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Pre-Service Teachers' Discourse Moves During Whole Class Mathematical Discussions: An Analysis and Proposed Framework

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ABSTRACT Learning to facilitate whole-class mathematical discussions is a complex task. Teachers must become adept at using discourse moves to elicit and extend student thinking and move discussions towards a mathematical point. Practice-based mathematics methods courses give pre-service teachers opportunities to rehearse these moves in a controlled setting. Providing opportunities for practice, often called rehearsals or approximations of practice, have been shown to be effective at developing pre-service teachers' abilities to use discourse moves, but we still do not have a clear understanding of the types of discourse moves pre-service teachers use during discussions or how these moves change over the course of a discussion. In this article, we analyze. whole-class discussions facilitated by pre-service teachers to determine the types of discourse moves used as the discussion progresses. Based on this data and relevant literature, we propose a framework for the structure of whole-class mathematical discussions.

KEYWORDS *discourse moves, whole-class discussion, pedagogies of practice, teacher education*

Introduction

Facilitating mathematical discourse among students is a key component of reform mathematics teaching (NCTM, 2014). The type and quality of classroom discourse can be associated with differences in student affect and self-regulation (Turner et al., 2003), as well as differences in learning (O'Connor et al., 2015). Developing effective discourse within a learning community takes time and requires attention to posing purposeful questions, developing students' abilities to explain their mathematical ideas, balancing classroom power dynamics, and sharing responsibility for learning among the teacher and students (Hufferd-Ackles et al., 2004). Teachers play an active role in facilitating productive mathematical discussions. They employ discourse moves, which Krussel et al. (2004) define as, "the deliberate actions taken by a teacher to mediate, participate, or influence discourse in mathematics education" (p. 307). These moves elicit student thinking, orient students to each other's ideas, and move the discussion towards a mathematical point. For instance, a teacher might ask the class to explain whether or not they agree with a student's reasoning and provide support for or against the student's argument, or they might restate a student's idea to ensure it is understood.

While there are many resources to support teachers in leading mathematical discussions (e.g., Lamberg, 2019; Smith & Stein, 2011), it is challenging for pre-service teachers (PSTs) to develop the skills necessary to lead rich, productive discussions. When facilitating mathematical discourse, PSTs struggle with anticipating student thinking (Yilmaz & Yetkin-Ozdemir, 2021), knowing when and what information to give students (Selling & Baldinger, 2016), and steering discussions to a mathematical point (Baldinger et al., 2016).

Practice-based teacher education models have the potential to address the difficulties many PSTs have integrating the instructional moves they are learning about into their teaching (Grossman et al., 2009;

Lampert, 2010). In a practice-based curriculum, PSTs rehearse and enact core teaching practices as part of their teacher education courses and fieldwork (Grossman et al., 2009). Approximations of practice play an important role in supporting PST's learning to facilitate meaningful discourse in practice-based mathematics methods courses (Baldinger et al., 2021; Ghousseini, 2009). These activities help PSTs deepen their understanding of discourse moves, recognize the importance of anticipating student thinking, realize the need to steer discussions to a mathematical point, and improve their overall ability to facilitate student-centered dialogue (Baldinger et al., 2016; David, 2020; Freeburn, 2015; Spangle & Hallman-Thrasher, 2014).

However, we do not yet have a clear picture of how PSTs in a practice-based course facilitate whole class discussions. What discourse moves do PSTs use, and how do the moves change over the course of the discussion? In this paper, we examine the types of discourse moves pre-service teachers implement while facilitating student-centered, whole class mathematical discussions, and we draw from our data and from relevant literature to propose a framework for structuring whole-class mathematical discussions.

Background

Discourse Moves

While teachers can enter discussions with a mathematical point in mind and a general plan for how the discussion might play out, specific discourse moves cannot be scripted in advance. In the midst of the classroom discussion, teachers must interpret the situation and draw on these routines used for specific purposes, such as eliciting student contributions, responding to student thinking, and ensuring the discussion stays on track. A number of researchers have identified various moves used by teachers to facilitate discourse (e.g., Chapin et al., 2003; Ellis et al., 2018; Franke et al., 2015; Herbel-Eisnmann et al., 2016; Steele, & Cirillo, 2013; Staples & Colonis, 2007). Ghousseini (2008) describes five types of discourse routines: *revoicing*, orienting students to each other's thinking, pressing students for explanations, *connecting* students' ideas, and modeling or pointing to specific aspects of discourse. O'Connor et al. (2015) found evidence that students learn more in classrooms where teachers use moves that encourage students to share their thinking and engage with their peers' ideas.

Through approximations of practice, PSTs can increase their ability to use discourse moves effectively (Baldinger et al. 2021; David, 2020; Spangle & Hallman-Thrasher, 2014). For instance, Freeburn (2015) found that PSTs in a practice-based mathematics methods course developed the ability to pose advancing questions that prompted mathematical connections. Also, David (2020) found that PSTs increased their skill in facilitating mathematical discussions over the course of a practice-based methods course, becoming more adept at using talk moves intentionally to support and extend student thinking.

Structuring Mathematical Discussions

As teachers use discourse moves to promote math talk, they need support in knowing how to structure whole class discussions (Walker, 2014). They must navigate the tension between valuing student ideas as the basis for mathematical discussions while ensuring the discussions are mathematically productive (Sherin, 2002). Multiple frameworks exist to support teachers in structuring discussions that lead to a mathematical point. Smith and Stein (2011) have outlined five practices for orchestrating productive mathematical discussions. In these practices, teachers anticipate student thinking prior to the discussion, actively monitor students while they work, select students to present their ideas, sequence the presentations in strategic order, and draw connections among ideas. In another framework, Lambert (2019) breaks down mathematical discussions into three "levels of sense making." The first level, making thinking explicit, is where students communicate their answers. The goal of this level is for students to understand each other's thinking. The second level, analyze each other's solutions, engages the whole class in critically evaluating the ideas presented, and finally, the third level, develop new mathematical insights, is where the teacher scaffolds students' thinking toward a "big idea."

While these frameworks are helpful in breaking down discussions into manageable parts, we know little about how PSTs discussions are structured in practice. Some studies have examined aspects of PST planning for whole class discussions. Tyminski et al. (2014) examined how PSTs using Smith and Stein's (2011) five practices framework planned discussions. They found that PSTs were able to identify mathematical goals for their discussions and plan the discussion in ways that could support those goals. Meikle (2016) looked at the rationale behind PSTs' selecting and sequencing strategies.

In terms of examining how PSTs structure their discussions in practice, Conner, Singletary, Smith, Wagner, and Francisco (2014) analyzed the ways two student teachers supported collective argumentation in their student teaching placements. They identified three types of support the student teachers used to promote argumentation: direct contributions to the argument, posing questions, and using other supportive actions. Still, we do not have a clear picture of how the pieces of a discussion practically fit together from start to finish.

Therefore, the purpose of this study is to identify the discourse moves PSTs use over the course of a whole class discussion and then build a new framework for discussions informed by this data and previous discussion models.

Methods

The study was conducted in a semester-long mathematics methods course required for secondary education PSTs. The ten PSTs in the class were all pursuing an initial teaching license. The class met once a week for three hours for 15 weeks. No compensation was provided to the participants in the study. The second author was the course instructor. The first author was a student in the course; however, the first author did not join the research team in any way until after the course was complete.

Methods Course Instruction

One of the primary goals of this practice-based course was for the PSTs to develop their ability to plan and facilitate mathematical discussions. The students had been introduced to discourse moves and structuring student-centered discussions in a previous course, but the purpose of this course was to help students bring these skills together to facilitate high quality discussions. The instructor used the teacher learning cycle (McDonald et al., 2013) to facilitate this learning. This cycle has four phases (introduce, prepare, enact, and analyze) that help PSTs learn to enact core instructional practices. To provide context for this study, we describe all four phases of instruction. The data for our research was drawn from the mathematical discussions that PSTs facilitated during the *enact* phase.

In the *introduce* phase, the instructor modeled three different examples of mathematical discussions. The class analyzed these discussions. They also read about a range of discourse moves and about Smith and Stein's (2011) five practices. The instructor shared ideas for structuring whole class discussions drawn from Smith and Stein (2011) and Lamberg (2019), and the class engaged in an approximation of practice in which they were given student work and had to plan how they would facilitate a discussion centered on this work to highlight a given mathematical concept. Groups of students shared their plans, the class discussed them, and the teacher educator provided targeted feedback.

In the *preparation* phase of the learning cycle, the PSTs worked with a partner to design a problem-based lesson to teach to their classmates. Each lesson was centered on a single mathematical task and was composed of a launch, explore, and summarize portion. The PSTs did not teach the entire lesson at once. Instead, it was divided into sections to allow the PSTs to focus on each part individually. First, the pairs launched their lessons in class. During the launch the PSTs made sure their peers understood the problem the lesson was centered on. Following the launch, each PST completed the problems from each of their peers' lessons on their own for homework. The next class, each teaching pair reviewed all of their classmates' work and planned their whole class discussion, selecting and sequencing the strategies they wanted to highlight (Smith & Stein, 2011). The teacher educator provided feedback during this process.

In the *enact* phase, during the following class, each pair of PSTs led a mathematical discussion in which they selected and sequenced three different student (i.e. PSTs assuming the role of student) presentations of their work. Each discussion was video recorded and lasted about 25 minutes. Finally, in the *analyze* phase, the PSTs watched, analyzed, and reflected on the recordings of their discussions.

Data Collection and Analysis

The data for this study consisted of the five discussions from the enact phase. The video recordings of these discussions were transcribed for the analysis.

In order to determine the discourse moves PSTs use to facilitate mathematics discussions, we conducted a qualitative content analysis of the transcripts of the discussions. A qualitative content analysis uses a qualitative approach to assign codes to data, followed by a quantitative analysis of the frequencies of those codes (Mayring, 2015). We began the analysis by closely reading the transcripts multiple times to become familiar with the content to be analyzed. Then, we used a combination of deductive and inductive codes. We then created an initial list of 16 codes that described the discourse moves the PST utilized. Next, we combined these codes into groups of similar actions. This resulted in six categories of discourse moves described below.

Next, we found the frequency of each discourse move for each discussion. In order to determine if the types of discourse moves varied over the course of the discussion, we divided each discussion into three parts (Part 1, Part 2, and Part 3). The beginning of each part was marked by a new student presenter sharing their work with the class. For example, Part 1 of a discussion consisted of the first student's presentation of their solution method as well as all the discussion that followed up until the second student presenter began to share.

We then calculated the frequency of each discourse move for each part of each discussion. Then we considered how these frequencies aligned with components of discussions found in existing literature. We used these alignments to develop a framework for showing the ways discourse moves support whole class discussions. Last, we examined how closely our framework aligned with the five discussions and made final adjustments.

Categories of Discourse Moves in the PSTs' Discussions

Clarifying. This discourse move refers to questions posed by the PST to clarify student statements, as well as questions posed by the PST to prompt students to seek clarification. For example, to clarify a student's idea, one PST posed the question, "Like this or do you want me to add the ones in the middle?" By presenting this question to the student, the PST is asking the student to clarify which numbers to add in a problem to ensure the student's idea is properly represented in the discussion.

Revoicing. We categorized a discourse move as *revoicing* if the PST repeated or rephrased a student's idea or if they asked another student to repeat or rephrase the idea. Revoicing can be used to give the class more processing time and ensure all students understand an idea (Chapin et al., 2003). For instance, a PST asked the class, "Could I have someone..um..kind of restate what she said?" Another time the PST revoiced a student saying, "So she's asking, 'How do I use this to find the midpoint?'"

Probing Questions. We categorized moves as *probing questions* if the PST posed a question to assess or advance student thinking. Assessing questions are used by teachers to determine what students understand, while advancing questions aim to extend student thinking (Freeburn & Arbaugh, 2017; Smith et al., 2008). Both types of questions probe student thinking. For example, one PST asked, "Why did you divide by one fourth?" Another advanced student thinking by asking, "Can we [find the distance] diagonally without a measuring tool?"

Comparing and Evaluating. PSTs used *comparing and evaluating* moves when they asked students for, or provided, a comparison or an evaluation of student work. For instance, one PST prompted a turn and talk saying, "So let's take a minute and have everybody, in groups of two, just kind of discuss the similarities and differences between [this] method and [that] method." The same PST later posed the question, "So to kind of go off of that, which one would you guys say is more efficient?"

Initiating. The PST engaged in *initiating* by introducing a new mathematical idea into the discussion. For example, one PST described the idea of using a visual diagram to help with a fraction division problem saying, "We can use a visual diagram to split it up into fourths." While some teachers attempting to facilitate inquiry-based learning try to refrain from telling students anything, Lobato et al. (2005) argue that teachers can strategically insert new ideas into discussions to stimulate student thinking.

Connecting. A discourse move was coded as *connecting* if the PST prompted students to make a connection among two or more mathematical ideas or if they explicitly stated a connection. For instance, in one discussion a PST asked, "So how does that connect to the $3(4^n)$ that you have below?"

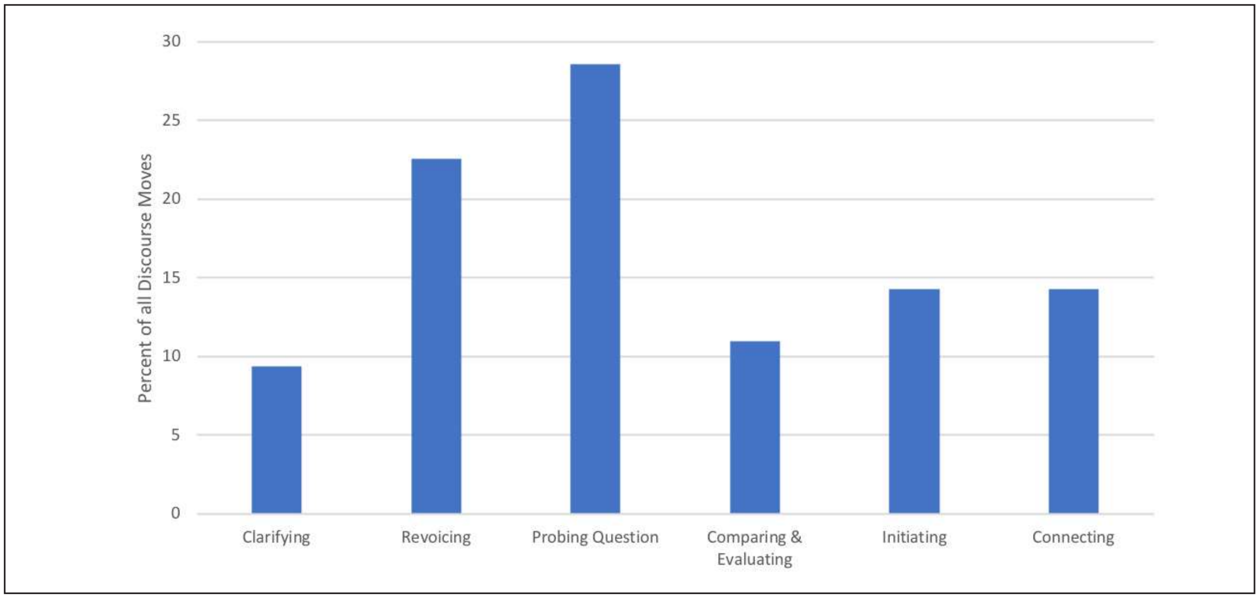
Results

Discourse Moves Used Overall

On average, each teaching pair of PSTs used 36.4 discourse moves over the course of their discussion (SD = 12.6). They used some types of moves more frequently than others. Figure 1 shows the percentage of discourse moves by category across all discussions. The most common discourse moves were *probing questions* (28.6%) and *revoicing* (22.5%), and the least frequent moves were *clarifying* (9.3%) and *comparing and evaluating* (11.0%).

Figure 1

Percentage of discourse moves by category



How Discourse Moves Changed Over Time

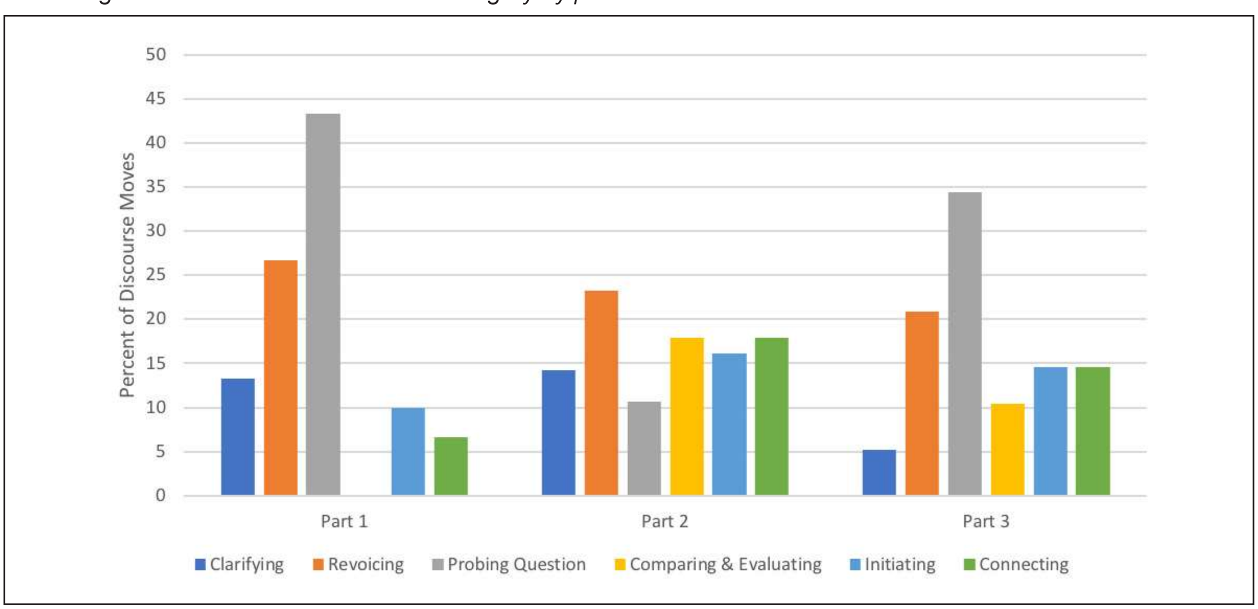
The way PSTs used discourse moves varied over the course of the discussion. In terms of number of moves, the PSTs used more discourse moves as the discussion progressed. They used an average of 6.0 discourse moves during Part 1 (SD = 2.1), 11.2 moves during Part 2 (SD = 7.6), and 19.2 moves during Part 3 (SD = 7.9). In other words, on average just over 50% of the discourse moves took place in Part 3. This demonstrates how the discussion builds as it progresses and how the PSTs did the most "work" in terms of discourse moves during Part 3.

In order to compare the types of discourse moves PSTs used in each part of the discussion, we computed the percentage of each category of discourse moves for each part of the discussion (see Figure 2). For instance, the first column on the left shows that *clarifying* moves made up 13.3% of all discourse moves in Part 1.

There are three results from Figure 2 that we would

Figure 2

Percentage of discourse moves for each category by part of discussion



like to highlight. First, when looking at the results for *comparing and evaluating*, we see that this move did not emerge until Part 2 where it accounted for 17.9% of the discourse moves used. Compare and evaluate was also used in Part 3, making up 10.4% of the discourse moves. Second, all of the other discourse moves, including *connecting* moves, were used throughout the three parts of the discussion. Finally, looking closely at *initiating* and *connecting* moves, we see that they make up similar proportions of the discussion as each other in each of the three parts of the discussion. For example, *initiating* moves make up 16.1% of the moves in Part 2, while *connecting* moves account for 17.9%.

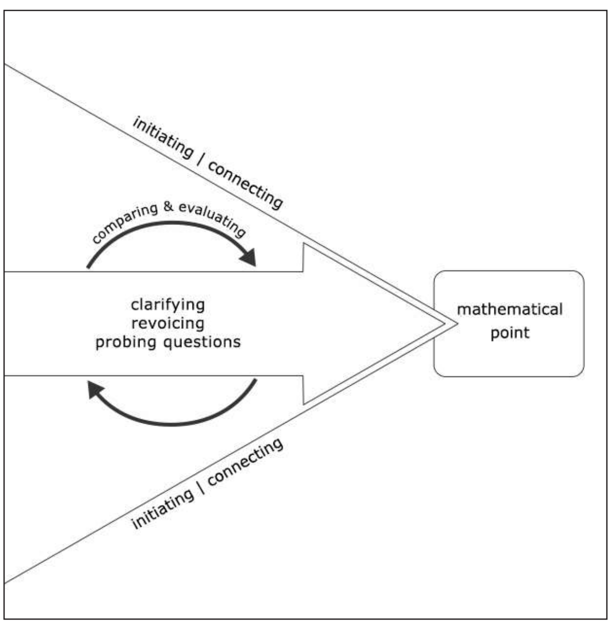
Discussion Framework

Examining how PSTs' use of discourse moves aligned with components of mathematical discussions identified in the literature has led us to propose a framework characterizing the way discourse moves support whole-class, inquiry-oriented discussions (see Figure 3).

Before the discussion begins, the teacher strategically selects and sequences student presentations in an order that will move the conversation toward the mathematical point of the lesson. In our study, each teaching pair selected three different students to share their strategies. However, this number may vary depending on

Figure 3

Framework for a Whole-Class, Inquiry-Oriented Discussion



the discussion. The PSTs elicited and advanced student thinking throughout each part of the discussion, using three discourse moves: *clarifying*, *revoicing*, and *probing questions*. These moves helped make student thinking explicit by ensuring that the teacher and the rest of the class understood what the student was saying, and they advanced student thinking by pressing them to think more deeply about their ideas. Taken together, these discourse moves are an example of how teachers can engage students in responsive listening (Empson & Jacobs, 2008) to support and extend student thinking. We represent them in our framework in Figure 3 by placing them inside an arrow to show how *clarifying*, *revoicing*, and *probing questions* serve to move the conversation forward.

Beginning in Part 2 of the discussion, we see PSTs prompt the class to compare and evaluate different ideas. This is not a separate, distinct phase of the discussion. Instead, *comparing and evaluating* moves are interwoven throughout Parts 2 and 3 as the teacher circles back to pull together strategies previously discussed with those currently being discussed. In Figure 3, this is represented by the circular arrows. These moves orient students toward others' thinking and prompts them to think analytically.

Finally, throughout the discussion, the teacher uses *initiating* and *connecting* to steer the discussion toward the intended mathematical point. Lobato et al. (2005) hypothesize that *initiating* moves can help students focus on important mathematical ideas. When used in the context of eliciting student thinking, *initiating* can play an important role in stimulating new ideas and helping students make sense of others. The PSTs in our study interwove *initiating* moves through all three parts of the discussion to help focus the discussion and move toward the mathematical point. Similarly, the PSTs used *connecting* moves in all three parts of the discussion. *Connecting* moves help students recognize relationships between mathematical ideas, often prompting the development of conceptual understanding (Gil, Zamudio-Orozco, & King, 2019). By intentionally and strategically *initiating* and *connecting*, teachers can "filter" (Sherin, 2002) through student ideas, sifting out those to focus on and use to build towards the mathematical point. This is represented in Figure 3 by two lines converging on the mathematical point showing how *initiating* and *connecting* serve to focus the conversation toward a learning goal.

Discussion

In summary, we found that PSTs used six different types of discourse moves to facilitate discussions: *clarifying*, *revoicing*, *probing questions*, *comparing and evaluating*, *initiating*, and *connecting*. The frequency of these moves varied over the course of the discussion, and the PSTs used more discourse moves as the discussion progressed.

We used this data to develop a framework for structuring discussions in practice. This framework has similarities to previous discussion models. First, our framework aligns with and builds on the work of Smith and Stein (2011). In preparation for a whole-class discussion, teachers engage in the first four of Smith and Stein's five practices—anticipating, monitoring, selecting, and sequencing. The framework proposed in this work focuses on Smith and Stein's fifth practice, *connecting*. By examining the primary discourse moves used during this phase and developing the framework to show how they work together, our work offers additional details on how teachers can structure the final *connecting* phase of the five practices.

Additionally, we see similarities between our framework and that of Lamberg (2019). The moves *clarifying*, *revoicing*, and *probing questions* align with the "making student thinking explicit" phase, *comparing and evaluating* has similarities to "analyze each other's solutions," and *initiating* and *connecting* is comparable to "develop new mathematical insights." However, while Lamberg (2019) describes the three levels of sense making primarily as distinct, sequential phases, our data shows that the PSTs interweave discourse moves with various purposes throughout the discussion. In this regard, our findings are more similar to that of Sherin (2002) who analyzed the discussions of a middle school teacher over the course of a school year and found that while the teacher included three structures similar to Lamberg (2019), the teacher progressed through the structures in a fluid, often cyclical manner.

While the PSTs in our study used a range of productive discourse moves, we did not see instances of teachers *modeling* or *pointing to specific aspects of discourse.* Ghousseini (2008) describes this meta-level discourse move as the teacher stepping outside of the conversation and commenting on it for the purpose of developing students' skills for participating in discussions. We hypothesize that students in our study did not do this for one of two reasons. First, it may be that this discourse move develops later after PSTs become more familiar with other discourse moves and facilitating whole class discussions. Second, it may be that the particular approximation of practice the PSTs were engaged in (i.e. teaching their peers in a methods class) removed the need for them to engage in this move because the PSTs acting in the role of students already had strong discussion skills. Given that Sherin (2002) describes the challenge of teachers navigating the tension between supporting the discussion process and ensuring students learn the content, it may be beneficial for PSTs to engage in approximations of practice similar to the one in our study prior to attempting to facilitate whole class discussions in an actual classroom.

We hope that the framework presented here will be helpful to both teacher educators and PSTs teaching and learning how to facilitate productive discussions. The work of leading discussions is complex, and by decomposing this practice, we hope to demystify the process. As we move forward with our research, we plan to examine how teacher educators can use this framework as a teaching tool to help PSTs develop their abilities to facilitate whole class mathematical discussions.

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