrea Serena Godbout
Angela Gonzales
Chloe Chandler Goodreau
Dakota Makua Gorman
Donald Jesse Greer
Michael Marie Grindy
Erin Cassidy Groswird
Krista Marie Grunberger
Tyler Alan Hagen
Hazel Gladys Jane Haikkila
Marcus Taylor Halversen

Nicholas Gregory Hargraves
Alexander Samuel Harnett
David Michael Hart
Melissa Lorraine Harvey
Olivia Louise Herrera
Kasia Adair Hill
Ludmila Eliana Hipsley
Jazelle Gina Hooper
Mariah Donna May Hopkins
Lindsey Marie Hoppin
Bryn Kathryn Horton
Thomas John Housek
Ellen Joanne Innis
Jordan Isaacson
Joseph Clifford Jansen
Allison Kirstine Janus
Nettie Pearl Mitchell Johnson
Lacey Marie Johnson
Felipe Johnson
Shelby Danielle Johnson
Zachary David Johnson
Dakota Shea Jones
Tanner William Jones
Clara Joy Kamau
Mia Anne Kellogg
Maxine Elaine Kelly
Ryan Scott Kennedy
Jenny Rae Kersten
Andrew Benjamin King
Russell Theodore Klair
Zachariah Waya Tsgili Klus
Christina Rose Knoll
Jessica Danielle Kraft
Jessica Ann Lacy
Joseph Allan Landry
Abbie Mae Leveque

Matthew Alexander Lipperd
Madeline Elizabeth Lopez
Emily Marie Madison
Rebecca Lynn Makita
Savanna San Mangelsen
Zane Michael Markham
Alfredo Martinez
Tyler Cameron Mattson
Wayne Thomas McCarthy
Theos Christopher McClish
Annelise Marie McFarland
Alexander Michael McIntosh
Dominique Sharee McIntosh
Victoria McKenzie
Violett Josette McNally
Kendra Dee Messimer
Marisa Grace Brudnick
Sage Nicole Monack
Thomas Ocean Moreno
Rainbow Roxanne Muchamuel
Nicholas Jefferey Newberry
George Austin Norfleet
Travis James Nugent
Joseph Carl Olson
Lacie Marie Orlando
Mary Elaine Ivy Orr
Austin Leonard Overton
Haley Lauren Pace
James Evan Paolini
Timothy William Parker
Carson Taylor Paynter
Zachary Alan Peabody
Katrina May Pearce
Samuel Jackson Pelphrey
Chandler Elizabeth Perazzo
Sebastian Thomas Peterson

SOURCE: ILLUSTRATIVE MATHEMATICS

The episode illustrates three important ideas. First, the students were individually and collectively looking for themselves, illustrative of the need to verify the artifact's authenticity or believability. Alternatively, the search for one's name might have been symbolic of students' desire to be "seen" within the mathematics classroom. That is, finding one's name—particularly if it was unique or culturally specific—validated that this was not the teacher's mathematics but, in fact, a part of students' own mathematics. Second, the girls were beginning to make their "noticings" public and shared. Noticing, as a valued mathematical behavior, was a new idea and allowed them to move beyond the "right-wrong" paradigm of mathematics. There was no risk of being "wrong" because "wrong" did not exist within this context. As such, noticing provided all participants a way into the conversation, regardless of their perceived status. Third, this phase of modeling, *problem posing,* led to the codifying of the students' questions. Those questions, in turn, would later structure how they mathematized the situation. But, the questions needed to be grounded in the context, and therefore the students needed to examine and make sense of the program as they were doing here.

Debra changed the direction of the conversation entirely when she declared, "These names don't seem real." Encouraged to explain, she said, "These names are foreign." She began to read aloud a few names to justify her belief: *Ringelman, Rinker, Peabody.* The students burst into laughter at the absurdity of these names. I paused to acknowledge that "these names don't feel like real people to you," and many agreed. In fact, this moment illustrated the importance of culturally relevant artifacts as springboards for learning. Debra, who identifies as African American, was asserting that these White-sounding names fell outside of her experience and world. Her use of the word "foreign" reinforced the strangeness of these names for her. She named the cultural mismatch between this artifact and her lived experiences. Our conversation about the believability of the program, and the students' skepticism about it, shifted into something broader:

Danya:

What is this supposed to show us?

Jaila:

Well, what relation does this have with math?

I reminded everyone of the graduation stories we had just shared and encouraged them to think of this graduation, strange names and all, as one we could imagine together. My efforts were grounded in ideas about story: We began with real stories from the students' lives, transitioned into a context *not believable/imaginable* to the students. To reconcile this mismatch, we were now in a new space of stories that could be *imagined.* As a white teacher-researcher, I knew that my chosen artifact had failed to connect with the students' lives and, in fact, had the potential to further alienate them from the mathematics. Yet, their critique of the "realness" of the artifact was consistent with initial interview data, in which the girls articulated a desire to be seen within mathematics and to create mathematics related to their own lives.

Beyond the artifact: The telling and re-telling of high school graduation

Once in small groups, the students investigated a question they had generated together in the problem-posing phase of modeling. For many, this meant beginning to model the situation of the graduation ceremony and, later, to build a function of it. Keisha, Jaila, and Danya wondered when the graduation program began. They used mathematical details in the program to develop a generalizable rate of five seconds per name. When Jaila doubted this rate, her partners re-told the name-calling portion of the graduation, using what Keisha and Danya called its "regular beat" and "correct" tempo to attempt to convince her that this mathematical claim was reasonable. Not yet convinced, Jaila asked them to "do it one more time," this time noting that "you sound like the people," presumably the teachers or administrators calling graduates to the stage. This exchange, where mathematics was performed, not written, was one of many throughout the study and was consistent with Hammond's (2015) theories about culturally relevant teaching within African American traditions. "By telling stories and coding knowledge into songs, chants, proverbs, and poetry," she wrote, "groups with a strong oral tradition record and sustain their cultures and cultural identities by word of mouth" (p. 15). That Danya, who identified as Afro-Caribbean, and Keisha, who identified as African American, engaged in modeling as a form of performance or re-enactment, supported them to be active, creative, confident participants within their math class.

Later, the students revised their rate to five names per minute or twelve seconds per name. Wanting to know how this revision occurred, I probed further:

Miss Imm:

Does that seem right?

Danya:

Yeah.

Keisha:
Yep.

Miss Imm:
Do you think you could convince the group that it's twelve seconds per name?

To do so, Keisha embellished the story of the graduation:

Keisha:
She's in heels. That's right 'cause when you're in heels…And then most auditoriums are slanted. So, you have your heels, and it's kind of….[gesturing a slight slope with her hands]

Keisha also wondered whether they call your "full name." When Danya confirmed that they do, they searched for long names in the program (e.g., Netty Pearl Mitchell Johnson) to illustrate that these students, coupled with walking in heels on a slanted surface, might really take twelve seconds to cross the stage.

Taken together, these interactions demonstrate how story building and storytelling provide opportunities for students to make sense of mathematics from their lived knowledge. The "graduation" task was chosen to launch the study to uncover participants' prior knowledge and to connect that knowledge with new algebraic concepts. But more importantly, it gave the students a vivid experience of mathematics that was different than any they experienced before. The iterative "dance" of modeling between the context and the mathematics through story—the way they envisioned, reenacted, critiqued, embellished, and invented—allowed each of them to co-construct this mathematics with meaning. Because the central activity was modeling—where the context cannot be trivialized or separated from the mathematics—this narrative illustrates a critical feature of what it means to do mathematics. As illustrated previously, each time the girls returned to the story of the graduation, they used both the narrative *and* mathematical details to test, express or revise their model. As one example, Keisha adds her own details to the story that are not expressed in the artifact (e.g., heels, slanted auditorium) to justify the rate. This need for congruence between mathematical and lived worlds is at the heart of modeling.

Modeling as a collaborative process, not a singular product

A few days later, one student per group was chosen to defend their models to the class to discuss and verify them together. Each group made a poster of their thinking, displayed prominently as they presented. With her partner absent, Khadijah was asked to represent her group.

Khadijah:
So, I'm by myself?

Miss Imm:
Do you feel like you could represent your thinking? What's on the poster?

Khadijah:
I'm pretty sure I could do it… I'm pretty sure…

Then, tapping on the various ratio tables on her poster, she notes, "I've probably forgotten one thing, but I think I've got the process, so I should be fine." That the model on her poster is a result of a process is further evidence that modeling supported students like Khadijah to deepen her ideas about rate. When she first began modeling, she, like her peers, over-relied on social knowledge—shared or collective ideas developed in relationship to others—satisfied by anything that "felt" or "seemed" reasonable and not interested in justifying her instincts with mathematics. Here, however, social knowledge played a different role in her sense-making; she used it to bolster, not replace, mathematical claims. She trusted the unit rate that she and Natasha constructed could explain how it was equivalent to other rates in the graduation program and began to connect rate to the idea of function by identifying the two variables (*names called* and *minutes*) and explaining their linearity on a graph.

Modeling across these episodes provided Khadijah a chance to build on her emergent ideas about rate in three significant ways. First, that the mathematics of the context made sense to her allowed her to continually check her understanding against the "common sense" of the story. Each time she expressed hesitancy about the mathematics, she returned to the context to justify her ideas. Second, because she was not problem solving independently but modeling in a partnership, her thinking about rate was co-constructed. Third, because modeling was iterative and included a public defense, Khadijah's ideas (and confidence about those ideas) deepened over the study. She had multiple opportunities over several weeks to construct (and reconstruct) ideas about rate, ratio, equivalence, and function. She came to appreciate that our goals had shifted: from getting the right answer (e.g., *product*) to modeling as a generalizable *process,* as illustrated by her statement, "I think I've got the process, so I should be fine."

Several weeks later, in a new modeling task, Khadijah was again asked to present to the class on behalf of her partner. She noted that many students were absent:

Khadijah:
I think we should wait 'til everyone else comes because everyone else has to be here, too, as a class, to know.

Miss Murray:
We can wait if you want.

Khadijah:
Because we have to have the whole class here, too, so they would understand it.

The episode illustrates an important turn—that participants were now producing mathematics for each other and for the class. As such, it was important that "everyone else"—the "whole class"—was present. The subtlety of their language is critical in demonstrating how much their view shifted. Though their teacher referred to the phase of modeling as "present your information"—potentially a one-way transmission, not a discussion—Khadijah understood that the class must be present "so they would understand it." In framing the presentation as a chance for her peers to discuss and make sense of their thinking, she went beyond a one-way presentation to her peers. She knew that she had to convince the class of the viability of their model and be willing to defend it when her peers scrutinized it. This is consistent with Anderson (2007), who noted the importance of the collective: "Learning mathematics involves the development of each student's identity as a member of the mathematics classroom community" (p. 7). Collaborative activity, such as modeling, has particular value among adolescent girls who tend to enjoy and become skilled at working together, learning to trust one other, and ultimately elevating each other's voices within the classroom space (Kuriloff et al., 2017). Studies confirm that modeling is best enacted as a group activity (Ikeda et al., 2007) where discussions elevate the collective understanding.

This exchange also suggests that modeling helped to reposition the girls relative to mathematical knowledge. Specifically, it suggests the ways that modeling can support students to be *creators* and not simply *receivers* or *reproducers* of mathematical ideas. Boaler (2002) found that students, particularly in traditional classrooms, were often given a limited way of participating within math class— what Belenky et al. referred to as "received knowing" (1986, p. 4). This was the result of having mathematical ideas presented to them by teachers, or textbooks, as the girls described throughout the study. When they were modeling, as I have shown, the girls were neither receiving nor reproducing mathematical knowledge; they were creating it themselves. This supported them to interpret ideas, make choices together, exercise their own thoughts, and exhibit agency, all of which contributed to more integrated and positive mathematical identities.

The importance of explanatory narratives: Cell phone task

Later in the study, the students were given a set of cell phone images shown in Figure 2 (next page) and invited to notice, name, and wonder. The task emerged from my observations of their early-afternoon scurry to find empty classroom outlets to charge their phones before dismissal. The task was designed to a) provide deeper opportunities to model and make sense of function, b) build upon a well-known cultural object (e.g., iPhone), and c) allow the students to generate interpretative stories as part of the modeling process.

When we first introduced the cell phone images, we were careful to say little about the owner of the phone. Yet, almost instantly, the students had gendered the cell phone owner as male, based largely on his perceived importance:

Natasha:
Does he have a job, like a serious job? That's probably why he has so many emails.

Miss Murray:
Most definitely has a serious job.

Sam:
Is he like a businessman or like a lawyer or like —

Miss Imm:
More serious than that.

Khadijah:
A CEO or something?

Jaila:
He works for the government?

Danya:
The President?

Only a busy or important person (e.g., man), they reasoned, would have 31,000 emails. This association of important people with maleness provided a window into how gender had shaped their own identities as girls of color. Within the context of mathematics education, this is consistent with other analyses such as Mednick's (2005) study of how the construct of being "good at math" was shaped by gender and, as a result, taken up less frequently by girls. Further, Rodd & Bartholomew (2006) noted how participation within college mathematics was complex for young women, who experienced both invisibility and "specialness" related to their participation.

After realizing no more "character details" would be provided, the students moved towards understanding the users' behavior and usage patterns. They became

Figure 1

Cell Phone Images

COURTESY OF THE AUTHOR

focused on determining why the battery was declining unevenly over two intervals, which later drove their mathematical modeling. Natasha first described the difference:

Natasha:

I was saying—because from 4:52 to 5:06 is only 14 minutes, and his phone went down 4 percent. So I say he was on his phone, 'cause from 5:51 to 7:13 his phone only went down 5 percent, and that's a longer, like, time distance there. You know what I'm trying to say? Yeah.

Knowing that the rate of battery decline was not steady across the intervals (i.e., a nonlinear function), the students began to develop explanatory narratives about *why* this difference existed:

Keisha:

It's probably…you know what I feel like? And I think people who have iPhones would agree—it was probably unlocked, and the screen was on, but he just wasn't doing anything to actually kill the battery as quick as he did the first.

Jaila:

Mm-hmmm, 'cause grownups, I see them on the train. They don't lock their screen! I'm just sitting there like… "Can you lock your phone?" [throws hands up, eyes widen]

Keisha:

[laughing] Yeah, like they'll sit there and with their phone unlocked, like, brightness up and everything.

The exchange was important for several reasons. First, when Keisha first proposed an explanatory narrative, she treated her peers as fellow iPhone owners, noting that her expertise was shared since "we all have iPhones...we *know* them." Second, Jaila bolstered Keisha's argument with observations from her daily commute—a lived experience that gave her credible knowledge about how adults behave on their phones. Her exasperation at adults' refusal to lock their phones generated laughter among the group. Unlike kids, Jaila claimed, these "grownups" simply did not understand how to use their phones. She demonstrated *how* her knowledge of iPhones was deeper than that of the adults around her. Third, there was no teacher facilitating or structuring this dialogue, and no need for an adult to interrupt or guide this conversation. The artifact of the cell phone images alone served as a springboard for authentic, imaginative storytelling. Because of my insistence that modeling was a collaborative endeavor and that we make mathematics for each other (not the teacher), stories such as these were co-constructed—it was the students' narrative to craft and later justify.

To a skeptic, episodes like these might suggest students were "off task"—further from the mathematical goals of the algebra course, but this is where modeling is uniquely positioned to be cast as story-building and storytelling mathematics. There can be no mathematics in modeling without an imaginable, full-of-common-sense context in which to situate the ideas. Unless students can really immerse themselves in a situation—to personalize it and play an active role in the construction of meaning—then these contexts are just as flimsy (and useless) as the ones they had experienced before.

Discussion and implications

Stories reflect not only what we believe about ourselves as math learners but also how others may see us in relation to mathematics. Many influences—teachers, families, peers, schooling, and testing—play a role in the shaping of these identities over time. Mathematics identities are always co-constructed with racial, gender, and class identities. In this paper, I situated the analyses in Natasha's and Danya's experiences of math class—consistent with many pieces of data that suggested that the participants were both disengaged and dis-identified with school mathematics. I posited that modeling could provide an alternative under two important conditions: that each modeling cycle would center around a culturally relevant artifact or shared embodied experience and that students took up these artifacts and experiences as opportunities to create and tell stories. This linking of story-making with mathematics was consistent with the literature and particularly relevant for students who have never experienced mathematics as related, personal, or sensible. According to Su (2020):

> A story creates a narrative from disparate events and connects listeners to itself and to one another. It is not different with mathematics. Connecting ideas is essential for building meaning in mathematics, and those who do it become natural story builders and storytellers. (p. 39)

As the analyses here show, re-framing modeling as mathematical story-making and storytelling positioned the students to take up new roles, and by extension, to deepen their mathematical identities.

Modeling, no matter how broadly conceived, is not a panacea and promises neither a more inclusive experience nor better outcomes for students. It cannot disrupt the large inequitable systems (e.g., tracking, high-stakes testing, systemic racism) that shape students' access to high-quality mathematics. Yet, when modeling is reconceived as being designed for and with students, who will become the story builders and tellers of mathematics, it has the potential to disrupt existing inequitable patterns within classrooms. As such, this study suggests that a vital (but under-theorized) relationship between mathematical modeling, culturally relevant pedagogy, and identity development exists and is worthy of continued study.

References

Aguirre, J., Mayfield-Ingram, K., & Martin, D. (2013). *The impact of identity in K-8 mathematics: Rethinking equity-based practices.* The National Council of Teachers of Mathematics.

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *The Mathematics Educator, 17*(1), 7–12.

Belenky, M. F., Clinchy, B. M., Goldberger, N. R., & Tarule, J. M. (1986). *Women's ways of knowing: The development of self, voice, and mind* (Vol. 15). Basic books.

Berry, R. Q. (2008). Access to upper-level mathematics: The stories of successful African American middle school boys. *Journal for Research in Mathematics Education, 39*(5), 464–488.

Blum, W. (2002). ICMI Study 14: Applications and Modelling in Mathematics Education: Discussion Document. *Educational Studies in Mathematics, 51* (1-2), 149–171.

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* Routledge.

Boaler, J. (2008). *What's math got to do with it?: Helping children learn to love their most hated subject—and why it's important for America.* Penguin.

Boaler, J., & Sengupta-Irving, T. (2016). The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. *The Journal of Mathematical Behavior, 41*, 179–190.

Cirillo, M., Bartell, T.G., & Wager, A. (2016). Teaching mathematics for social justice through mathematics modeling. In C. Hirsch and A. McDuffie (Eds.), *Annual Perspectives on Mathematics Education 2016: Mathematical Modeling and Modeling* (pp. 87–96). National Council of Teachers of Mathematics.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1), 9–13.

Cortes, K., Goodman, J., & Nomi, T. (2013). A double dose of algebra. *Education Next, 13*(1), 71–76.

Dyson, A. H., & Genishi, C. (1994). T*he Need for Story: Cultural Diversity in Classroom and Community.* National Council of Teachers of English.

Gann, C., Avineri, T., Graves, J., Hernandez, M., & Teague, D. (2016). Moving students from remembering to thinking: The power of mathematical modeling. In C. Hirsch and A. McDuffie (Eds.), *Annual Perspectives on Mathematics Education 2016: Mathematical Modeling and Modeling* (pp. 97–106). National Council of Teachers of Mathematics.

Gay, G. (2000). *Culturally responsive teaching: Theory, research, and practice.* Teachers College Press.

Gilligan, C. (1990). Teaching Shakespeare's sister: Notes from the underground of female adolescence. In C. Gilligan, N. Lyons, & T. Hanmer (Eds.), *Making connections: The relational worlds of adolescent girls at Emma Willard School* (pp. 6–29). Harvard University Press.

Gravemeijer, K. (1997). Solving word problems: A case of modelling? *Learning and Instruction, 7*(4), 389–397.

Greer, B. (2008). Algebra for all. *The Montana Mathematics Enthusiast, 5*(2&3), 423–428.

Hammond, Z. (2015). *Culturally responsive teaching and the brain: Promoting authentic engagement and rigor among culturally and linguistically diverse students.* Corwin Press.

Ikeda, T., Stephens, M., & Matsuzaki, A. (2007). A teaching experiment in mathematical modelling. In C. Haines P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics* (pp. 101–109). ICTMA 12, Horwood Publishing.

Jacobs, H. R. (1994). *Mathematics: A human endeavor.* Macmillan.

Joseph, N. M., Hailu, M. F., & Matthews, J. S. (2019). Normalizing Black girls' humanity in mathematics classrooms. *Harvard Educational Review, 89*(1), 132–155.

Kuriloff, P., Andrus, S., & Jacobs, C. (2017). *Teaching girls: How teachers and parents can reach their brains and hearts.* Rowman & Littlefield.

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Education Research Journal, 32*, 465–491.

Ladson-Billings, G. (1998). Teaching in dangerous times: Culturally relevant approaches to teacher assessment. *Journal of Negro Education*, 255–267.

Lee, V. E., Smith, J. B., & Croninger, R. G. (1997). How high school organization influences the equitable distribution of learning in mathematics and science. *Sociology of Education,* 128–150.

Lesh, R. A., & Doerr, H. M. (2003). Foundations of a model and modeling perspective on mathematics teaching, learning, and problem solving. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective on teaching, learning, and problem solving in mathematics education.* Lawrence Erlbaum Associates.

Martin, D. (2007). *Mathematics learning and participation in African American context: The co-construction of identity in two intersecting realms of experience.* In N. S. Nasir & P. Cobb (Eds.), Diversity, equity, and access to mathematical ideas (pp. 146–158). Teachers College Press.

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influences, and individual agency.* Lawrence Erlbaum.

Mendick, H. (2005). A beautiful myth? The gendering of being/doing 'good at maths.' *Gender and education, 17*(2), 203–219.

Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Civil rights from Mississippi to the Algebra* Project. Beacon Press.

Nasir, N. S., & Saxe, G. B. (2003). Ethnic and academic identities: A cultural practice perspective on emerging tensions and their management in the lives of minority students. *Educational Researcher, 32*(5), 14–18.

National Research Council. (1998). *The nature and role of algebra in the K-14 curriculum.* National Academy Press.

Noguera, P. A. (2003). The trouble with Black boys: The role and influence of environmental and cultural factors on the academic performance of African American males. *Urban Education, 38*(4), 431–459.

Oakes, J. (2005). Keeping track. Yale University Press.

Pollak, H. (2012). What is mathematical modeling? In H. Gould, D. Murray, & A. Sanfratello (Eds.), *Mathematical modeling handbook* (pp. viii–xi). Consortium for Mathematics and Its Applications.

Powell, A. G., Cohen, D. K., & Farrar, E. (1985). *The shopping mall high school: Winners and losers in the educational marketplace.* National Association of Secondary School Principals (US): National Association of Independent Schools & Commission on Educational Issues.

Radovic, D., Black, L., Salas, C. E., & Williams, J. (2017). Being a girl mathematician: Diversity of positive mathematical identities in a secondary classroom. *Journal for Research of Mathematics Education, 48*(4), 434–464.

Rodd, M., & Bartholomew, H. (2006). Invisible and special: Young women's experiences as undergraduate mathematics students. *Gender and Education*, *18*(1), 35–50.

Rubel, L. H. (2017). Equity-Directed Instructional Practices: Beyond the Dominant Perspective. *Journal of Urban Mathematics Education, 10*(2), 66–105.

Shavelson, R. J., Phillips, D. C., Towne, L., & Feuer, M. J. (2003). On the science of education design studies. *Educational Researcher, 32*(1), 25–28.

Su, F. (2020). *Mathematics for Human Flourishing.* Yale University Press.

Tatum, B. D. (1997). Racial identity development and relational theory: The case of Black women in White communities. In J. V. Jordan (Ed.), *Women's growth in diversity: More writings from the Stone Center,* (pp. 91–106). Guilford Press.

Waller, W. (1932). *The sociology of teaching.* New York: John Wiley & Sons.

Williams, K., & Miner, B. (2012) Failing our kids: Why the testing craze won't fix our schools. In Au, W., & Tempel, M. B. (Eds.). *Pencils down: Rethinking high-stakes testing and accountability in public schools, 9–12.* Rethinking Schools.