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Mathematical Identity and the Role of the Educator

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ABSTRACT Mathematical identity is a socio-motivational construct known to be a predictor of mathematical achievement. Students who identify positively with mathematics are more likely to pursue advanced courses and Science, Technology, Engineering, and Mathematics (STEM)-related occupations. Although mathematical identity is shaped by a myriad of internal and external factors on both a small and large scale, educators play a significant role in the formation of their students' mathematical identities. This paper presents an overview of research and theory regarding those pedagogical practices for teaching mathematics that can foster the formation of positive mathematical identities.

KEYWORDS *mathematical identity, mathematical learning, pedagogical practices*

Mathematical Identity and Mathematical Learning

Lauded by Gee (2000) as an analytic lens for research in education, mathematical identity is a burgeoning topic of study that has become increasingly prominent over the past two decades. Since its emergence on the scene during what has been coined by Lerman (2000) as the "social turn in mathematics education," an operational definition for mathematical identity has evolved to a "narrative rendering" of participative experiences that equates storytelling with identity-building (Sfard & Prusak, 2005). As a result, researchers have come to define mathematical identity as a socio-motivational construct that refers to the dispositions and deeply ingrained beliefs regarding one's ability to participate and perform effectively in mathematical contexts as a learner and user of mathematics (Bohrnstedt et al., 2020; Dingman et al., 2019; Martin, 2009).

Since identity-making is a "communicational practice" over which humans are active agents (Sfard & Prusak, 2005), social interactions are critical to the formation and perpetuation of mathematical identities. Every occasion for communication is an opportunity to (re)construct and (re)negotiate self-images through the discursive positioning of self and others (Davies & Harré, 2001; Waring, 2018). Although moments for mathematical identity formation are ubiquitous to daily life, of particular importance are those experiences that occur within the classroom setting (Anderson, 2007). In fact, teachers' roles in shaping their students' mathematical identities can be dramatic (Martin, 2009) because teachers communicate to their students what mathematics is, what learning mathematics entails, and who is considered a doer of mathematics. Indeed, extant research in sociocultural learning theory emphasizes the significant impact, both short term and long term, that teachers have in shaping identity, suggesting that learning and identity development are intrinsically intertwined (Martin, 2009). The National Council of Teachers of Mathematics (NCTM) believes mathematics educators can leverage students' identities to enhance mathematical learning (2014). As a result, in 2016 the NCTM renewed its focus on the ability and responsibility of teachers to foster positive student mathematical identity by reframing their principle of access and equity to "capture the additional and critical constructs of students' mathematical identities, students' sense of agency, and the teaching of mathematics for social justice" (Larson, 2018, para. 2).

Anderson (2007) describes learning mathematics as a "complex endeavor" consisting of three dimensions: (1) the development and application of skills, algorithms, and procedures; (2) the construction and acquisition of mathematics knowledge; and (3) the participation in social interactions that influence thoughts, actions, and membership within communities. Identity formation is a critical component of the third dimension wherein students "must participate within mathematical communities in such a way as to see themselves and be viewed by others as valuable members of those communities" (Anderson, 2007, p.8). Teachers are both architects and stewards of mathematical communities in the classroom, responsible for designing and supervising an environment that encourages students to view themselves and others as valued members and contributors.

Boaler (2002), likewise, views learning as a process composed of three components in which knowledge, classroom practice, and identity are interrelated in a disciplinary relationship. In her study with Advanced Placement Calculus students, Boaler (2002) found that classes with teacher-centered instruction positioned students as "received knowers," in which the role of the student was to passively receive mathematical knowledge. The teacher and the textbook were described as the mathematical authority, meaning that they were relied on as the holders of all mathematical knowledge (Boaler, 2002). Students classified as received knowers were more likely to dislike and disengage from mathematics, often seeking out academic opportunities in other disciplines that offered more interpretation and freedom to express their ideas (Boaler, 2002). Ultimately, the lack of classroom discourse left students with few opportunities to develop a positive mathematical identity. In contrast, students in discussion-centered classes formed a relationship with mathematics that gave them agency and authority over the learning process (Boaler, 2002). Rather than viewing mathematical learning as a mere reproduction of standard procedures, they understood their active role in the process and made plans to continue studying mathematics; notably, mathematical authority was conferred to the students whose opinions, ideas, and conjectures served as valuable contributions to the learning of mathematics (Boaler, 2002). When faced with challenging problems, those students performed a "dance of agency" in which they moved freely between established methods and their own creative process (Boaler, 2002). As a result, they not only mastered the content but, more importantly, appropriated the mathematical knowledge – a key component to the formation of positive mathematical identities. Fortunately, the positive results elicited by the discussion-centered pedagogical practices of the educators in Boaler's (2002) study can be replicated in any mathematics classroom through the adoption of similar practices.

Pedagogical Practices That Promote Positive Mathematical Identities

"Students do not just learn mathematics in school classrooms, they learn to be" (Boaler & Greeno, 2000, p.188). Teaching mathematics is more than just the dissemination of content and the development of mathematical skills. It is also about empowering students to see themselves as participants and doers of mathematics, facilitating motivation and interest through opportunities of engagement and discourse in the classroom, and helping students to understand the value and relevance of mathematics in their own lives (Miller & Wang, 2019; Wang, 2012). Since the role of the teacher is critical to the formation of mathematical identities, educators should regularly engage in practices that foster positive associations with mathematics. Several of these pedagogical practices are described below.

Fostering the Four Components of Identity

According to Anderson (2007), there are four components of identity associated with mathematical learning that a teacher can enforce through intentionally designed mathematical tasks: engagement, imagination, alignment, and nature. He argues that students engage in mathematics when given the opportunity to explore and develop their own problem solving strategies. Through this experience, they not only connect with their classroom community but are also recognized as members and contributors to the greater mathematical community. Anderson (2007) describes imagination as the way in which students envision those engaging mathematical activities as fitting into their daily lives, as well as their future educational pursuits and careers. Anderson (2007) believes that when students respond to the imagine component of identity, they align their preconceived notions to conform with institutional expectations and behave in ways that align with a specific type of person (i.e., a "mathematics person"). Finally, Anderson (2007) describes the nature component of identity as being directly related to fixed mindset beliefs and the existence of a mathematics gene. To aid in the development of positive mathematical identities for all students, Anderson (2007) argues that teachers should discount the nature component, which can often lead to unfounded explanations for student participation or success in the mathematical community. Several of Anderson's (2007) recommendations for educators to foster positive mathematical identities are described in Table 1.

Table 1

Methods for Fostering Positive Mathematical Identities

Anderson's (2007) Recommendation	General Strategy	Specific Examples
Use meaningful tasks	Incorporate open-ended questions or projects that allow students to develop strategies, make use of mathematical tools, and focus on explanations over the brevity of responses.	A swimming pool project in which students explore concepts of surface area and volume through the design of a swimming pool according to particular standards or constraints. An exercise in concept attainment that asks students to look at examples of polygons and non-polygons to generalize a rule for what constitutes a polygon (Brahier, 2016).
Establish classroom norms centered on discourse	Incorporate classroom expectations that encourage collaboration wherein the teacher comprises the role of facilitator.	Questions can be elicited from students by providing them with "Student Question Cards" that model various types of questioning techniques, such as "Why do you do that?" and "How does this compare to …?" (Brahier, 2016). Teachers can demonstrate ways for students to listen with intention by creating a poster that describes specific actions students can take to (1) pay close attention, (2) show that they are listening, (3) provide feedback, and (4) respond appropriately (Hattie et al., 2016). Teachers can encourage student contributions through the utilization of checklists that prompt students to consider ways that they can contribute to the learning process, such as "Have you considered looking for another way to solve the task?" (Hattie et al., 2016).
Provide occasions for students to reflect upon their mathematical journey	Incorporate success criteria, journal prompts, or concept maps.	At the culmination of a lesson, teachers can ask students to respond to "I can…" statements, such as "I can explain why a zero exponent produces a value of one," to make their learning, and the progress they've made, more visible (Hattie et al., 2016). At the end of an assessment, teachers can ask students to write in their journals in response to the prompt, "How do you think you performed on today's test? What was the easiest part? What was the hardest part? Would you make any adjustments to how you prepared for the test? Why?" (Brahier, 2016). In a Discrete Mathematics course, students can create a concept map for proof-writing strategies based on the logical form of the conclusion.

Table 1 (Continued)

Methods for Fostering Positive Mathematical Identities

Anderson's (2007) Recommendation	General Strategy	Specific Examples
Keep students abreast of the role that mathematics plays in their success as a student and future employee	Consistently remind students that mathematics is integral to college matriculation and their careers.	Create a reference sheet for, or display on a bulletin board, the progression of mathematics courses in their school with a particular emphasis on those courses that are required for entrance into college. Invite professionals from outside the school to discuss ways in which they use mathematics in their professional lives (Anderson, 2007). Include applications of mathematics that are relevant to the students or their prospective careers, such as using graph theory for the scheduling of final exams or modeling a mock election using different voting systems.
Make students aware of high expectations	Enforce the belief that all students can learn mathematics.	Maintain expectations that all students will continue their study of mathematics in every year of high school and beyond (Anderson, 2007). Consistently communicate growth mindset beliefs and practices that emphasize effort attributions, learning goals, and a mastery-oriented response to failure (Dweck, 2000).

Fostering Identity Through Effective Mathematics Teaching Practices

According to Kebreab et al. (2021), a positive mathematical identity can be fostered in the classroom in accordance with the NCTM' *Effective Mathematics Teaching Practices* (MTPs) (2014). Kebreab et al. (2021) based their concept of identity on Gee's (2000) four perspectives: natural, institutional, discursive, and affinity. Gee (2000), like Anderson (2007), describes the nature perspective of identity as a view developed from natural forces beyond one's control, such as genetics. The institution perspective of identity is how one derives their identity from authorities within institutions, such as how students derive their identity as mathematicians from their teachers. The discourse perspective is described as a social construct recognized through dialogue, such as through social interactions in the classroom that influence views regarding who is a learner and user of mathematics. Finally, the affinity perspective is derived from shared experiences with "affinity groups," such as the members of a mathematics class or group (Kebreab et al., 2021).

Kebreab et al. (2021) argue that natural identities can be developed when teachers support students in productive struggle (MTP 7). In particular, when educators afford opportunities for students to persevere in problem solving, it communicates confidence in students' natural ability to overcome challenges and reaffirms their agency over mathematical learning. Kebreab et al. (2021) suggest students' institutional identities can be developed through the implementation of tasks that promote reasoning and problem solving (MTP 2), as well as using and making connections between mathematical representations (MTP 3). For instance, educators can position students as capable knowers and doers of mathematics when using guiding questions such as "How would you describe this?" or "Where would be a good place to start?" Such questions should be used in conjunction with rich mathematical tasks, which include nonroutine problems that can be solved using a variety of methods (Brahier, 2016). This serves to shift the mathematical authority to the student which, therefore, encourages them to make their own connections and justify their thinking. Kebreab et al. (2021) posit that the discursive identity can be developed by posing purposeful questions (MTP 5) and building procedural fluency from conceptual understanding (MTP 6). For example, when educators question or prompt students they are sending the message that student thinking is valued which may encourage them to further engage with their peers and teachers. Finally, when educators establish clear mathematics goals to focus learning (MTP 1), Kebreab et al. (2021) postulate that students reestablish their roles and responsibilities as affinity members of the classroom mathematical community. Thus, the affinity perspective of identity can be fostered by the teacher via frequent redirection towards the learning goals and classroom expectations.

Fostering Positive Mathematical Identities Through Discussion-Centered Pedagogical Practices

According to Boaler and Selling (2017), "the idea that students develop different mathematical identities in mathematics classrooms that include beliefs about oneself, ideas about mathematics, and an eagerness to engage actively with mathematics draws from a situated perspective that attends to forms of engagement" (p. 83). Boaler and Selling (2017) delineate two forms of engagement: active and passive. Active engagement in the mathematics classroom occurs when students are engaged in problem solving, discourse, and the application of mathematical methods. In contrast, passive engagement takes place when students are positioned as "received knowers" in which their role in problem solving is to listen to the authority of the teacher and reproduce the teacher's methods (Boaler & Selling, 2017).

Educators that employ discussion-centered pedagogical styles can consistently offer opportunities for active engagement that encourage students to have agency over validating mathematics methods, generating questions, and developing ideas. For instance, consider placing students in cooperative learning groups and presenting them with a challenging mathematical task situated in the real world. As the students work, encourage them to make conjectures, justify their reasoning, and critique the approaches and methods of their peers (Seeley, 2004). Additionally, when instructional strategies are employed that shift the mathematical authority to the students, educators can enhance the autonomy of their students by welcoming questions, acknowledging frustrations, and encouraging independent investigations (Skilling, 2014). This can be done through the establishment of classroom discussion norms that create a mathematical community which values inchoate ideas and making mistakes. Educators may also consider utilizing flipped or blended classroom approaches to afford more opportunities for inquiry-based, constructivist learning. When the classroom structure is reversed, students can take advantage of classroom time to build conceptual understanding and engage in activities that lead them to discover that mathematics is a process and not a "universal truth handed down by some disembodied, non-human force" (Becker, 1995, p.168). Ultimately, when educators adopt these practices they "share the process of mathematical problem solving with students … making mathematics more equitably accessible, and also encouraging larger numbers of students to explore mathematics as a career" (Boaler & Greeno, 2000, p.189). Unsurprisingly, positive mathematical identities flourish in these types of learning environments because students are given agency over the mathematical process, are encouraged to view themselves as mathematicians, and experience first-hand the value of mathematics in their lives (Boaler, 2002; Boaler & Greeno, 2000).

Conclusion

The significance of the relationship between mathematical identity and mathematical learning is underscored by its increasing prominence in research. Mathematical identity is considered an indicator of academic performance, persistence, and success (Bohrnstedt et al., 2020; Cribbs et al., 2015; Marsh et al., 1988). Additionally, forming positive mathematical identities can empower students, especially those marginalized by race, gender, class, or ethnicity (Larson, 2016). Moreover, those students that positively identify with mathematics are more likely to pursue future mathematics courses as well as STEM-related occupations (Cribbs et al., 2015; Watt et al., 2017). According to the National Science Foundation (NSF), perpetual innovations in STEM fields are critical for maintaining a competitive edge in the increasingly knowledge-based, technological, and global economy in which we live (2007). The NSF (2007) contends that increasing the STEM literacy of citizens is crucial to ensuring their full and active participation in sustaining a high quality of life.

This paper presented an overview of research and theory regarding pedagogical practices for teaching mathematics that can foster the formation of positive mathematical identities. To begin this work it is important that we, as educators, first self-assess and reflect upon our own beliefs that are both intentionally and unintentionally communicated to students. As stewards of the classroom mathematical community, in what ways are we communicating ideations of who is a learner and doer of mathematics? Likewise, it is important that we analyze our classroom norms and expectations in consideration of the message they send to students regarding the nature of mathematical learning. Are we presenting mathematics in such a way so as to foster intuition, creativity, and connectedness (Burton, 1998, 1999)? The 70 research mathematicians in Burton's (1998, 1999) study identified these three notions as consistent with the discipline of mathematics itself. Further, they described feelings of euphoria in mathematical exploration and a distinct need for collaboration with others (Burton, 1998, 1999). As teachers evaluate their own pedagogical practices, they should be asking themselves: Are we eliciting positive feelings in the classroom? To present mathematics in any other way would be to disenfranchise students from the joy of identifying with the subject and, therefore, developing a positive mathematical identity.

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