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NOTES FROM THE FIELD

Playing with Push Toys and Technology: Solving a System of Linear Equations

Kelly W. Remijan
Illinois Mathematics and Science Academy
Center for Teaching and Learning

Mathematical Action Technologies

When students are given the opportunity to utilize technology and engage in hands-on activities within a mathematics class, they can experience mathematics in action. Mathematical action technologies "offer students opportunities to interact…in ways that are not possible (alone) with paper and pencil" (McCullough et. al, 2021, p. 739). One such mathematical action technology, for instance, involves a calculator-based ranger (CBR) which collects and displays motion data in real-time. As CBR activities impact students' abilities to interpret and model "physical phenomena" which enhances graphical understanding (Kwon, 2010), I have incorporated various CBR activities that involve students walking in front of a motion detector to create or replicate a particular given graph (Remijan, 2019). After engaging students with CBR activities within my classroom, as well as outdoor activities involving crash reconstruction and the Illinois State Police (Remijan, 2017), I developed an additional activity to model a "crash" within the classroom using push toys and a CBR.

As such, the following is an example of a technology-based activity involving push toys which I have personally conducted with eighth grade students in Honors Algebra and ninth grade students in Algebra 1A, a first-year course of a two-year Algebra I sequence. I have also performed this activity with students as a mathematics teacher in O'Fallon, Illinois and as a Curriculum Specialist with the Illinois Mathematics and Science Academy conducting statewide outreach. The lesson activity presented seeks to reinforce students' understanding of linear equations, graphing linear equations, and solving a system of equations by graphing.

Figure 1
Push Toys

To begin, students were told that two push toys, Turtle and Mickey (Figure 1), would be modeling a crash where they would travel in front of a motion detector before crashing into each other.

Additionally, I explained and showed students that the CBR was connected to a graphing calculator and would collect and graph distance and time data automatically. Lastly, students were informed that Turtle can travel 5 feet in 3 seconds while Mickey can travel 7 feet in 3 seconds.

Students were then given the following scenario:

Scenario 1

Turtle is placed 2 feet from the motion detector and travels away from the motion detector.

a) Write an equation to model the path of Turtle in terms of distance versus time.

   Expected Answer: $y = \frac{5}{3}x + 2$ where the slope is positive since Turtle is moving away from the motion detector and increasing its distance from the motion detector.

b) Graph the line representing the path that Turtle takes in terms of distance versus time.

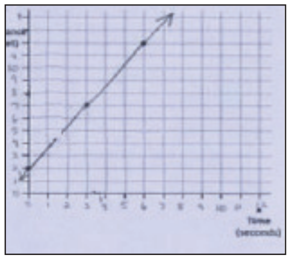
   Expected Answer: A linear graph found in Figure 2.

c) Model the situation involving Turtle using a CBR and a graphing calculator.

   Expected Result: A video of my former 9th grade Algebra 1A students modeling this situation can be found at https://youtu.be/sTMtWfvhlbc.

Figure 2

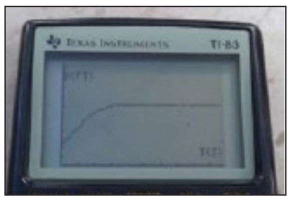
Graph of the Line Representing Turtle's Movement



After conducting the hands-on experiment involving Turtle, and reviewing the result acquired from the graphing calculator (see Figure 3), I asked my students "Why is the calculator showing something different than what you graphed?"

Figure 3

CBR Model of Turtle's Path on Graphing Calculator



With various students sharing their thoughts, students eventually came to a consensus that the graph on the calculator shows that Turtle eventually stopped while the original, hand-drawn graph suggests that Turtle never stopped. Students were guided to recognize that the oblique line on the calculator respresents Turtle moving away from the motion dector at the constant speed of 5 feet per 3 seconds, and that the horizontal line represents Turtle stopping. Additionally, the students determined that the slope of Turtle's path represented Turtle's speed. Thus, the slope of the horizontal line formed by Turtle's lack of movement is consistent with Turtler's speed being that of zero. This activity reinforces the concepts of speed comparing change in distance to change in time, as well as what it means for a linear function to be increasing or constant. Furthermore, the activity may help students build connections between objects in motion in relation to time, as well as the graphical representation and analysis of such a situation.

Next, students were given a second scenario:

Scenario 2

Mickey is placed 8 feet from the motion detector and travels towards the motion detector.

a) Write an equation to model the path of Mickey in terms of distance versus time.

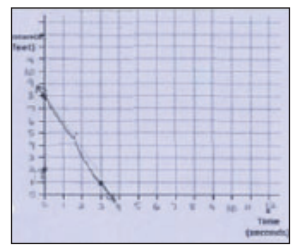
   Expected Answer: $y = \frac{-7}{3}x + 8$ where the slope is negative since Mickey is moving towards the motion detector and decreasing its distance from the motion detector.

b) Graph the line representing the path that Mickey takes in terms of distance versus time.

   Expected Answer: A linear graph found in Figure 4.

Figure 4

Graph of the Line Representing Mickey's Movement



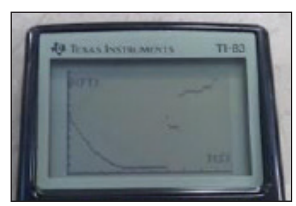
c) Model the situation involving Mickey using the CBR and a graphing calculator.

   Expected Result: A video of my former 9th grade Algebra 1A students modeling this situation can be found at https://youtu.be/ZHNUxfq-7RQ.

After conducting the hands-on experiment involving Mickey, and reviewing the result acquired from the graphing calculator (see Figure 5), I asked my students again "Why is the calculator showing something different than what you graphed?"

Figure 5

CBR Model of Mickey's Path on Graphing Calculator



After a brief time of reflection, students explained that Mickey stopped right before the motion detector, making his speed zero, and showing a constant function. Similarly to Scenario 1, by guiding students to make these connections between the experiment and the algebraic and graphical representations, this activity may help to reinforce the concepts of speed, a decreasing function, and a constant function.

Lastly, students were presented with a third scenario:

Scenario 3

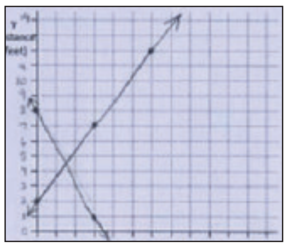
If both Turtle and Mickey leave at the same time, when and where will they "crash?"

a) Graph the two lines representing the paths of both Turtle and Mickey on the same coordinate plane.

   Expected Answer: Two intersecting linear graphs as shown in Figure 6.

Figure 6

Graph of the Two Lines Representing Turtle and Mickey's Movements



b) Model the situation involving Mickey and Turtle using the CBR and a graphing calculator.

   Expected Result: A video of my former 9th grade Algebra 1A students modeling this situation with the CBR can be seen here at https://youtu.be/QMFeBmIgm2g.

Figure 7

Intersection of Turtle and Mickey's Paths

After analyzing the graph, students determined that Turtle and Mickey will "crash" 1.5 seconds after they both leave and at a point which is 4.5 feet away from the motion detector. Additionally, after conducting the hands-on experiment, students analyzed the data captured from the motion detector (see Figure 7).

With the calculator's graph again displaying a different result than the hand-drawn graph, I followed up with the question "Why is the decreasing function not visible?" In response, some students were able to use critical thinking skills to determine that the motion detector was only able to see Turtle because the motion detector could not "pick up" Mickey due to Turtle blocking the "laser beam." Lastly, students determined that the point shown on the calculator where the increasing and constant functions meet is the precise point in which the crash occurred. This point indicates the time and distance from the motion detector at which Turtle and Mickey crash into each other. Throughout this process of critical thinking, analysis, and discussion, students were able to acquire conceptual understanding and greater meaning behind the functions forming the graphs as well as the solution to the system of linear equations.

Conclusion

With the help of CBR technology and push toys, students were able to model a situation involving the intersection of two objects in motion and collect real-time data. With such an activity, students were encouraged to connect graphing and solving a system of linear equations to a real-world situation. Hands-on activities, like this one, can make mathematical concepts come to life for students. Consequently, these activities may be able to promote students' ability to make connections between real-world actions and mathematical concepts that, potentially, build their conceptual understanding of the underlying mathematics. Moreover, as graphing and solving systems of linear equations appear as early as middle school mathematics curriculum all the way to post-secondary mathematics, mathematics teachers at all levels are encouraged to consider integrating activities that utilize CBR's and toys. Naturally, all teachers are recommended to use their professional discretion on the amount of guidance that is provided to help students gain conceptual understanding of the topic during the activity.

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