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How Much Reform? An Analysis of a Chapter in a Reform-Based Calculus Textbook

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ABSTRACT Though the curriculum reform movement that began in 1989 has had a significant impact on calculus pedagogy and content (Windham, 2008), the question remains to what extent reform-based calculus textbooks reflect these reforms. This paper explores this question by analyzing the mathematical content and pedagogical approach of the chapter on functions in Hughes-Hallett et al.'s (2018) reform-based college textbook *Calculus: Single and Multivariable (CS&M)*. To conduct this analysis, the paper first synthesizes Hwang et al.'s (2021) framework for analyzing mathematics textbooks along with Sood and Jitendra's (2007) framework for comparing traditional and reform-based mathematics textbooks. The paper then applies the synthesized frameworks to identify elements of reform from the big ideas, contextual features, mediated scaffolding, problem solving activities, and STEM integration found in the *CS&M* chapter. The results suggest that, although the chapter integrates many reform-based principles, there may be scope for further integration. Such efforts, however, may be constrained by influential stakeholders.

KEYWORDS *calculus, reform, textbook, curriculum, mathematics pedagogy*

Introduction

Considering that mathematics textbooks exert significant influence on pedagogy as well as the topics teachers present in class (Chang et al., 2016; Johansson, 2005), they may also help teachers implement educational goals promoted within a curriculum (Hwang et al., 2021). In recent decades, debate has centered on whether to adopt traditional mathematics curriculum and textbooks or reformed curriculum and textbooks grounded in *Principles and Standards for Mathematics Education* (NCTM, 2000). Traditional curriculum and textbooks emphasize systematic explanation of algorithms, practice of problems to demonstrate concepts, teacher-centered instruction, memorization, and procedural knowledge (Schoenfeld, 2002; Sood & Jitendra, 2007; Waite, 2000). In contrast, reform-based curriculum and textbooks emphasize conceptual knowledge and critical thinking, engage students in real-world problem solving, focus on explanation, and encourage active learning (Sood & Jitendra, 2007; Waite, 2000). Reform-based approaches may also encourage Science, Technology, Engineering, and Mathematics (STEM) integration further to promote real-world problem solving (Bybee, 2013).

With regard to calculus, reform arose out of concern that students had a weak understanding of the subject and lacked interest in pursuing higher mathematics (Todd, 2012). Characteristics of traditional calculus include a highly rigorous and rigid curriculum, a pen-and-paper approach to problem solving, a heavy emphasis on theorems and proofs, memorization, and a format in which the teacher is the primary source of knowledge (Garner & Garner, 2001; Windham, 2008). Situated as the primary source of knowledge, the teacher explains concepts and provides examples during lectures, while students take notes, ask clarifying questions, and study from textbooks in a mostly individual experience (Garner & Garner, 2001; Windham, 2008). Defenders of reformed calculus contend that finding correct answers to procedural questions has little value if one cannot explain why algorithms work and therefore cannot develop conceptual understanding. Accordingly, reformed calculus courses often place less emphasis on doing proofs and more on understanding what proofs mean, as well as on applications of calculus (Garner & Garner, 2001; Windham, 2008). These courses often employ group work to help students construct their own meaning of concepts and performance-based assessments that push students to reason and justify their ideas (Garner & Garner, 2001).

When decisions about adoption of reform-based textbooks are made with the idea of integrating reform-based pedagogical principles in calculus courses, it is useful to know to what extent these textbooks actually reflect such principles. This paper begins exploring this issue by analyzing the mathematical content and pedagogical approach of Chapter 1, "Foundation for Calculus: Functions and Limits," in Hughes-Hallett et al.'s (2018) reform-based college textbook *Calculus: Single and Multivariable (CS&M)*. This textbook was chosen because the lead author is a primary proponent of reformed calculus (e.g., Hughes-Hallett, 2006), and their textbook would likely reflect reform-based principles. Additionally, this textbook, in its various editions, has been one of the most popular reform-based calculus textbooks (Bressoud, 2011; Windham, 2008). Chapter 1 was chosen because functions display the relationship between variables and are essential for learning algebra as well as calculus (Chang et al., 2016). The analysis of the mathematical content of the chapter focuses on big ideas, context, and STEM integration. The analysis of the pedagogical approach of the chapter focuses on mediated scaffolding and problem solving activities. The rationale for these choices will be provided later in this paper. Ultimately, this paper seeks to answer the question: To what extent does Chapter 1 of *CS&M* reflect the principles of a reform-based curriculum?

Literature Review

Although many forces, including standards and assessments, shape the curricula that classes adopt, textbooks play a central role (Stein et al., 2007). This is partly because they are concrete objects rather than abstract ideals that teachers and students can use in the classroom (Ball & Cohen, 1996). Given the influence mathematics textbooks have on curriculum and teaching (Chang et al., 2016; Johansson, 2005), a reform-based calculus textbook may be able to promote reform-based teaching and learning practices. To demonstrate further the influence of textbooks on the choices both teachers and students make, the following sections outline the relationships between mathematics textbooks and teachers, as well as between mathematics textbooks and students.

Relationships Between Mathematics Textbooks and Teachers

According to Ball and Cohen (1996), there is a close relationship between mathematics textbooks and teachers' practice. While teachers rely on their professional experiences and beliefs to interpret the content within a textbook (Liakos et al., 2021), they still look to textbooks as a "dialogic partner" (Dietiker et al., 2018, p. 522) that contains vital material to be used in the classroom. Liakos et al. (2021) conducted a qualitative study of the curriculum planning activities of a seasoned teacher who switched from teaching calculus with a traditional approach to teaching reform-based calculus. Observing the teacher using the textbook extensively in his planning and adding his own related materials, they concluded that the textbook significantly influenced his teaching. In addition, research suggests that in more recent years, despite the increased use of digital resources, textbooks continue to play a vital role in influencing how and what teachers teach (Glasnović Gracin & Jukić Matić, 2021; Polikoff, 2015).

Research suggests that, during periods of reform, mathematics textbooks are especially influential among teachers (Glasnović Gracin & Jukić Matić, 2021; Howson, 2013). This may be due to the fact that mathematics textbooks often serve as an authority not only on reformed mathematical content, but also on new curriculum (Glasnović Gracin & Jukić Matić, 2021; Valverde et al., 2002). Moreover, often new textbooks are published during times of reform as a cost-effective way to implement new curriculum (Polikoff, 2015), as teachers often seek help in the implementation of a new curriculum in textbooks and corresponding teacher guides (Howson, 2013). New textbooks under a reformed curriculum may also encourage teachers to undergo professional development so that they can use the textbooks effectively (Glasnović Gracin & Jukić Matić, 2021). Thus, while textbooks may not directly determine teachers' pedagogical approaches or strategies, they may influence the mathematical content taught in the course.

Relationships Between Mathematics Textbooks and Students

Research into traditional and reformed calculus is motivated not just by the relationship between calculus textbooks and teachers, but also by the relationship between calculus textbooks and students (Bressoud et al., 2016). Research reveals that the interactions among mathematics textbooks, teachers, and students are complex (Sevimli, 2016). Additional research suggests that most of the mathematical tasks students undertake and most of what they learn are influenced by the content of the textbook (Begle, 1973; Thompson et al., 2012). This may be because textbooks are written for students with tasks specifically addressed to them (Glasnović Gracin & Jukić Matić, 2021; Valverde et al., 2002). Unfortunately, scant research explores mathematics textbooks from the perspective of students. Nonetheless, it is evident that textbooks may influence students as significantly as they do teachers.

Research on Reformed Calculus Textbooks

Although reform in calculus has garnered much attention from researchers, particularly in relation to pedagogy and curriculum (Bressoud et al., 2016; Dunnigan & Halcrow, 2020; Garner & Garner, 2001; Keynes & Olson, 2000), most of these researchers provide little analysis of textbooks. For example, Dunnigan and Halcrow (2020) describe a restructuring of the course Applied Calculus at their university, which focused on increasing students' conceptual understanding and eliminating large lectures, but failed to address the role of textbooks. Similarly, Keynes and Olson (2000) describe changes in the content and pedagogy of the calculus sequence at the University of Minnesota, devoting their attention primarily to such innovations as the development of group work among students and the use of new technologies, but saying little about the use of textbooks.

What little research explores the impact that reform-based calculus textbooks have on teaching and learning focuses on a few specific issues. For instance, Chang et al. (2016) investigate uses of coordinated multiple representations in calculus textbooks for pedagogical and scaffolding purposes. Özgeldi and Aydin (2021) explore the levels of competency demand used by three calculus textbooks, including traditional and reformed. Neither Chang et al. (2016) nor Özgeldi and Aydin (2021) address other issues, such as STEM integration in calculus textbooks or the use of understanding, estimating, exploring, resolving, and explaining in the solving of mathematical problems. To address this gap in the literature, this paper provides a more comprehensive evaluation of a chapter in a reform-based calculus textbook that focuses not only on content, but also on pedagogical characteristics reflecting the principles of the reformed calculus movement.

Specifically, this paper seeks to answer the questions:

1. To what extent do the big ideas in the *CS&M* chapter on functions reflect the principles of a reform-based mathematics curriculum?
2. To what extent does the context in the *CS&M* chapter on functions reflect the principles of a reform-based mathematics curriculum?
3. To what extent do the examples and problems in the *CS&M* chapter on functions integrate the STEM disciplines?
4. To what extent do the examples and problems in the *CS&M* chapter on functions use multiple representations?
5. To what extent do the examples and problems in the *CS&M* chapter on functions require different problem solving activities, such as understanding, estimating, exploring, resolving, and explaining?

Methodology

Materials

CS&M is a product of the reform movement that has its roots in the 2000 publication of the NCTM's *Principles and Standards for Mathematics Education* (Windham, 2008). The data source of this study consists of materials from the chapter on functions, including definitions of concepts, illustrations of these concepts, practice exercises, and end-of-chapter exercises.

Analytical Framework

Two frameworks are synthesized to evaluate the content and pedagogical approach of the *CS&M* chapter on functions, henceforth referred to as "Chapter 1." These two frameworks are chosen because they allow for an analysis of the presence or absence of a wide array of features that are central to the 1989 reform movement. This synthesis of the two frameworks allows not only an analysis of the content of Chapter 1 for evidence of reform-based principles, but also expounds on how that chapter integrates reform-based pedagogical approaches. The first framework, developed by Hwang et al. (2021), is a commonly used framework to analyze mathematics textbooks by distinguishing two dimensions. The first is a horizontal dimension, which includes topic sequence and frequency. The second is a vertical dimension, which includes contextual features, cognitive demands, and problem solving activities. The second framework, developed by Sood and Jitendra (2007), is used to compare number sense between traditional and reform-based mathematics textbooks. It is rooted in the principles of effective instruction for students at risk in mathematics and includes criteria such as the teaching of big ideas, conspicuous instruction, mediated scaffolding, and judicious review (Sood & Jitendra, 2007).

The present study focuses on big ideas (Sood & Jitendra, 2007) and context (Hwang et al., 2021) to evaluate the mathematical content of Chapter 1. Unlike Sood and Jitendra (2007), who, when analyzing teachers' manuals and other instructional materials, assume a teacher's perspective, the present study analyzes the student edition and thus assumes a student's perspective. STEM integration, a feature of interest in the 1989 mathematics reform movement (Maass et al., 2019; Williams, 2011), is additionally included to analyze the content of Chapter 1 for its use of examples and problems relating mathematics with science, technology, or engineering. The use of mediated scaffolding (Sood & Jitendra, 2007) and problem solving activities (Hwang et al., 2021) are used to analyze the pedagogical approach within Chapter 1. The following is a summary of the analytical framework the present study adopts.

1. Mathematical content
   A. Big ideas: Collections of related concepts that help students acquire a broad set of skills and knowledge (Sood & Jitendra, 2007)
   B. Context: How a textbook illustrates math problems (Hwang et al., 2021)
   C. Integration with STEM: The use of examples and problems relating mathematics with science, technology, or engineering
2. Pedagogical approaches
   A. Mediated scaffolding: Support provided to students through teachers (e.g., instructional feedback), materials (e.g., visual prompts and representations), or tasks (e.g., the systematic introduction of more difficult materials) (Sood & Jitendra, 2007)
   B. Problem solving activities: The use of understanding, estimating, exploring, resolving, and explaining in the solving of mathematical problems (Hwang et al., 2021)

Mathematical Content: Big Ideas, Context, and STEM Integration

Big ideas are what the authors of a textbook consider important (Sood & Jitendra, 2007). They are discernible through chapter headings, the amount of space in the chapter devoted to them, and the number of problems and examples that illustrate them (Sood & Jitendra, 2007). Figure 1 provides an example of a big idea that illustrates the use of different representations to understand, interpret, and analyze functions.

Figure 1

Example of a Big Idea in Chapter 1

Big Idea: Understanding, interpreting, and analyzing functions using different representations

1.1 FUNCTIONS AND CHANGE

In mathematics, a *function* is used to represent the dependence of one quantity upon another.

Let's look at an example. In 2015, Boston, Massachusetts, had the highest annual snowfall, 110.6 inches, since recording started in 1872. Table 1.1 shows one 14-day period in which the city broke another record with a total of 64.4 inches.

Table 1.1

Daily snowfall in inches for Boston, January 27 to February 9, 2015

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Snowfall	22.1	0.2	0	0.7	1.3	0	16.2	0	0	0.8	0	0.9	7.4	14.8

You may not have thought of something so unpredictable as daily snowfall as being a function, but it is a function of day, because each day gives rise to one snowfall total. There is no formula for the daily snowfall (otherwise we would not need a weather bureau), but nevertheless the daily snowfall in Boston does satisfy the definition of a function: Each day, t, has a unique snowfall, S, associated with it.

We define a function as follows:

> A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

Example 1

The function $C = f(T)$ gives chirp rate as a function of temperature. We restrict this function to temperatures for which the predicted chirp rate is positive, and up to the highest temperature ever recorded at a weather station, 134°F. What is the domain of this function f?

Solution

If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables C and T, any T value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then C cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40°F$, we see that T cannot be less than 40°F. (See Figure 1.2.) In addition, we are told that the function is not defined for temperatures above 134°F. Thus, for the function $C = f(T)$ we have

$$\begin{aligned} \text{Domain} &= \text{All } T \text{ values between } 40°\text{F and } 134°\text{F} \\ &= \text{All } T \text{ values with } 40 \le T \le 134 \\ &= [40,134] \end{aligned}$$

Example 2

Find the range of the function f, given the domain from Example 1. In other words, find all possible values of the chirp rate, C, in the equation $C = f(T)$.

Note. Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, pp. 2-3. Copyright 2018 by John Wiley & Sons, Inc.

Context refers to how a textbook illustrates math problems (Hwang et al., 2021). *CS&M* explicitly states that it uses the "Rule of Four," illustrating concepts geometrically (G), numerically (N), algebraically (A), and verbally (N) (Hughes-Hallett et al., 2018). See Table 1 for examples.

Figure 2 provides an example of a STEM integration problem that connects the use of functions with the discipline of physics.

Table 1

Examples of Context in Chapter 1

<table>
<tr><th>Context</th><th>Example</th></tr>
<tr><td>Algebraic (A)</td><td>Find the domain and range in Exercises 24–25.
24. $y = x^2 + 2$

Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
<tr><td>Verbal (V)</td><td>Problems 39–42 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."

39. Driving speed against time on the highway

Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 9. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
<tr><td>Geometric (G)</td><td>For Exercises 20–23, give the approximate domain and range of each function. Assume the entire graph is shown.

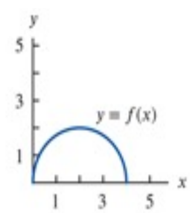
Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.

</td></tr>
<tr><td>Numeric (N)</td><td>16. Find a linear function that generates the values in Table 1.3.

Table 1.3
<table>
<tr><td>x</td><td>5.2</td><td>5.3</td><td>5.4</td><td>5.5</td><td>5.6</td></tr>
<tr><td>y</td><td>27.8</td><td>29.2</td><td>30.6</td><td>32.0</td><td>33.4</td></tr>
</table>
Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
</table>

Figure 2

Example of STEM Integration in Chapter 1

75. When Galileo was formulating the laws of motion, he considered the motion of a body starting from rest and falling under gravity. He originally thought that the velocity of such a falling body was proportional to the distance it had fallen.

What do the experimental data in Table 1.7 tell you about Galileo's hypothesis? What alternative hypothesis is suggested by the two sets of data in Table 1.7 and Table 1.8?

Table 1.7

Distance (ft)	0	1	2	3	4
Velocity (ft/sec)	0	8	11.3	13.9	16

Table 1.8

Time (sec)	0	1	2	3	4
Velocity (ft/sec)	0	32	64	96	128

Note. Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc.

Pedagogical Approach: Mediated Scaffolding and Problem Solving Activities

Mediated scaffolding provides students with support through teachers, materials, or tasks (Sood & Jitendra, 2007). Chapter 1 contains materials, such as multiple representations and it contains tasks, such as the systematic introduction of more difficult problems. Providing multiple representations of mathematical concepts counts as a form of scaffolding (Ngin, 2018), as it can support students' understanding of new or difficult concepts (Sood & Jitendra, 2007), particularly when students are directed to reason how different representations relate to each other (Chang et al., 2016). See Figure 3 for an example of a problem that uses mediated scaffolding by employing both algebraic and geometric representations.

Hwang et al. (2021) adapted Pólya's (1945) model for problem solving, maintaining that problem solving activities include understanding, estimating, exploring, resolving, and explaining. Understanding involves making sense of a problem, estimating involves approximating an answer or problem solving strategy, exploring involves investigating an answer, resolving involves finding an answer, and explaining involves providing the rationale behind an answer or problem solving strategy (Hwang et al., 2021). The present study adopts Hwang et al.'s (2021) classification to reveal the types of problem solving activities used in Chapter 1. See Table 2 for examples.

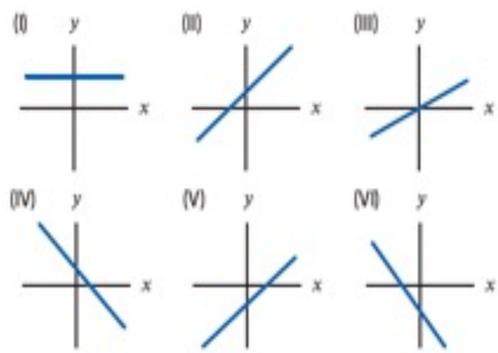
Figure 3

Example of Mediated Scaffolding Using Multiple Representations in Chapter 1

Match the graphs in Figure 1.9 with the following equations. (Note that x and y scales may be unequal.)

a. $y = x - 5$
b. $-3x + 4 = y$
c. $5 = y$
d. $y = -4x - 5$
e. $y = x + 6$
f. $y = x/2$

Figure 1.9



Note. Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc.

Table 2

Examples of Problem Solving Activities in Chapter 1

Problem Solving Activity	Example
Understanding	None in Chapter 1
Estimating	44. A certain region has a population of 10,000,000 and an annual growth rate of 2%. Estimate the doubling time by guessing and checking. **Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 23. Copyright 2018 by John Wiley & Sons, Inc.
Exploring	Investigate $\lim_{x\to\infty} \frac{1}{x}$ and $\lim_{x\to-\infty} \frac{1}{x}$ **Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 73. Copyright 2018 by John Wiley & Sons, Inc.
Resolving	The functions in Exercises 5–8 represent exponential growth or decay. What is the initial quantity? What is the growth rate? State if the growth rate is continuous. 5. $P = 5(1.07)^t$ **Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 20. Copyright 2018 by John Wiley & Sons, Inc.
Explaining	In Problems 83–88, is the statement true or false? Give an explanation for your answer. 83. For any two points in the plane, there is a linear function whose graph passes through them. **Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc.

Procedure

Using the above analytical framework, the author and a colleague independently examined Chapter 1 with the following aims:

1. to identify the big ideas;
2. to count the number of examples and problems illustrated algebraically, the number illustrated verbally, the number illustrated geometrically, and the number illustrated numerically;
3. to count the number of examples and problems that relate functions to either science, technology, or engineering;
4. to count the number of examples and problems that use multiple representations, noting which multiple representations are used;
5. to count the number of examples and problems that focus on understanding, the number that focus on estimating, the number that focus on exploring, the number that focus on resolving, and the number that focus on explaining.

After completing their independent examinations, the author and colleague compared their preliminary results, checking for any instances of disagreement. Two such instances were found. The author and colleague then consulted with experts, and, after discussion, reached unanimous agreement.

Results

Mathematical Content: Big Ideas

In Chapter 1, the big ideas include: (1) understanding, interpreting, and analyzing functions using different representations, (2) building functions, (3) constructing and comparing linear, logarithmic, trigonometric, power, polynomial, and rational functions, and (4) understanding limits and continuity using different representations (Hughes-Hallett et al., 2018). Chapter 1 includes no proofs, and it often introduces concepts with applications, suggesting that applications are the reason for, as well as the result of, doing calculus (Garner & Garner, 2001).

Mathematical Content: Context

As Table 3 indicates, the problems in Chapter 1 are predominantly illustrated algebraically (A), followed by verbally (V), geometrically (G), and numerically (N).

Table 3
Results for Context

Representation Type	Percent of the 854 Total Examples/Problems
Algebraically (A)	71.2%
Verbally (V)	24.2%
Geometrically (G)	17.1%
Numerically (N)	4.6%

These results are noteworthy considering that traditional calculus textbooks use mostly algebraic representations (Todd, 2012), while reform-based textbooks, with their emphasis on multiple representations, would be expected to have a more even distribution of representations. Todd (2012) similarly found that the Hughes-Hallett (2009) text they analyzed, included few problems with numerical representations and, therefore, an uneven distribution of representations. The heavy emphasis on algebraic representations in Chapter 1 may result from the fact that asking and answering questions by mathematical means is among the chief purposes of mathematical activity (Niss & Højgaard, 2019). According to Niss and Højgaard (2019), such questions are about mathematical thinking, mathematical problem solving, mathematical modeling, or mathematical reasoning. Moreover, mathematical activity, by its nature, involves the ability to handle algebraic representations, which are connected with "mathematical language and tools" (Özgeldi & Aydin, 2021, p. 186). Thus, the widespread use of algebraic representations in Chapter 1 may be the authors' attempt to develop students' competency in mathematical language and tools.

Mathematical Content: STEM Integration

Although Chapter 1 contains some problems and examples relating to science, technology, and engineering, these represent only a small percentage of all the problems and examples (see Table 4). These results run contrary to reform-based principles, which emphasize STEM integration.

Table 4
Results for STEM Integration

Discipline	Percent of the 854 Total Examples/Problems
Science	6.0%
Technology	1.0%
Engineering	0.2%

Pedagogical Approach: Mediated Scaffolding

Some problems in Chapter 1 contain more than one of the four types of representations: geometric (G), numeric (N), algebraic (A), and verbal (V). Considering that the use of multiple representations can be a form of mediated scaffolding (Ngin, 2018), problems were evaluated for the type and number of distinct representations used. Therefore, if a problem used both a geometric and algebraic representation, it was coded as GA. As Table 5 shows, of problems containing multiple representations,

Table 5
Results for Mediated Scaffolding

Representation Types	Percent of the 854 Total Examples/ Problems
Geometric and Algebraic (GA)	8.0%
Algebraic and Verbal (AV)	2.1%
Geometric and Verbal (GV)	1.4%
Numeric and Verbal (NV)	1.4%
Numeric and Algebraic (NA)	0.4%
Geometric and Numeric (GN)	0.2%
Geometric, Algebraic, and Verbal (GAV)	0.2%
Geometric, Numeric, and Algebraic (GNA)	0.1%

the most frequent contain geometric and algebraic (GA) representations, followed by algebraic and verbal (AV) representations.

One possible explanation for these results is that Chapter 1 may be attempting to target areas that students find more difficult when studying calculus. For example, research suggests that students often have difficulty with coordinating multiple representations, particularly those that include graphical representations (Chang et al., 2016). Perhaps the prevalence of GA representations is intended to help students overcome this difficulty. The prevalence of AV representations may indicate the authors' attempt to help students review previously learned concepts and introduce more difficult or unfamiliar concepts (Chang et al., 2016). The prevalence of both GA and AV representations may also indicate coordination of the process and object perspectives of functions. A process perspective focuses on a function's procedural characteristics, with each x value linked to a y value, whereas an object perspective views functions as entities (Chang et al., 2016; Moschkovich et al., 1993). Therefore, students may tend to think of algebraic representations from a process perspective, but verbal or geometric representations from the object perspective (Chang et al., 2016; Moschkovich et al., 1993).

Pedagogical Approach: Problem Solving Activities

Results reveal that most problems in Chapter 1 focus on resolving, followed by explaining, estimating, and exploring, and no problems focus on understanding (see Table 6).

Table 6
Results for Problem Solving Activities

Representation Type	Percent of the 854 Total Examples/Problems
Resolving	82.1%
Explaining	15.3%
Estimating	2.2%
Exploring	0.4%
Understanding	0.0%

These results imply a structure in the lessons that is geared toward problem solving activities, which is consistent with the reformed curriculum approach (Sood & Jitendra, 2007). Moreover, the frequency of explaining in Chapter 1 is aligned with a reform-based pedagogical approach to teaching. Explaining is often associated with group activities and thus represents a more student-centered approach to teaching calculus (Sood & Jitendra, 2007). The absence of problems on understanding stands contrary to the goal of reformed-based mathematics textbooks to help students understand concepts (Sood & Jitendra; Waite, 2000). The unequal distribution of problem solving activities also stands contrary to the goal of reform-based mathematics textbooks to provide students with the opportunity to engage in multiple problem solving activities (Sood & Jitendra, 2007; Waite, 2000).

Discussion

The above analysis reveals that significant steps seem to have been taken to adapt Chapter 1 to the principles of reformed calculus. This is evident, for example, from Chapter 1's emphasis on problem solving skills and multiple representations to help students visualize concepts. At the same time, however, Chapter 1 exhibits some similarities with traditional calculus. These similarities include a heavy emphasis on symbolic and algebraic representations, and less emphasis on STEM integration. The similarities between the traditional and reformed calculus approaches in Chapter 1 may stem from the fact that the subject of functions requires students to acquire competency in mathematical language that serves as a foundation for future topics (Niss & Højgaard, 2019). On the other hand, a variety of representations to illustrate function concepts may appeal to students who prefer more numerical or geometric representations. As Bressoud et al. (2016) acknowledge, calculus reform has led to "the recognition in almost all textbooks and most universities of the importance of graphical and numerical in addition to algebraic representations of derivatives and integrals" (pp. 17–18).

The publishers of *CS&M* may have had reasons not to incorporate more reform-based principles in Chapter 1.

For one thing, textbooks are commercial and political enterprises with various stakeholders, including government officials, influencing the selection of their content (Polikoff, 2018; Shapiro, 2012). For example, to reach a broader audience and larger market, textbooks must deliver the curriculum content that adoption states specify (Batista Oliveira, 1995). Thus, state regulations may influence design criteria, topics, objectives, and other important components of calculus textbooks (Batista Oliveira, 1995). Beyond the adoption criteria of states, publishers must also appeal to teachers (Batista Oliveira, 1995). Some of those teachers may be pedagogically conservative, and prefer textbooks with a more traditional approach (Batista Oliveira, 1995). The persistence of traditional calculus methods in a reform-based textbook may be an indication of the influence of multiple stakeholders who prefer traditional methods of calculus instruction.

Three limitations of the present study are worth noting. First, the present study analyzes just one chapter of one reform-based textbook. One might question how representative this chapter and this textbook are. Thus, an investigation of other chapters and other reform-based textbooks is recommended. Second, while the analytical framework the present study adopts is useful for analyzing functions, it may require expansion when analyzing derivatives, integrals, or other complex concepts in single variable calculus. Third, the present study focuses only on a reform-based calculus textbook and does not compare and contrast it with a traditional calculus textbook. Although Todd (2012) makes such a comparison between one reform-based and one traditional calculus textbook, a comparison and contrast of a larger sample of reform-based and traditional single variable calculus textbooks might shed further light on how much reform reform-based textbooks actually incorporate.

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