

A Century of Leadership in Mathematics and Its Teaching

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# Jumping on the Shower Curtain: Using the Hundred Chart Kinesthetically to Embody Quantity Sense in Elementary Students 

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#### Abstract

In this paper I discuss the possibilities for exploring the hundred chart kinesthetically with elementary preservice teachers using a life-size hundred chart-an innovative tool, homemade from a shower curtain - to build quantity sense. Preservice teachers explored creative approaches to teach mathematics using the life-size hundred chart and planned engaging movement activities for young students. I detail the activity prompts for students, sample lesson ideas from preservice teachers, and reflections from those who implemented the tool in their instruction. I provide additional suggestions for teacher educators to use this tool in building quantity sense with their students.


KEYWORDS hundred chart, movement, preservice, elementary

## Introduction

Supporting children in building number sense, or the "general understanding of number and operations along with the ability and inclination to use this understanding" (McIntosh et al., 1992, p. 3), features prominently in elementary mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000; NSDEECD, 2019). As Jordan et al. (2007) synthesized in their survey of research on children's number sense development, "Most agree that number sense involves abilities related to counting, number patterns, magnitude comparisons, estimating, and number transformation" (p. 36). Some studies (Gerofsky, 2010; Wagner \& Davis, 2010; Zevenbergen, 2004) suggest benefits in opening this concept further to include an appreciation of number, to promote flexibility with number, and to provide opportunities for "feeling quantity" through performance and experiential activity. In this paper, I conceptualize number flexibility and this expressive nature as quantity sense.

Lakoff and Nuñez (2001) argued that mathematical thinking is embodied, meaning that for children, cognition is rooted in their bodily-kinesthetic interactions with
the physical world. Kinesthetic approaches to learning mathematics in elementary school range from students' finger counting (Soylu et al., 2018) and skip counting through rhythm and dance (Lindt \& Miller, 2017), to indicating similarity and balance through gestures (Thomas \& Dueber, 2020). Efforts to make mathematics learning tangible for children with an emphasis on hands-on activities using concrete materials show positive results for developing quantity sense (Swan \& Marshall, 2010). Current research also shows increased engagement and conceptual benefits for children who participate in fullbody movement and gestural mathematics in playful spaces of learning (Edwards et al., 2014). Tracing graphs in the air with arms and making geometric shapes or angles from students' bodies are good examples of this. Additionally, asking students to walk number pathways on a hundred chart or make jumps to determine a skip counting pattern may show potential for students to feel or embody their developing quantity sense. Learning mathematics playfully sustains engagement and opens students to additional benefits for problem solving including collaboration with peers, opportunities to correct mistakes efficiently, and space for creativity in mathematics to emerge (Featherstone, 2000).

In this paper I demonstrate possibilities for playful learning by embodying quantity sense through a tool that uses a household shower curtain as a hundred chart. The hundred chart is a $10 \times 10$ grid showing numbers from 1-100 (or 0-99) that students in Grade 1 often see for the first time as they transition from the focus on numbers $1-20$ in kindergarten. In Grades 2 and 3, students continue to develop understanding of numbers to 100 and 200 using a variety of counting strategies. For example, students count forward and backward from different starting points and skip count in different ways to demonstrate problem solving (NSDEECD, 2019). Conventionally, teachers use a paper hundred chart for students to show skip counting patterns, calculate two-digit addition and subtraction problems or play "four in a row"-style games. More recently, online versions of hundred charts provide interaction for students through a touch screen. However, a life-size hundred chart allowing students to develop quantity sense through movement may more fully support their mathematical thinking through embodied cognition arising from interactions with the physical world (Lakoff \& Nuñez, 2001).

## Participants and Context

I work with elementary school preservice teachers (PSTs) enrolled in a two-year post-degree program at a Faculty of Education located in a small university in rural Eastern Canada. The participants I describe in my classroom were Year 1 students completing their first curriculum and instruction course in mathematics education. This 3-credit course met twice weekly for nine weeks and introduced PSTs to the philosophy and strategies for teaching early elementary mathematics (kindergarten to Grade 2). The course focused on methods for engaging students in developing concrete and pictorial representations of preand early number concepts through hands-on activities. By instilling positive beliefs about learning and teaching mathematics, I also aimed to boost content knowledge and confidence levels for PSTs.

Before jumping on the shower curtain, PSTs explored pre-number concepts of sorting, seriating, and patterning. This fostered their understanding of one-to-one correspondence and counting. PSTs used counters, ten frames, Cuisenaire rods, and many partitioning activities including hands-on tasks like Bears in a Cave (using bear counters and an overturned bowl as a cave to play with part-whole concepts) and Making Trains (making a train by connecting rods end-to-end to visualize parts of the whole) to familiarize themselves with number families
(e.g., $1+9=10,9+1=10,10-9=1$, and $10-1=9$ ) and to consolidate the "make-ten" facts. The central focus was to demonstrate conceptual understanding of joining and separating structures by first manipulating concrete materials and then naming the concepts and articulating the procedures.

To continue this pedagogical focus and to extend the numbers to 100, I presented the PSTs with the hundred chart drawn on a shower curtain. I chose the shower curtain to enlarge the typical $10 \times 10$ grid printed on paper to "life-size" so that students could walk, skip, or jump on it. I explained how I made the hundred chart on a plastic shower curtain for minimal cost using a straight edge and a marker after carefully measuring the sides to ensure my lines would make one hundred squares. I also noted that once the template was made, subsequent charts could be drawn with relative ease. I asked the PSTs to observe the new tool and to tell me what they noticed and wondered (Figure 1).

Figure 1
PSTs Observe the Blank Shower Curtain Hundred Chart


Immediately, PSTs responded that there were no numbers on the shower curtain-only empty squares. They wondered how students would make sense of the open chart. This was the beginning of our exploration with the hundred chart and my opportunity to model the tool's versatility and the flexibility of the open design. In the next section, I describe how PSTs used the shower curtain to embody their emerging quantity sense and show their understanding of number to 100 .

## Jumping on the Shower Curtain

When they made their own shower curtain hundred chart, I asked PSTs to leave the cells blank to promote flexibility with the tool and strengthen students' quantity sense at the same time. I explained to the class that the option to place number cards or objects on the shower curtain as part of activities could highlight specific locations and numbers to support students' learning.

## Identifying the Corners

Through careful deliberation, PSTs suggested that given a blank chart, students could determine the numbers and therefore the corner numbers would become quite significant. One corner would have to be " 1 " and another, " 100 ." I then asked, "What numbers would we put in the other two corners?" Finding answers to this question became our first whole-class activity. Choosing four volunteers, I asked each PST to claim a corner. I watched as the four PSTs stepped up to the shower curtain but did not step on it. I encouraged them to jump right on and to walk across it, not around it, if necessary. My intention was for students to use the tool fully and to move their body on the grid just as they might move a counter on a paper chart. Once I saw PSTs standing on their chosen corners, I turned to one and asked, "What number are you?" The response was quick: "I'm number one!" Turning to the second PST, I asked about her number and, after some thought and support from peers, she said, "I must be ten!" I then asked how she knew she was ten and she pointed at the first square. With her finger, she then counted the spaces between them. I accepted her response but as a follow-up, I asked, "How can you prove you are number ten with your body?" Immediately the PST walked across the top row of the shower curtain and jumped from one space to another until she arrived back at her corner. The proof from her body movement and jumps was all we needed, and we were ready to find out about the other two corners. After much discussion, the class discovered that if someone skipped by tens down the "ten" column, they would arrive at the 100 square (Figure 2).

Figure 2
PSTs "Walk" the Shower Curtain


The PSTs also found that if someone "walked backwards" from 100 they would find 91 in the opposite corner. As PSTs identified each number for the corner squares on the grid, with bodily-kinesthetic proof, I asked others to show me different ways to find the numbers for the corner squares. Volunteers quickly jumped on the shower curtain and skipped, walked, and stretched their bodies to show a variety of number pathways including counting by tens starting on one, skip counting by fives, counting on, and counting back.

One PST commented, "This is like the hopscotch activity," referencing a previous class in which we explored a variety of ways for students to count to ten while they hopped on numbers. Once corner squares were identified, we continued to play with the corners by asking volunteers to switch places (and numbers) and new volunteers to take up positions on the corners. This orientation to the corners of the blank chart assured PSTs that students could identify any number they wanted to once the corners were determined.

A new conceptualization of the chart emerged when PSTs started to change the position of 1 and 10 . When one PST claimed the square on the lower left side of the shower curtain as 1 , many peers wondered if this was correct, or even possible. As PSTs explored further, they realized that not all hundred charts are created the same way, and some could start at the bottom and go up, start on the left and go right, or start on the right and go left. Additionally, PSTs wondered if we could incorporate zero onto the shower curtain and, if we did, how it would change the corners and the patterns found in the chart. Based on PSTs' explorations and questions, I decided to progress the lesson to guide the PSTs to find more meaningful ways to interact with the hundred chart.

## Playing on the Shower Curtain

I divided the PSTs into six small groups of six, each with their own shower curtain to play on. Previously we had reviewed the curriculum expectations for Grades 1 and 2 as a class, so I challenged the groups to brainstorm ideas for engaging, interactive tasks with young children that would meet these learning outcomes. All groups initially stood around the edges of the shower curtain, pointing to certain spots, gesturing to rows and columns, and discussing ideas for activities. I circulated among the groups, asking questions to prompt discussion, and offering additional materials that might be considered for activities. These included counters, small stuffed animals, small cards, frames to highlight squares, number cards, and large foam dice (Figure 3). Before long, PSTs started to walk on the shower curtain, many of them manipulating the objects I had distribut-

Figure 3
PSTs Designing Activities on the Shower Curtain

ed. The PSTs placed objects and cards on blank squares to identify the numbers and rolled the dice to generate random numbers to find on the grid. I observed several groups identifying the corners and other groups returning to the outcomes to work on adding and subtracting problems. All groups eventually used additional materials on the shower curtain after successfully creating new activities to share with others. I describe these activities in the following section.

## PST Suggestions for Shower Curtain Math

Table 1 lists the shower curtain activities created by PSTs to build students' quantity sense kinesthetically, the learning outcomes (NSDEECD, 2019) for students, directions, and the necessary materials:

In all cases I noticed participants' emphasis on embodying quantities for students-for example, walking the space between numbers, jumping from row to

Table 1
Shower Curtain Activities Designed by PSTs

| Learning Outcome | Activity | Directions | Materials |
| :---: | :---: | :---: | :---: |
| Grade 1 |  |  |  |
| Students will be expected to say the number sequence by <br> - 1s, forward and backward between any two given numbers, 0 to 100, <br> - 2 s to 20 , forward starting at 0 , <br> - 5 s to 100 , forward starting at 0, using a hundred chart or a number line, <br> - 10s to 100, forward, starting at 0 , using a hundred chart or a number line. | What's Your Number? | Throw an object (e.g., a counter or block) on the shower curtain. Where did it land? Tell the number. Walk the pathway from 1 to find the number of the square where the counter landed. Find a different pathway to your number. | - 100 chart shower curtain <br> - Small objects (e.g., counters, blocks, etc.) |
|  | Help Teddy Find Home | Toss the teddy bear on the shower curtain. Designate another square as "home" with a card and describe how Teddy will walk home. Find another possible route Teddy can take to get home. | - 100 chart shower curtain <br> - Teddy bear <br> - Home" card |
|  | Hop, Skip, Jump | Roll a die or two dice depending on the student's level of understanding. Starting at one, hop, skip, or jump according to the number rolled. Start at a different number other than one. Count backward from 100 or 50. | - 100 chart shower curtain <br> - Two dice |
| Grade 2 |  |  |  |
| Students will be expected to demonstrate an understanding of addition (limited to one- and two-digit numerals) with answers to 100 and the corresponding subtraction by <br> - Using personal strategies for adding and subtracting with and without the support of manipulatives <br> - Creating and solving problems that involve addition and subtraction <br> - Explaining and demonstrating that the order in which numbers are added does not affect the sum <br> - Explaining and demonstrating that the order in which numbers are subtracted matters when subtracting. | Shower <br> Curtain <br> Addition | Toss a counter on the shower curtain. Roll two dice. Add the number rolled to the counter and find the sum on the chart. Walk the pathway to find the sum. Walk a different pathway to find the same sum. Modify this to try subtraction sentences. | - 100 chart shower curtain <br> - Two dice |
|  | Arrow Arithmetic | Draw arrows on the blank cards. Down arrow means +10 , Up arrow means -10, right arrow means +1 , and left arrow means -1 . Shuffle the cards and lay four cards down in a row. Toss a counter on the shower curtain to find the starting number. Follow the arrows to determine the solution. Ensure students walk the solution pathway. | - 100 chart shower curtain <br> - Arrow cards |
|  | Close to One Hundred | Using a deck of playing cards, partners each turn over four cards to make two two-digit numbers. Each person walks the solution pathway, using addition or subtraction to get the closest to one hundred. | - 100 chart shower curtain <br> - Playing Cards |

row or column to column to represent skip counting, and representing the addend in an addition question by moving their body along a pathway to determine the sum. Participation in these activities involved seeing and doing mathematics concretely through movement. This allowed PSTs to experience and feel quantity by performing number operations with the body. PSTs visualized number relationships through the pathways they walked, at times following directions from peers who commented, for example, "Jump down a row, it means add ten." The activities addressed the development of quantity sense by promoting flexibility as PSTs found different pathways to solutions and represented quantities differently through movement (e.g., to represent 50 , start at the top of the " 10 " column and take 5 steps down, counting by 10 's, to land on 50 ; or, starting on the " 1 " square, take 10 jumps on the grid, counting by 5 's, to show the number sequence that begins: $5,10,15,20 \ldots$ etc.). PSTs also embodied partitioning to show numbers in parts to make them easier to work with, a key concept in developing quantity sense. For example, to show 64 as 6 tens and 4 ones, PSTs hopped by 10's to land on 60 and then jumped four more squares to land on 64. Moreover, they utilized estimation strategies as they made judgments based on visual cues and placement of materials (e.g., "I think it landed about halfway down the column," and "Those objects look farther apart, so about 40 or 50 in between"). Further, PSTs' activities showed options for students to feel the magnitude of numbers as they represented their estimates and operations with movement and gestures.

## Collaborating in Small Groups

Once each group had shared a lesson idea, I distributed a small booklet with contextual problems for PSTs to solve collaboratively. I wanted to focus attention on the
learning outcomes for Grade 2 with particular attention to solving a given story problem by modeling it with materials and by showing different ways to arrive at the same solution. I designed questions for adding and subtracting two-digit numbers to model for students solving problems with a variety of unknown numbers, either an initial amount, a change, or a result. Table 2 shows examples of these questions that asked students to show their thinking, model the problem concretely, and demonstrate possible solution pathways kinesthetically on the shower curtain.

I circulated among the groups observing the strategies that each group entertained to problem solve. All groups were completely engaged with their shower curtain as I observed PSTs walking solution pathways to arrive at the appropriate numbers (Figure 4).

I listened to conversations throughout the groups as PSTs discussed finding starting points, selecting materials to place on the shower curtain, and deciding where

Figure 4
PSTs Collaboratively Solve Problems on the Shower Curtain


Table 2
Adding and Subtracting Problems for Small Groups

| Learning Outcome | Question | Structure |
| :--- | :--- | :--- |
| You have 41 gummi bears. You eat 23 of them. | How many do you have left? | Result unknown |
| Molly is collecting baseball cards. She has collected 28, but <br> she wants to have a full deck and a full deck is 62 cards. | How many more cards would she <br> have to collect to have a full deck? | Change unknown |
| You keep some marbles in a bag. Kaveh, your friend, had <br> some doubles so he gave you 15 marbles. When you <br> combined all the marbles you found out that you now <br> have 86 marbles. | How many marbles were in your <br> bag before Kaveh gave you some <br> more? | Initial unknown |

they would go, as well as drawing their pathway in the booklet to show the answer pictorially (Figure 5).

In written reflections, many PSTs commented on the "fun" they had working together on problems and using the shower curtain to "prove" their solutions were correct. One student reflected on previous experiences learning mathematics and commented, "The activities were so different from my math class [in which] we were given worksheets to fill out and the teacher wanted us to work alone." Another PST commented on the shower curtain tool saying, "[The shower curtain] was so engaging. I just wanted to hop on it!" Reflections from students also showed how the 100 grid extended their mathematical thinking. One student wrote, "I could see numbers as tens and ones because we were going up and down the shower curtain counting by tens." One student reflected on the significance of the shower curtain activities for her future teaching by writing, "I learned about the importance of creating an engaging learning environment with the use of manipulatives and collaborative group work," while another student commented, "I learned to be more open-minded when it comes to math and math problems. I also learned that math doesn't have to be about worksheets or sitting at

Figure 5
PSTs Draw Solution Pathways in Problem Solving Booklets

your desk and doing work. There is so much more to it!" Hoping to build on the enthusiastic responses from the PSTs, I asked each group to brainstorm follow-up activities for students that could extend the use of the shower curtain. Below, I describe the possibilities that PSTs envisioned for their future classes.

## Future Possibilities

One group of PSTs extended the earlier idea to change the numbering on the shower curtain from 0-99, reorientating the numbers to start with 0 in the bottom right corner. This, they reasoned, would afford students more flexibility in visualizing numbers, and would also align with the understanding for students that "Numbers go up so we should go up the shower curtain as numbers get bigger." PSTs also considered doubling the shower curtain dimensions to represent numbers to 200 or renaming the chart from 101 - 200 to expand students' thinking with three-digit numbers. Another group proposed a Hidden Picture activity (Figure 6) in which each card of a set showed a computation question, a pictorial representation of number using base ten materials, or numbers written in words. Students would determine

Figure 6
PSTs Design 'Hidden Picture' Activities for Elementary Students

the number on the card, find the location on the hundred chart, and place the card on the appropriate square. Once cards were placed, students could turn each card over to reveal a picture or symbol created by the cards. If all cards were placed appropriately, the image would be clear. This activity would be self-correcting because misplaced cards would alter the final picture. Further, the PSTs considered extending the computation on the cards to include multiplication and division problems for the upper elementary students. In these activities, students would benefit from seeing multiple representations of numbers and from exploring computation concretely to promote flexibility with determining and visualizing number facts.

A third group decided to imagine the shower curtain as a tool for upper elementary students to explore decimals and decided that the entire chart would represent one unit, thereby showing students tenths (each column) or hundredths (each square). This reimagining highlighted the flexibility of the open 100 grid. Along with whole numbers, students could visualize rational numbers for comparing and ordering decimals as well as modeling equivalent decimals. The shower curtain would also serve as a concrete tool for solving problems involving operations with decimals from tenths to hundredths. In these ways, PSTs described experiential activities in which students could use their bodies to show rational number quantities. Such activities support quantity sense development generally as students engage expressively with a wide variety of number relationships found on the grid.

Some groups entertained learning outcomes outside the numbers and operations strands. Activity ideas included exploring measurement by using the squares on the shower curtain to show area and perimeter. One group decided to create giant polygons and irregular shapes cut from cardboard to place over the grid. Students would be encouraged to measure perimeter, first using non-standard units (e.g., lengths of string, dry spaghetti), then using standard units for length and area. Some tasks included extensions for students to find these measurements for compound shapes by deconstructing them in different ways. One group suggested that shower curtain geometry would benefit older students tasked with finding the area of triangles and trapezoids. One activity incorporated spatial reasoning and patterning as PSTs asked students to identify specific numbers with counters to reveal a 2D shape (e.g., place a counter on 25-27, 35-37, and 45-47 to create a square). Tasks involved identifying the shape and looking for patterns in the numbers that make up the
shape. PSTs observed, for example, that on one side of the square all the numbers ended in five, on the diagonal the tens and units digit increased or decreased by one, each column represented a pattern of skip counting by ten, and there was an odd-even-odd number pattern in each row. PSTs showed how spatial reasoning could integrate with data analysis when students created concrete graphs on the shower curtain by placing counters, shoes, or other objects in columns on the grid to represent the data using one-to-one correspondence. Students in upper elementary grades could also establish the first quadrant of the Cartesian plane by identifying the bottom of the shower curtain as the $x$-axis and the left side as the $y$-axis and create bar graphs representing many-to-one correspondence. With these ideas in mind, PSTs appeared confident to bring the hundred chart shower curtain into their practicum classroom to model interactive and engaging lessons. In contrast to more traditional classroom methods using handouts and seat work, the shower curtain afforded PSTs an opportunity to take advantage of the life-sized grid to support students' developing quantity sense through performance and experiential activity (Gerofsky, 2010; Wagner \& Davis, 2010).

## Conclusion

The introduction to shower curtain mathematics proved successful for PSTs in allowing them to engage fully in collaborative problem solving through embodied cognition. Although PSTs felt some trepidation at first to walk or jump on the hundred chart, they quickly became engaged and excited about using their bodies to show number pathways, operations, shape and more. Many PSTs reflected on the value of using such a tool to develop children's quantity sense by increasing their interaction, motivation, and enjoyment in mathematics and to appreciate quantities experientially by promoting number flexibility and kinesthetic activity during a lesson. By the end of our introductory class, it was a challenge getting PSTs to step off the shower curtain. This, for me, demonstrated the value in using a tool for kinesthetic learning in mathematics.

In my own practice teaching PSTs at the post-undergraduate level, I recognize value in modeling a new tool and then allowing students the room they need for exploring and imagining possibilities (Iversen et al., 2015). As I circulated the class, I observed how playful movement engaged all learners and stimulated conversation among group participants. Prompts and questions
for PSTs (e.g., What are you doing in your activity? What do you see children doing? How can you best support them on the shower curtain?) resulted in richer discussion and more meaningful exchanges (Throop Robinson, 2020) than in previous classes. This example of how to combine simple, everyday materials into an innovative teaching tool may be of interest to mathematics educators seeking to develop quantity sense in their preservice courses for several reasons. Firstly, using a life-size hundred chart appeared to engage students in kinesthetic mathematical activity involving full body movement. As Edwards et al. (2014) suggested, being fully engaged in a task, both mentally and physically, may also yield conceptual benefits for learners as they come to know and feel quantity with their bodies. Secondly, observations suggest that the tool prompted learners to demonstrate flexibility in representing quantity through their embodied cognition (Lakoff \& Nuñez, 2001). Finally, the experiential activity of jumping on the shower curtain was perceived to build confidence in mathematical thinking for PSTs who may, in the end, feel more prepared and empowered to teach mathematics to children with playful enjoyment.

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